on ϕ, the following constraints are found

$$(\phi_x^2 + 3\phi_t^2)^{-2}(2\phi_x\phi_t + 3\phi_t^2 - \phi_x^2)F[\phi] = 0, \tag{37}$$

and

$$(\phi^2 + 3\phi_t^2)^{-2}\phi_t\phi_x F[\phi] = 0, \tag{38}$$

where $F[\phi]$ is given by (15). The requirement $-2\partial v_0/\partial t = \partial w_0/\partial t - \partial w_0/\partial x$ follows from the requirement (35a) and (35b). A group-theoretical reduction of system (4) via

$$\begin{aligned} u(x, t) &= \frac{1}{x} f_1(s), \\ v(x, t) &= \frac{1}{x} f_2(s), \\ w(x, t) &= \frac{1}{x} f_3(s), \end{aligned} \tag{39}$$

($s = t/x$: similarity variable) yields a system of ordinary differential equations which pass the Painlevé test. The system is given by

$$\begin{aligned} (1-s)f_1' &= f_1 + \frac{1}{\varepsilon}(f_2^2 - f_1 f_3), \\ f_2' &= -\frac{1}{2\varepsilon}(f_2^2 - f_1 f_3), \\ (1+s)f_3' &= -f_3 + \frac{1}{\varepsilon}(f_2^2 - f_1 f_3). \end{aligned} \tag{40}$$

System (40) passes the Painlevé test. We can find a Bäcklund transformation for the system. This is given by

$$\begin{aligned} f_1(s) &= \varepsilon\phi' \frac{4(1+s)}{3+s^2}\phi^{-1} + f_{11}, \\ f_2(s) &= -\varepsilon\phi' \frac{2(1-s^2)}{3+s^2}\phi^{-1} + f_{21}, \\ f_3(s) &= \varepsilon\phi' \frac{4(1-s)}{3+s^2}\phi^{-1} + f_{31}, \end{aligned} \tag{41}$$

where ϕ, f_{11}, f_{21} and f_{31} satisfy the equations

$$\begin{aligned} \varepsilon(1-s^2)(3+s^2)\phi'' - 8\varepsilon s\phi' &= -(3+s^2)\phi'[(1-s)f_{11} + (1-s^2)f_{21} + (1+s)f_{31}], \\ (1-s)f_{11}' &= f_{11} + \frac{1}{\varepsilon}(f_{21}^2 - f_{11}f_{31}), \\ f_{21}' &= -\frac{1}{2\varepsilon}(f_{21} - f_{11}f_{31}), \\ (1+s)f_{31}' &= -f_{31} + \frac{1}{\varepsilon}(f_{21}^2 - f_{11}f_{31}). \end{aligned} \tag{42}$$

A special solution of system (40) can be given for $f_{11} = f_{21} = f_{31} = 0$. Then we have

$$\phi(s) = c_1 \left(2 \ln \frac{1+s}{1-s} - s + c_2\right), \tag{43}$$

so that

$$\begin{aligned} f_1(s) &= \frac{4\varepsilon}{1-s}\left[2 \ln \frac{1+s}{1-s} - s + c_2\right]^{-1}, \\ f_2(s) &= -2\varepsilon\left[2 \ln \frac{1+s}{1-s} - s + c_2\right]^{-1}, \\ f_3(s) &= \frac{4\varepsilon}{1+s}\left[2 \ln \frac{1+s}{1-s} - s + c_2\right]^{-1}, \end{aligned} \tag{44}$$

where c_1 and c_2 are constants of integration. Consequently, when taking into account (42), we have found a special solution for the one-space dimensional Broadwell model. The space-independent three-velocity model can be solved by quadratures as done for the two-velocity models.

Cornille (1987) studied the one-space dimensional Broadwell model. He found three classes of positive exact solutions by determining 'solitons' (one-dimensional shock wave solutions) and 'bisolitons' (two-dimensional, space plus time solutions) for the system.

4. Conclusions

We have shown that the Carleman model, McKean model and Broadwell model do not pass the Painlevé test. The 'nonpassing' of the Painlevé test agrees with the fact that the general solution (Cauchy problem) cannot be given. The constraints (see equation 15) which one finds at the resonance are the same for all three models. This constraint also appears in various nonintegrable relativistic field equations (Steeb and Euler 1988). Furthermore, the models studied do not admit Lie Bäcklund vector fields. We also gave the Lie symmetry vector fields and performed group-theoretical reductions. The resulting ordinary differential equations pass the Painlevé test and can be integrated.

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Manuscript received 31 March, accepted 26 August 1988