# Thermal Expansion of Fe<sub>2</sub>MnSi

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#### Abstract

Measurements of the thermal expansion of  $Fe_2MnSi$  in the temperature range 200-300 K are reported. A large, relatively narrow peak is found at the magnetic re-ordering temperature in contrast to a broad, weak anomaly at the Curie temperature. Values for the magnetic Grüneisen parameter  $\gamma_m$  are derived from the thermal expansion data and previously reported specific heat data following the subtraction of a non-magnetic background.

#### 1. Introduction

The Heusler alloy  $Fe_2MnSi$  is a member of the  $Fe_{3-x}Mn_xSi$  system which forms as a continuous pseudo-binary solid solution between  $Fe_3Si$  and  $Mn_3Si$  (Gladysevskij *et al.* 1956). The crystal structure of this system (L2<sub>1</sub>) may be described as a body-centred cubic lattice with a face-centred cubic superlattice or, equivalently, as four interpenetrating face-centred cubic sublattices, labelled A, B, C and D. In  $Fe_2MnSi$  the B and D sites are occupied by iron atoms, the C sites by manganese atoms and the A sites by silicon (Yoon and Booth 1974, 1977).

Fe<sub>2</sub>MnSi has two magnetic phase transitions below 300 K. The high temperature transition, at  $T_c$ , corresponds to a change from a paramagnetic state to a ferromagnetically ordered state. The low temperature transition, at the reordering temperature  $T_R$ , has been interpreted as a rearrangement of the atomic moments on the C sites to a rhombohedral anti-ferromagnetic arrangement (Yoon and Booth 1974).

Smith *et al.* (1980) measured the specific heat of  $\text{Fe}_{3-x}\text{Mn}_x\text{Si}$  compounds with x = 0.6, 0.8, 1.0, 1.2 and 1.6 over the temperature range 4-460 K. Distinct cusp-shaped peaks were found at  $T_{\text{R}}$ , whereas no gross features were evident at  $T_{\text{c}}$ . By using the data for the x = 0.6 and 1.6 compounds as a composite background, small 'step-like' anomalies were resolved in the specific heat close to  $T_{\text{c}}$  for x = 1.0, 1.2 and 1.6 compounds.

To complement these specific heat measurements, the thermal expansion of Fe<sub>2</sub>MnSi has been measured over the temperature range 2–300 K, with special attention being paid to the behaviour of the expansion coefficient  $\alpha$  in the vicinity of  $T_c$ .

#### 2. Experimental Details

Two  $Fe_2MnSi$  samples were measured, one of which was the same as that used in the earlier specific heat measurements. The second sample was prepared using the

procedure described in Smith *et al.* (1980), except that the heat treatment was followed by slow cooling rather than quenching. Both samples were roughly cylindrical, with lengths of approximately 20 mm and diameters of 8 mm. The end-faces of each sample were lapped flat to within  $0.25 \,\mu\text{m}$  and parallel to within 5 minutes of arc.

Electron microprobe analyses on a polished section of each sample were used to verify the compositions. X-ray diffraction measurements, performed on a Scintag PADV diffractometer, using both polished sections and powders produced by crushing alloy pieces, showed that the samples were all single phase and had the L2<sub>1</sub> structure. The lattice parameters, derived by extrapolating  $\cos^2\theta$  plots, were both  $5.663\pm0.001$  Å, which is in excellent agreement with the lattice parameter obtained by interpolating the values reported by Yoon and Booth (1977).

The thermal expansion measurements were made in a three terminal, capacitance dilatometer, identical to that described by White and Collins (1972). The samples were built up to a length of 5.07 cm using spring-loaded copper spacers. Measurements were made with liquid helium as a coolant for temperatures up to 80 K, and with liquid nitrogen from 80 to 300 K.

The same measuring procedure as that described by White and Collins (1972) was followed. For temperatures greater than 50 K, times of the order of one hour were required for the cell to reach thermal equilibrium.

The temperature of the capacitance cell was monitored using two thermometers, a germanium cryo-resistor for temperatures between 1.5 and 25 K, and a four-wire platinum resistor for temperatures between 13 and 300 K. Both thermometers had calibrations directly traceable to the CSIRO Division of Applied Physics, giving a precision of better than  $\pm 10$  mK for T and an accuracy better than 1% for  $\Delta T$ . The capacitance of the cell was measured using a ratio-transformer capacitance bridge with a resolution better than  $\pm 10^{-6}$  pF, giving a sensitivity of approximately  $\pm 0.1$  Å in the measurement of the change in length of the sample, relative to the cell.

Across the full temperature range, the thermal expansion coefficient could be determined with a precision of  $\pm 1 \times 10^{-8} \text{ K}^{-1}$ . The thermal expansion of a rod of 5 N copper, measured using this apparatus, agreed with published data (Kroeger and Swenson 1977) to better than 1% from 10 to 300 K.

The magnetic state of the sample was monitored during the thermal expansion measurements using an a.c. transformer method. A pair of coils was placed around the sample within the expansion cell. The primary coil was excited by an a.c. signal of constant amplitude and frequency (2 V, 350 Hz). The current induced in the secondary coil was measured using a phase-locked amplifier. This enabled detailed measurement of the thermal expansion in the vicinity of both transition temperatures to be made.

### **3. Results**

The linear thermal expansion coefficients of both samples of  $Fe_2MnSi$  as functions of temperature are shown in Fig. 1. Slow cooling the sample rather than quenching had no discernible effect on the thermal expansion over the temperature range 80 to 300 K. The significant anomaly in the vicinity of  $T_R$  and the relatively small kink at  $T_c$  are similar to the specific heat anomalies near the transition temperatures (Smith *et al.* 1980; Kalishevich *et al.* 1977). There were no apparent discontinuities in the sample length at the transition temperatures.



Fig. 1. Linear thermal expansion for  $Fe_2MnSi$  as a function of temperature: Pluses, quenched sample; squares, slow-cooled sample; triangles, secondary coil signal (arbitrary units); and solid curve, derivative of the interpolated coil signal.

Fig. 1 shows the secondary coil signal as a function of temperature (triangles). Low field a.c. susceptibility in the vicinity of a magnetic transition is a complex function of magnetic domain and demagnetisation effects. The assignment of values to transition temperatures on the basis of the secondary coil signal is relatively arbitrary (Simpson and Smith 1982; Maartense and Williams 1976; Ododo 1979). A similar situation exists in the measurement of other thermophysical and magnetic properties, as evidenced by the range of values for  $T_{\rm R}$  (59-69 K) and  $T_{\rm c}$  (214-24 K) reported in the literature (Yoon and Booth 1974, 1977; Ziebeck and Webster 1976; Lutskaya et al. 1976; Smith et al. 1980; Lutskaya 1985). We have chosen  $T_{\rm R}$  and  $T_{\rm c}$  to be the temperatures given by the inflection points in the change of coil signal with temperature at the magnetic transitions. The inflection points were determined by interpolating the coil signal measurements and taking the first derivative with respect to temperature. Fig. 1 shows the derivative as a function of temperature (solid curve). The transition temperatures correspond to the position of the peaks. This gives  $T_{\rm R} = 60$  K in excellent agreement with Lutskaya (1985) ( $T_{\rm R} = 59.5$  K) and with the position of the maximum in the low temperature expansion coefficient anomaly. Similarly we obtain  $T_c = 216$  K, in agreement with Ziebeck and Webster (1976)  $(T_{\rm c}=214~{\rm K}).$ 

The extraction of the magnetic contribution to the thermal expansion requires an estimate of the lattice and electronic contributions which are subtracted from the experimental values. The same approach is used to extract the magnetic contribution to the specific heat. Smith *et al.* (1980) used a background curve consisting of data for the specific heat of Fe<sub>2.4</sub>Mn<sub>0.6</sub>Si below 134 K and Fe<sub>1.4</sub>Mn<sub>1.6</sub>Si above 134 K. The assumptions were that the magnetic contribution to the specific heat ( $C_m$ ) associated with  $T_c$  at 450 K for the x = 0.6 compound is negligible below 134 K and that  $C_m$  associated with  $T_c$  at 134 K for the x = 1.6 compound is negligible above 134 K.

This approach has the drawback that, whereas molecular field theory predicts a sharp cutoff in the magnetic specific heat at  $T_c$ , measurements on many ferromagnetic materials, including iron, nickel and gadolinium (Hofmann *et al.* 1956), show that  $C_m$  generally persists well above  $T_c$ , reflecting the presence of short range order. This effect could significantly affect the background used by Smith *et al.*, making it too large above 134 K.

Alternatively, the specific heat of Fe<sub>3</sub>Si may be used as a background. This assumes that the magnetic contribution to the specific heat of ferromagnetic Fe<sub>3</sub>Si  $(T_c \sim 850 \text{ K})$  is negligible in the range 0-300 K and that the lattice and electronic contributions are the same for Fe<sub>3</sub>Si and Fe<sub>2</sub>MnSi. Support for the latter assumption is based, firstly, on the fact that Fe<sub>2</sub>MnSi is isostructural with Fe<sub>3</sub>Si and that iron and manganese have almost identical atomic weights, indicating a similarity of the lattice contribution for each compound. Secondly, the closeness of the total specific heats of Fe<sub>3</sub>Si and antiferromagnetic Mn<sub>3</sub>Si  $(T_N \sim 22 \text{ K})$  over the range 100-300 K indicates a closeness of both the lattice and electronic contributions to the specific heat of each compound (Kalishevich *et al.* 1977). It must be emphasised, however, that a similarity in the lattice and electronic contributions for the end members of the pseudo-binary alloy system Fe<sub>3-x</sub>Mn<sub>x</sub>Si does not guarantee that these contributions remain the same throughout the compositional range.

Measurements of the elastic constants of  $Fe_{3-x}Mn_xSi$  compounds (Lutskaya 1985) showed that the bulk modulus of  $Fe_2MnSi$  ( $B_T \sim 315$  GPa) is considerably higher than the bulk modulus of  $Fe_3Si$  ( $B_T \sim 180$  GPa) (Rausch and Kayser 1977). This implies a lower lattice contribution to the specific heat for  $Fe_2MnSi$  compared with that for  $Fe_3Si$ . However, in the absence of a more reliable method for separating the various contributions to the specific heat in a magnetic system, the use of  $Fe_3Si$  as a non-magnetic background over the temperature range investigated is taken to be a reasonable choice even though it probably underestimates the magnetic contribution.

Data for the specific heat of  $Fe_3Si$  were taken from several sources (Gel'd and Krentsis 1963; Touloukian and Buyco 1970; Kalishevich *et al.* 1977). The specific heat measurements of  $Fe_2MnSi$  by Smith *et al.* (1980) were combined with the measurements of Kalishevich *et al.* (1977) and with low temperature (1.8–40 K) specific heat measurements of the slow-cooled  $Fe_2MnSi$  sample (to be published), conducted at the CSIRO Division of Applied Physics. The Smith *et al.* results required a 7–8% shift in the temperature scale to bring them into agreement with the other measurements. The probable origin of this discrepancy is thermometry errors in their continuous calorimeter measurements.

Measured values for  $C_p$  were converted to  $C_v$  by the relationship

$$C_p - C_v = V T \beta^2 B_{\rm T}, \qquad (1)$$

where V is the molar volume,  $\beta = 3\alpha$  and  $B_{\rm T}$  is the isothermal bulk modulus.

The Fe<sub>2</sub>MnSi specific heat data are shown in Fig. 2, along with the Fe<sub>3</sub>Si background. The magnetic contribution to the specific heat of Fe<sub>2</sub>MnSi, obtained by subtracting the Fe<sub>3</sub>Si background, is shown in Fig. 3. It is apparent that there is a relatively flat region between  $T_{\rm R}$  and  $T_{\rm c}$ , possibly the result of combining a high temperature tail in the magnetic specific heat associated with the transition at  $T_{\rm R}$  with the rising magnetic specific heat associated with the transition at  $T_{\rm c}$ .



Fig. 2. Specific heat data: Crosses, Fe<sub>2</sub>MnSi; squares, Fe<sub>3</sub>Si.



Fig. 3. Magnetic contribution to the specific heat for  $Fe_2MnSi$ .

The total magnetic entropy was derived from numerical integration of  $C_m/T$  as a function of temperature. The value of  $S_m = 11 \text{ J mol}^{-1} \text{ K}^{-1}$  has a 10-20% uncertainty due to the uncertainty involved in estimating the non-magnetic background. It is apparent, however, that the total magnetic entropy is considerably smaller than the  $3R \ln 2 = 17 \cdot 3 \text{ J mol}^{-1} \text{ K}^{-1}$  associated with the ordering of 3N magnetic spins, each with  $s = \frac{1}{2}$ .



Fig. 4. Squares: linear thermal expansion coefficient data; crosses: non-magnetic background for  $Fe_2MnSi$  as a function of temperature.



Fig. 5. Magnetic contribution to the linear thermal expansion coefficient of  $Fe_2MnSi$  as a function of temperature.

In the case of the thermal expansion it is to be expected that the greater stiffness of the  $Fe_2MnSi$  lattice will result in a lower lattice contribution compared with  $Fe_3Si$ , and thus an overestimate of the non-magnetic contribution if the approach adopted for the heat capacity is extended to the thermal expansion. In fact, the thermal An approximate estimate of the non-magnetic background may be obtained by assuming that the total non-magnetic electronic and lattice Grüneisen parameter for  $Fe_2MnSi$ , given by

$$\gamma_{e,1} = \frac{3 \, V B_{\mathrm{T}} \, \alpha_{e,1}}{C_{e,1}}, \qquad (2)$$

is not strongly temperature dependent. This assumption was supported by determining the temperature dependence of the total Grüneisen parameter for  $Mn_3Si$  over the range 100-300 K. Using specific heat and thermal expansion measurements of  $Mn_3Si$ (Letun *et al.* 1965; Barmin *et al.* 1984) and the bulk modulus of Fe<sub>3</sub>Si, a total Grüneisen parameter of 2.2, varying by less than 1% from 100-300 K, was obtained.

A value for  $\gamma_{e,1}$  for Fe<sub>2</sub>MnSi may be obtained by assuming that for temperatures above 250 K the magnetic contribution to the specific heat and thermal expansion is negligible. This procedure gives  $\gamma_{e,1} = 3.4 \pm 1$  and since  $B_T$  varies by less than 3% from 0-300 K (Lutskaya 1985), equation (2) may be used to calculate the non-magnetic thermal expansion background from the specific heat data for Fe<sub>3</sub>Si. The results are shown in Fig. 4 and the magnetic contribution to the thermal expansion of Fe<sub>2</sub>MnSi, obtained by subtracting the non-magnetic background, is shown in Fig. 5.

The  $(\alpha_m, T)$  curve is similar to the  $(C_m, T)$  curve, with two peaks centred at  $T_R = 60 \pm 1$  K and  $T_c = 214 \pm 2$  K, in excellent agreement with Lutskaya (1985) and Ziebeck and Webster (1976). The negative values for  $\alpha_m$  between  $T_R$  and  $T_c$  demonstrate the shortcoming in the choice of background. However, it is not possible to obtain a better estimate for the non-magnetic thermal expansion background on the basis of current information.



Fig. 6. Magnetic Grüneisen parameter for Fe<sub>2</sub>MnSi as a function of temperature.

The magnetic Grüneisen parameter  $\gamma_m$ , calculated from the magnetic specific heat and thermal expansion data in Figs 3 and 5, is shown in Fig. 6;  $\gamma_m$  is not shown in the temperature ranges where the uncertainties due to the uncertainty in background are considered to dominate.

## 4. Discussion

A significant feature of both the thermal expansion and the specific heat measurements is the relatively small and broad anomaly at  $T_c$  compared with the large, narrow, cusp-shaped anomaly at  $T_R$ . The persistence of  $C_m$  and  $\alpha_m$  above the Curie temperature is usually ascribed to the presence of short-range order above  $T_c$ . This leads to the conclusion that the transition at  $T_R$  has a stronger long-range character than the transition at  $T_c$ .

The narrowness of the peak at  $T_{\rm R}$  for the thermal expansion measurements and the sharpness of the change in the a.c. coil signal at  $T_{\rm c}$  indicate that the broadening of the peak at  $T_{\rm c}$  is not due to chemical clustering or to the influence of local strains on the exchange parameters.

The observation that the total magnetic entropy is much smaller than the  $3R \ln 2$  associated with the ordering of 3N magnetic spins each with  $S = \frac{1}{2}$  is contrary to a model based on strictly local moments. Recent neutron diffuse scattering and saturation magnetisation measurements on  $\operatorname{Fe}_{3-x}\operatorname{Mn}_x$ Si alloys for x < 0.6 (Kepa *et al.* 1988) showed that the magnetic moment on the Mn atom, substituted on the C sites, is strongly temperature dependent. The magnetic moment at low temperature was reported to be  $2 \cdot 2\mu_B$  (Yoon and Booth 1974), whereas Kepa *et al.* proposed a value of  $0.6\mu_B$  at room temperature. A reduction of the Mn magnetic moment with increase of temperature could be a significant factor in explaining the differences at  $T_R$  and  $T_c$  in the shape of the thermal expansion and specific heat anomalies and would account for the low magnetic entropy.

The high values of  $\gamma_m$  at  $T_R$  and  $T_c$  indicate a strong volume dependence of the magnetic entropy, with the volume dependence at the reordering temperature being slightly greater than that at the Curie temperature. The absence of discontinuities in the sample length temperature dependence indicates that both magnetic transitions are second order. The pressure dependence of the transition temperatures may therefore be obtained using Ehrenfest's equation (Krishnan *et al.* 1979):

$$\frac{\mathrm{d}T_{\mathrm{m}}}{\mathrm{d}P} = \frac{T_{\mathrm{m}}\Delta\beta V}{\Delta C_{\mathrm{p}}} = \frac{\Delta\chi_{\mathrm{T}}}{\Delta\beta},\qquad(3,4)$$

where  $\Delta\beta = 3\Delta\alpha$  and  $\Delta\chi_T$  is the change in the isothermal compressibility at the transition temperature.

Using equation (3) we obtain  $dT_R/dP = 1 \cdot 1 \pm 0 \cdot 2 \text{ K GPa}^{-1}$  at the reordering temperature, while  $dT_c/dP = 4 \cdot 0 \pm 2 \text{ K GPa}^{-1}$  at the Curie temperature. Here  $\Delta \chi_T$  was derived from the changes in Poisson's ratio and Young's modulus reported by Lutskaya (1985). This was only possible at  $T_R$  as the elastic constant measurements at  $T_c$  did not have sufficient resolution to obtain a reliable value for  $\Delta \chi_T$ . The value for  $\Delta \chi_T$  at  $T_R$  was  $(1 \cdot 5 \pm 2) \times 10^{-5} \text{ GPa}^{-1}$ , giving  $dT_R/dP = 1 \cdot 3 \pm 1 \text{ K GPa}^{-1}$  using equation (4), in agreement with the value obtained using (3).

The quantity  $dT_m/dP$  and the thermodynamic magnetic Grüneisen parameter are related subject to the condition that the magnetic entropy can be represented by

 $S_{\rm m} = S(E_{\rm m}/T)$ , where the magnetic interaction energy  $E_{\rm m}$  alone determines the magnetic transition temperature (Barron *et al.* 1980). It follows that

$$\gamma_{\rm m} = \left( -\frac{\partial \ln T_{\rm m}}{\partial \ln V} \right)_T,\tag{5}$$

and hence that

$$\frac{\mathrm{d}\,T_{\mathrm{m}}}{\mathrm{d}\,P} = \frac{\gamma_{\mathrm{m}}\,T_{\mathrm{m}}}{B_{\mathrm{T}}}\,.\tag{6}$$

Using the values for  $\gamma_m$  derived above, we obtain  $dT_R/dP = 1 \cdot 1_4 \pm 0 \cdot 2 \text{ K GPa}^{-1}$  and  $dT_c/dP = 3 \cdot 4_0 \pm 0.4 \text{ K GPa}^{-1}$ , in good agreement with the values calculated using Ehrenfest's equation. It follows that  $E_m$  alone determines the transition temperatures  $T_R$  and  $T_c$ .

Agreement between direct measurements of  $dT_m/dP$  and values calculated from magnetic Grüneisen parameters has been demonstrated for several alloy systems, specifically spin glass alloys (Simpson *et al.* 1981; Smith 1981) and iron-nickel alloys (Barron *et al.* 1980). One exception is the behaviour of nickel-chromium alloys close to the critical composition for ferromagnetism (Simpson and Smith 1982). Direct measurements of  $dT_m/dP$  for Fe<sub>2</sub>MnSi are currently being conducted to verify the values obtained from the thermal measurements.

#### 5. Conclusions

Measurements of the thermal expansion of  $Fe_2MnSi$  confirm the general features of the specific heat measurements, namely a large cusp-shaped peak at  $T_R$  in contrast to a weaker and broader anomaly at  $T_c$ . The anomaly at  $T_c$  has a more conventional shape in the thermal expansion, rising to a maximum and then slowly falling above  $T_c$ . The persistence above  $T_c$  is indicative of short-range order.

Values for the magnetic Grüneisen parameters indicate a strong volume dependence of the magnetic entropy at  $T_{\rm R}$  and  $T_{\rm c}$ . Agreement between  $dT_{\rm m}/dP$  values calculated from Ehrenfest's equation and those obtained from the magnetic Grüneisen parameters indicates that the magnetic entropy has the form  $S_{\rm m} = S(E_{\rm m}/T)$ , i.e.  $E_{\rm m}$  is the major factor determining  $T_{\rm R}$  and  $T_{\rm c}$ .

We are extending our thermal expansion measurements to more alloys in the  $Fe_{3-x}Mn_xSi$  series. This should enable us to determine the composition dependence of the magnetic Grüneisen parameters.

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