Aust. J. Phys., 1988, **41**, 729–33

The Knee of the Cosmic Ray Energy Spectrum

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Abstract

An examination is made of possible primary cosmic ray energy spectra which are consistent with observed air shower size spectra. At the size spectrum knee, it is necessary to have a very sharp primary spectrum break in order to be consistent with both the size spectrum and also air shower fluctuations. Such a break may be due to energy loss at the cosmic ray source.

1. Introduction

The measured spectra of cosmic rays from 10^{14} to 10^{20} eV are remarkably featureless. The only prominent features are found between 10^{15} and 10^{16} eV where there is a steepening of the power law energy spectrum and at about 10^{19} eV where there is a flattening (Wolfendale 1984). The steepening feature is often referred to as the knee and is associated with a number of controversial and apparently contradictory measurements of associated cosmic ray parameters.

At energies of about 10^{15} eV, the most readily measurable and numerous component of cosmic ray showers is the electromagnetic cascade which is measured usually as a shower size (N_e) approximating the total number of electrons at the detecting array. Below $\sim 10^5$ particles (at sea-level), the rate of events above a particular shower size and at a particular zenith angle as a function of shower size, the integral size spectrum, is well approximated by a power law (Fomin *et al.* 1987):

$$I(>N_e) = kN_e^{-\gamma} \qquad (\gamma \sim 1\cdot 5).$$

At sea-level sizes above 10^6 particles, a power law is also a good fit but the index γ is now greater by $\sim 0\cdot 5$–$0\cdot 6$ with an apparently sharp change at a vertical sea-level size of $\sim 6\times 10^5$ particles (see e.g. Vernov *et al.* 1968). Roughly speaking, the energy of the primary particle is about 10^{10} times the vertical sea-level shower size in this energy range although fluctuations in shower development result in significant deviations for individual showers. Thus, the power law size spectrum probably results from a power law energy spectrum.

The origin of this break or 'knee' is unknown but suggestions have been made that it may be associated with a loss of containment of primary particles in the galactic magnetic field (Peters 1961) or perhaps the upper energy limit to a component of cosmic rays accelerated by pulsars (Karakula *et al.* 1974). It is usually assumed that the size spectrum break corresponds to a similar power law break (of $0\cdot 5$ to $0\cdot 6$)

0004-9506/88/050729$03.00

in the primary energy spectrum. It is certainly true that for featureless spectra the indices of the size and energy spectra are the same. This will not be so if there are features in the spectra.

2. A Model Relating the Primary Proton Energy Spectrum and Observed Size Spectrum

Cosmic ray air showers are complex statistical cascades in which the energy of a primary particle is progressively degraded to produce mainly (in numerical terms) secondary electromagnetic particles. The overall dependence of the number of electrons in the superimposed electromagnetic cascades on atmospheric depth takes the form of a progressive increase to reach a peak, shower maximum, followed by an almost exponential attenuation (Greisen 1956; Clay and Gerhardy 1982) (asymptotically) with an attenuation length of $\sim 185\ \mathrm{g\,cm^{-2}}$. Mean shower maximum occurs at about half-way through the atmosphere for showers in the energy range of interest here.

It is important to recognise (a) that for a given primary particle energy, the actual depth of shower maximum is largely influenced by the earliest interactions of the primary particle (Lagutin *et al.* 1987), (b) that at shower maximum there is almost always a simple proportionality between shower size and primary energy (Wolfendale 1984), and (c) that because of fluctuations in the depth of maximum between showers of similar primary energy (Clay 1985) there will be large fluctuations in shower size (for most arrays which are at altitudes well below typical shower maxima) for a given primary energy (Baltrusaitis *et al.* 1984). The composition of the primary cosmic ray beam is uncertain but there is general agreement that at 10^{15} to 10^{16} eV there is a significant proton component (Clay 1985). This is important since the protons are expected to have the longest interaction mean-free-path and hence the greatest fluctuations in shower size at a given primary energy (Clay 1985). With a steeply falling energy spectrum, the result is that the proportion of proton initiated showers at a fixed shower size in any data set will be much higher than the proportion of protons in the primary beam at fixed energy. We can therefore reasonably expect our observations to be dominated by proton initiated showers (Clay 1985). This is the basis for recent measurements of the proton–air cross section at these energies (Baltrusaitis *et al.* 1984). Thus, in considering the knee of the size spectrum, we need to consider primarily a knee in the size spectrum of proton initiated showers.

At sizes below the knee, there is an extended size region with a power law spectral index of $\sim 1\cdot 5$. If there is an extended range of sizes with a fixed spectral index, the size spectrum is a good measure of the primary energy spectral index. It is of interest to examine the expected size spectrum resulting from a proton primary energy spectrum containing a break, i.e. with a change to a steeper differential power law index at a certain energy.

Such a Monte Carlo model has been investigated assuming a proton depth of maximum distribution given by Ellsworth *et al.* (1982). It was assumed that, at shower maximum, shower size was proportional to primary particle energy and that observations were made well past shower maximum so that an attenuation length of $185\ \mathrm{g\,cm^{-2}}$ could be assumed to describe shower development near ground level. Showers were selected from an energy spectrum with a differential index of $-2\cdot 5$ truncated at about 10^{16} eV, allowed to fall near the array and checked to determine whether they satisfied the array detection criteria (based on the Adelaide University Buckland Park array). If so, they were analysed to determine an observed integral size spectrum. These size spectra all showed a knee followed by a good approximation to a power law for at least a decade above the knee. An example is shown in Fig. 1. Results from this model are shown in Table 1 with the power law index above the primary energy of the knee used as a variable.

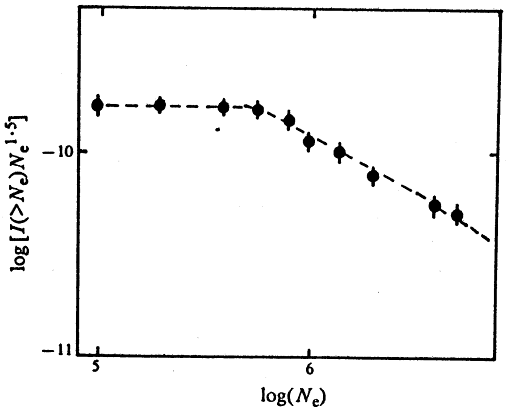

Fig. 1. Integral size spectrum resulting from a primary proton power law energy spectrum of differential index 2·5 truncated at about 10^{16} eV. Experimental data from Vernov *et al.* (1968) are included with appropriate intensity normalisation.

Table 1. Steepening of the observed shower size spectrum for various increases in the power law index of the primary proton spectrum

Primary spectral index increase	Resulting shower size spectral index increase
0·5	0·23±0·03
1·0	0·36±0·03
1·5	0·53±0·06
∞	0·53±0·04

3. Discussion of the Model Results

It can be seen from Table 1 that, in order to obtain an observed size spectral index increase of ~0·5–0·6, it is necessary to have a proton primary energy spectral index increase greater than 1·0 and probably at least as high as 1·5. Also, the measured spectra do not preclude a sharp cut-off to the spectrum. Such primary energy spectral changes are significantly greater than those previously assumed in modelling possible origins of the knee. It is of paramount importance to recognise that most energy dependent distributions obtained in air shower physics are flawed in the sense that they are statistical averages obtained through first obtaining statistical distributions as a function of shower size. One expects any measurement of an energy dependent distribution based on a direct energy determination for individual events to disagree with less direct observations. It is always necessary to examine the experimental technique employed before judging and comparing results.

We can examine some consequences of a primary proton energy spectrum with a sharp cut-off at ~10^{16} eV. There will be a knee in the vertical sea-level size spectrum at about 6×10^5 particles, with a change in slope to a power law of index ~2 over the next decade of size, even if there are no primary particles above this energy. At larger sizes, lacking higher energy particles, the spectrum steepens as fluctuations to sea-level occur. One might expect discrepancies in observational changes to result from such an extreme model when measurements are made as a function of shower size (or energy parameters derived from it), but in fact a number of observations at $\sim10^{16}$ eV support such a simple model.

The depth of shower maximum normally increases progressively with energy so that more energetic showers develop lower in the atmosphere. The rate of change of this depth with energy (or shower size) is known as the elongation rate. This is normally much less than 100 $\mathrm{g\,cm^{-2}}$ per decade of energy (or shower size). In the present model with a sharp cut-off, the depth of shower maximum would progressively increase through sizes of about 10^6 particles with a large elongation rate. It is possible that such an increase has been observed in controversial results by the Adelaide group (Thornton and Clay 1981) which based its measurements on shower sizes measured by a scintillator array. These results were not supported by measurements using Cerenkov radiation, a more direct primary energy estimator (Aliev *et al.* 1985). Such conflict would be expected on the basis of this model provided that there is not a complete cut-off in the primary energy spectrum, but there is a step with a reduced flux immediately above the break. The predicted elongation rate with shower size in this model is ~200 $\mathrm{g\,cm^{-2}}$ per decade, in agreement with the Adelaide observations.

The relationship between total electron shower size and muon shower size should show structure at these energies. These measurements are difficult and controversial but certainly one measurement (Rao 1983), at least, of N_μ against N_e shows a flattening between $N_e\sim3\times10^5$ and 10^6 as one would expect if, for a fixed N_μ (or energy parameter), one had a significant variation in shower size.

As one moves to higher altitude and closer to shower maximum, one would expect to see a more complex picture as fluctuations will not then preferentially pick out the proton component. Interestingly, data from Mt Chacaltaya (Bradt *et al.* 1965) show an index change in the size spectrum of $\sim1\cdot0$ compared with sea-level measurements of $\sim0\cdot6$, as one would expect if the proton component were still important at high altitudes as a significant component of the beam at fixed energy.

An energy spectrum at $\sim10^{15}$ to 10^{17} eV compatible with the observed size spectra can then contain a steepening which occurs over a limited energy range. This energy range would appear to be less than a factor of 3 for compatibility with Chacaltaya data. The steepening must be greater than 1 in the power law index of the spectrum and is associated with an intensity reduction by a factor of at least 10. This effect cannot be due to an absorption band since the size spectrum does not return to an extrapolation of the lower energy spectrum. There would thus appear to be an effect for primary cosmic rays such as an absorption threshold edge in the region of 10^{16} eV.

4. Possible Interpretation of the Results in Terms of an Absorption Feature

It is known through observations of ultra-high energy gamma-rays from neutron star binary systems that such sources are capable of accelerating particles to $\geqslant10^{17}$ eV. Energetic considerations suggest also that such sources are capable of producing all galactic cosmic rays at energies in the region of the knee (Hillas 1984). Protheroe (1984) has shown that protons remaining in the vicinity of at least one significant neutron star binary system for the order of hours will be severely attenuated by photopion production in the source radiation field and that there will be a very sharp absorption energy threshold for a fixed source temperature. In the case of Cygnus X-3 this threshold is at $\sim 10^{16}$ eV and a source of this nature would then appear to be a strong candidate for both cosmic ray production and attenuation to produce the observed spectrum. It is significant that the gamma-ray spectrum from Cygnus X-3 steepens above $\sim 10^{16}$ eV, and this would be compatible with a model of this object as a cosmic ray energy source with a generally steepening cosmic ray energy spectrum above the knee superimposed on the absorption edge such as observed.

The observed knee in the cosmic ray shower size spectrum thus corresponds to a very considerable steepening in the parent energy spectrum which is masked by the effect of fluctuations in the cascades initiated by the proton component in the beam. This primary energy spectrum steepening is of a form similar to an absorption edge and may be due to photopion absorption effects with black body photons at the cosmic ray source. Such a model may explain some perplexing observations resulting from air shower measurements made as a function of shower size rather than primary energy.

Acknowledgment

This work was supported by the Australian Research Grants Scheme.

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Manuscript received 25 March, accepted 15 June 1988