# Coulomb Excitation of ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ 

R. H. Spear, ${ }^{\text {A }}$ W. J. Vermeer, ${ }^{\text {A }}$ S. M. Burnett, ${ }^{\text {B }}$ G. J. Gyapong ${ }^{\text {A,C }}$ and C. S. Lim $^{\text {A }}$<br>A Department of Nuclear Physics, Research School of Physical Sciences, Australian National University, G.P.O. Box 4, Canberra, A.C.T. 2601, Australia.<br>${ }^{B}$ Department of Physics and Theoretical Physics, Faculty of Science, Australian National University, G.P.O. Box 4, Canberra, A.C.T. 2601, Australia.<br>C Present address: Department of Physics, University of York, York, U.K.


#### Abstract

The $N=84$ nuclei ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ have been Coulomb-excited using ${ }^{4} \mathrm{He},{ }^{12} \mathrm{C}$ and ${ }^{16} \mathrm{O}$ projectiles. Scattered particles were detected at a mean laboratory angle of $170 \cdot 6^{\circ}$ with an annular silicon surface-barrier detector. The static quadrupole moment of the $2_{1}^{+}$state was determined for both nuclei. In addition, various electromagnetic transition probabilities involving the $2_{2}^{+}, 2_{3}^{+}, 3_{1}^{-}$and $4_{1}^{+}$states were measured. Taken in conjunction with previous information, the results show that the properties of ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ are well described by the vibrational $U(5)$ limit of IBM-2, and strongly support the proposition that the $2_{3}^{+}$level in each nucleus is an essentially pure one d-boson mixed-symmetry state.


## 1. Introduction

This paper presents the results of Coulomb-excitation studies of the $N=84$ nuclei ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$. The main objective of the work was the determination of electromagnetic matrix elements relevant to the identification of the collective $2^{+}$states of mixed proton-neutron symmetry expected on theoretical grounds to occur at an excitation energy $E_{\mathrm{x}}$ of about 2 MeV in both nuclei.

The initial predictions of mixed symmetry states were made primarily, but not exclusively, within the framework of the interacting boson model (IBM) (see e.g. Iachello 1984 and references therein). Unlike the original version of the model (IBM-1), the version known as IBM-2 distinguishes between neutron ( $\nu$ ) and proton ( $\pi$ ) bosons, and predicts the existence of states of mixed proton-neutron symmetry, i.e. states which are not fully symmetric with respect to the interchange of proton and neutron bosons. Because IBM-1 has had widespread success in describing the low-lying states of nuclei, the mixed-symmetry states will usually be expected to occur at higher excitation energies than the totally symmetric states.

As indicated above, the IBM is not the only model which predicts the occurrence of mixed-symmetry states. For example, treatments of symmetry mixing have been published based on the vibrational model (Faessler and Nojarov 1986), the shell model and the particle-core coupling model (Heyde and Sau 1986). Indeed, the latter authors have argued that mixed-symmetry modes are a general feature of any two-component nuclear system.

Most of the initial interest in mixed-symmetry states centred on the $J^{\pi}=1^{+}$ mode expected (Iachello 1981, 1984) to occur in well deformed axially-symmetric
nuclei at $E_{\mathrm{x}} \approx 3 \mathrm{MeV}$ with large M1 strength to the ground state. Experimental evidence for the existence of these $1^{+}$states has been obtained in strongly deformed nuclei ranging from ${ }^{46} \mathrm{Ti}$ to ${ }^{238} \mathrm{U}$, mainly from electron and photon scattering (Bohle et al. 1984a, 1984b; Berg et al. 1984; Heil et al. 1988; Hartmann et al. 1987). The excitation has been pictorially described as a type of 'scissors' mode, involving small-amplitude oscillations of the angle between the symmetry axes of the deformed proton and neutron distributions; however, this interpretation has recently been disputed (Speth and Zawischa 1988; Freeman et al. 1989).

Iachello (1984) pointed out that for spherical nuclei a collective $J^{\pi}=2^{+}$mixedsymmetry state would be expected at $E_{\mathrm{x}} \approx 2-3 \mathrm{MeV}$, with $B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2^{+}\right) \approx 3 \mathrm{~W} . \mathrm{u}$. He showed that in the $U(5)$ limit (purely vibrational motion)

$$
\begin{equation*}
B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2^{+}\right)=\left\{5 N_{\pi} N_{\nu} /\left(N_{\pi}+N_{\nu}\right)\right\}\left(e_{\pi}-e_{\nu}\right)^{2}, \tag{1}
\end{equation*}
$$

where $e_{\pi}$ and $e_{\nu}$ are the proton- and neutron-boson effective charges. Thus, experimental investigation of the mixed-symmetry $2^{+}$state would be of great importance since it could provide a direct measure of the difference between $e_{\pi}$ and $e_{\nu}$. In a geometrical picture this mode can be described (Faessler and Nojarov 1986) as a low-lying isovector quadrupole vibration with the protons and neutrons oscillating out of phase. An enhanced M1 strength to the lowest fully symmetric $2^{+}$state would also be expected (Van Isacker et al. 1986). There is very little experimental information regarding mixed-symmetry states in spherical nuclei.

Hamilton et al. (1984) have suggested that the $2_{3}^{+}$levels in the $N=84$ isotones ${ }^{140} \mathrm{Ba},{ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ are good candidates for vibrational mixed-symmetry states. They carried out calculations in the U(5) limit of IBM-2 and obtained good agreement with experimental branching ratios and E2/M1 mixing ratios for the $2_{3}^{+}$levels; they concluded that for the nuclei concerned $e_{\pi}=0.12 \mathrm{eb}$ and $e_{\nu}=0.24 \mathrm{eb}$, i.e. that $e_{\nu}>e_{\pi}$, which is rather surprising since neutrons carry no charge. However, as stressed by, for example, Faessler and Nojarov (1986), it is the absolute transition strengths which are of crucial importance in testing the theory. This is true not only for the $2_{3}^{+}$level but also for other levels in order to fix the parameters of the theory. For ${ }^{140} \mathrm{Ba}$ and ${ }^{142} \mathrm{Ce}$ there is no previous information on absolute electromagnetic transition strengths apart from $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)$.

In the present work we have used Coulomb excitation to study levels in ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$. Energy-level diagrams are shown in Fig. 1. Among the information obtained we have been able to determine $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{3}^{+}\right)$for ${ }^{142} \mathrm{Ce}$. This value can be combined with previous measurements of branching ratios and mixing ratios (Peker 1984) to determine the M1 and E2 strengths for the proposed mixed-symmetry to full-symmetry transition $2_{3}^{+} \rightarrow 2_{1}^{+}$. In the case of ${ }^{144} \mathrm{Nd}$ these transition strengths were already known from the nuclear-resonance-fluorescence work of Metzger (1969) and the branchingand mixing-ratio measurements of Snelling and Hamilton (1983). We are therefore able to compare the theoretical predictions with results obtained from two completely different experimental techniques. We have also carried out improved determinations of the static quadrupole moment, $Q\left(2_{1}^{+}\right)$, of the $2_{1}^{+}$


Fig. 1. Level schemes of ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ taken from Peker (1984) for ${ }^{142} \mathrm{Ce}$, and from Tuli (1979), Snelling and Hamilton (1983) and Krane et al. (1983) for ${ }^{144} \mathrm{Nd}$. Excitation energies are given in keV .
states of ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ (Hamilton et al. used this quantity as an important test of their model predictions), and have determined various other transition strengths in the two nuclei. Taken in conjunction with previous experimental work (e.g. Metzger 1969; Snelling and Hamilton 1983; Fahlander et al. 1980), the results permit a comprehensive comparison of the properties of ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$. Hence they provide a crucial test of the IBM- $2 \mathrm{U}(5)$ calculations of Hamilton et al., which predict not only the absolute strengths of transitions involving symmetric and mixed-symmetry states, but also that these strengths should be equal in these two nuclei. Finally, we have determined $B\left(E 3 ; 0_{1}^{+} \rightarrow 3_{1}^{-}\right)$ for both ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$. A brief report covering part of the present work has appeared elsewhere (Vermeer et al. 1988).

## 2. Experimental Procedure

Principles of the experimental procedure have been given in previous publications (see e.g. Esat et al. 1976; Fewell et al. 1979). In the present case, beams of ${ }^{4} \mathrm{He},{ }^{12} \mathrm{C}$ and ${ }^{16} \mathrm{O}$ projectiles, obtained from the 14UD accelerator at


Fig. 2. Spectra obtained from bombardment of a ${ }^{142} \mathrm{Ce}$ target with ${ }^{4} \mathrm{He},{ }^{12} \mathrm{C}$ and ${ }^{16} \mathrm{O}$ projectiles at a mean laboratory scattering angle of $170 \cdot 6^{\circ}$. States of ${ }^{142} \mathrm{Ce}$ are indicated by their spin and parity.
the ANU, were used to bombard targets of ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$. The beam energy was known to an accuracy of better than $0 \cdot 1 \%$ (Spear et al. 1977). Targets were prepared by evaporating $\mathrm{CeF}_{3}$ and $\mathrm{NdF}_{3}$ onto thin carbon foils. The isotopic enrichments were $93 \cdot 4 \%$ and $97 \cdot 5 \%$ for ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ respectively. Target thicknesses were measured by Rutherford scattering. It was found that the targets used for ${ }^{4} \mathrm{He}$ bombardment had partial thicknesses of 58 and $38 \mu \mathrm{~g} \mathrm{~cm}^{-2}$ for ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ respectively, and those used for ${ }^{12} \mathrm{C}$ and ${ }^{16} \mathrm{O}$


Fig. 3. As for Fig. 2, but for a ${ }^{144} \mathrm{Nd}$ target. In extracting the area of the $3_{1}^{-}$peak the small contribution (a few per cent) of the partially resolved $2_{2}^{+}$peak was calculated using known $B(E 2)$ values (see text).
bombardment were 4.8 and $3.0 \mu \mathrm{~g} \mathrm{~cm}^{-2}$ respectively. Backscattered particles were detected at a mean laboratory scattering angle of $170 \cdot 6^{\circ}$ using an annular silicon surface-barrier detector. Typical spectra are shown in Figs 2 and 3. Rutherford backscattering measurements with low-energy carbon beams showed no evidence of significant target contaminants, except for a small amount of Mo in the ${ }^{142} \mathrm{Ce}$ targets. Scattering from Mo isotopes produces


Fig. 4. Spectra obtained with ${ }^{4} \mathrm{He}$ projectiles on ${ }^{142} \mathrm{Ce}$, showing cases where the Mo contaminant peak is (a) resolved from the ${ }^{142} \mathrm{Ce}$ $2_{1}^{+}$peak, and (b) not resolved. In case (b) the contribution attributed to Mo is calculated as described in the text. In both cases the full curves show fits to the data, and the dashed curve shows the underlying tail from the $0_{1}^{+}$peak.
peaks in sensitive regions of the spectra only for ${ }^{4} \mathrm{He}$; for ${ }^{12} \mathrm{C}$ and ${ }^{16} \mathrm{O}$ the Mo peaks occur at energies which are too small to be troublesome. For some ${ }^{4} \mathrm{He}$ spectra the peak corresponding to the Mo contaminant was clearly resolved (Fig. 4), providing further information on the quantity of Mo present. In order to permit accurate subtraction of Mo contributions when determining excitation probabilities (Section 3), the variation of yield for Mo as a function of bombarding energy was determined by bombarding a target consisting of natural Mo deposited on a thin Au backing with ${ }^{4} \mathrm{He}$ projectiles at the energies actually used in the experiment.

## 3. Analysis and Results

Spectra were analysed using well established lineshape fitting procedures (Esat et al. 1976; Fewell et al. 1979) to extract excitation probabilities $P_{\exp }\left(J_{n}^{\pi}\right)$


Fig. 5. Portion of spectrum obtained with $33-\mathrm{MeV}{ }^{12} \mathrm{C}$ projectiles on ${ }^{142} \mathrm{Ce}$. The full curve shows a fit to the data obtained as described in the text. Peaks corresponding to states in ${ }^{142} \mathrm{Ce}$ are indicated by their appropriate $J_{n}^{\pi}$ values. The dashed curve represents the tail of the $0_{1}^{+}$ peak. Also shown are contributions from ${ }^{140} \mathrm{Ce}$.
for various states, where the experimental excitation probability is defined as

$$
\begin{equation*}
P_{\exp }\left(J_{n}^{\pi}\right)=A\left(J_{n}^{\pi}\right) /\left\{A\left(0_{1}^{+}\right)+A\left(U_{n}^{\pi}\right)\right\}, \tag{2}
\end{equation*}
$$

the quantities $A$ being the areas of the appropriate spectral peaks. Allowance was made for small contributions to the spectra from minor isotopes of the target element ('isotopic impurities') using the supplier's isotopic assay in conjuction with $B(E 2)$ values for excited states of these isotopes obtained from the literature. An example of the fits obtained is shown in Fig. 5. Values of $P_{\exp }\left(2_{1}^{+}\right)$were determined for all spectra. Excitation probabilities for the $4_{1}^{+}, 2_{2}^{+}$, $3_{1}^{-}$and $2_{3}^{+}$states of ${ }^{142} \mathrm{Ce}$ and for the $4_{1}^{+}$and $3_{1}^{-}$states of ${ }^{144} \mathrm{Nd}$ were determined from some spectra where the corresponding peaks were sufficiently prominent to provide useful results. The values obtained are listed in Tables 1 and 2.

It is essential for the valid application of Coulomb-excitation theory that the data analysed should be obtained at bombarding energies sufficiently low that nuclear interference is negligible (Spear et al. 1978). 'Unsafe' energies may be detected by plotting the ratio $P_{\exp } / P_{\text {coul }}$ as a function of bombarding energy $E$, where $P_{\text {Coul }}$ is the excitation probability calculated assuming pure Coulomb excitation. The onset of nuclear interference usually manifests itself in a decrease of $P_{\exp } / P_{\text {coul }}$ below a hitherto constant value as $E$ is increased. Safe-energy plots for the $2_{1}^{+}$data are shown in Fig. 6; $P_{\exp } / P_{\text {Coul }}$ is plotted as a function of $E$ and of $s$, the distance of closest approach of the nuclear surfaces, defined by the expression

$$
\begin{equation*}
s\left(\theta_{\text {c.m. }}\right)=\frac{0 \cdot 72 Z_{1} Z_{2}}{E}\left(1+\frac{A_{1}}{A_{2}}\right)\left(1+\operatorname{cosec}{ }^{\frac{1}{2}} \theta_{\text {c.m. }}\right)-1 \cdot 25\left(A_{1}^{\frac{1}{3}}+A_{2}^{\frac{1}{3}}\right) \mathrm{fm}, \tag{3}
\end{equation*}
$$

Table 1. Measured excitation probabilities $P_{\exp }\left(2_{1}^{+}\right)$for ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$
Bombarding energies $E$ have not been corrected for energy loss in the target

| Projectile | $E(\mathrm{MeV})$ | 142 <br> Ce <br> $P_{\exp }\left(2{ }_{1}^{+}\right) \times 10^{2}$ | $E(\mathrm{MeV})$ | 144 Nd <br> $P_{\exp }\left(2_{1}^{+}\right) \times 10^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{4} \mathrm{He}$ | $10 \cdot 0$ | $0 \cdot 2494(27)$ | $10 \cdot 5$ | $0 \cdot 2406(29)$ |
|  | $11 \cdot 0$ | $0 \cdot 4042(28)$ | $11 \cdot 5$ | $0 \cdot 3924(47)$ |
|  | $11 \cdot 2$ | $0 \cdot 4458(40)$ | $11 \cdot 8$ | $0 \cdot 4497(54)$ |
|  | $11 \cdot 4$ | $0 \cdot 4836(43)$ | $12 \cdot 1$ | $0 \cdot 5086(61)$ |
|  | $11 \cdot 6$ | $0 \cdot 5312(47)$ | $12 \cdot 4$ | $0 \cdot 5659(68)$ |
|  | $11 \cdot 8$ | $0 \cdot 5645(53)$ | $12 \cdot 7$ | $0 \cdot 6326(76)$ |
|  | $12 \cdot 0$ | $0 \cdot 6127(53)$ | $13 \cdot 0^{\mathrm{A}}$ | $0 \cdot 6942(83)$ |
|  | $32 \cdot 0$ | $2 \cdot 479(21)$ | $32 \cdot 0$ | $1 \cdot 892(20)$ |
|  | $33 \cdot 0$ | $2 \cdot 893(24)$ | $33 \cdot 0$ | $2 \cdot 233(25)$ |
|  | $34 \cdot 0$ | $3 \cdot 303(26)$ | $34 \cdot 0$ | $2 \cdot 586(30)$ |
|  | $35 \cdot 0$ | $3 \cdot 777(33)$ | $35 \cdot 0$ | $3 \cdot 039(33)$ |
|  | $36 \cdot 0^{\mathrm{A}}$ | $4 \cdot 240(32)$ | $36 \cdot 0$ | $3 \cdot 430(35)$ |
|  | $37 \cdot 0^{\mathrm{A}}$ | $4 \cdot 739(36)$ |  |  |
|  | $45 \cdot 0$ | $4 \cdot 863(40)$ | $44 \cdot 0$ | $3 \cdot 380(44)$ |
|  | $46 \cdot 0$ | $5 \cdot 426(42)$ | $45 \cdot 0$ | $3 \cdot 844(71)$ |
|  | $47 \cdot 0$ | $5 \cdot 998(48)$ | $46 \cdot 0$ | $4 \cdot 294(55)$ |
|  | $48 \cdot 0$ | $6 \cdot 486(57)$ | $47 \cdot 0$ | $4 \cdot 741(62)$ |
|  | $49 \cdot 0$ | $7 \cdot 151(59)$ | $48 \cdot 0$ | $5 \cdot 264(68)$ |
|  | $50 \cdot 0^{\mathrm{A}}$ | $7 \cdot 693(61)$ | $49 \cdot 0$ | $5 \cdot 742(75)$ |

${ }^{\text {A }}$ These energies were deemed to be unsafe and data were not used in Coulomb-excitation analysis.

Table 2. Measured excitation probabilities $P_{\exp }\left(J_{n}^{\pi}\right)$ for ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ for states other than $\mathbf{2}_{1}^{+}$
Bombarding energies $E$ have not been corrected for energy loss in the target

| Target | Projectile | $\begin{gathered} E \\ (\mathrm{MeV}) \end{gathered}$ | $P_{\exp }\left(J_{n}^{\pi}\right) \times 10^{4}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $2_{2}^{+}$ | $2_{3}^{+}$ | $3-$ | $4_{1}^{+}$ |
| ${ }^{142} \mathrm{Ce}$ | ${ }^{4} \mathrm{He}$ | 11.8 | 0.055(28) | 0.25(3) | 0-585(37) | 0.067(28) |
| ${ }^{142} \mathrm{Ce}$ | ${ }^{12} \mathrm{C}$ | $35 \cdot 0$ | 1.04(24) | 1-84(16) | 4.569(28) | 2.57(21) |
| ${ }^{144} \mathrm{Nd}$ | ${ }^{12} \mathrm{C}$ | $32 \cdot 0$ |  |  | 3-0(2) | 0.56(8) |
|  |  | $33 \cdot 0$ |  |  | 3-7(2) | 0.94(12) |
|  |  | $34 \cdot 0$ |  |  | 5-2(3) | 1.02(12) |
|  |  | 35.0 |  |  | 6.1(3) | 1.66(18) |
|  |  | $36 \cdot 0$ |  |  | 9•1(4) | 1.76(17) |
| ${ }^{144} \mathrm{Nd}$ | ${ }^{16} \mathrm{O}$ | $46 \cdot 0$ |  |  | 7-2(5) |  |
|  |  | $47 \cdot 0$ |  |  | 10.1(5) | 3-1(4) |
|  |  | $48 \cdot 0$ |  |  | $12 \cdot 7(7)$ | 4-7(4) |
|  |  | $49 \cdot 0$ |  |  | 14.2(8) | 4.9(4) |

where $Z_{1}, A_{1}$ and $Z_{2}, A_{2}$ are the atomic numbers and masses of projectile and target respectively, $\theta_{\text {c.m. }}$ is the scattering angle in the centre-of-mass system, $E$ is the laboratory bombarding energy in MeV , and the nuclear radius is taken to be $1 \cdot 25 A^{1 / 3} \mathrm{fm}$. The values of $P_{\text {Coul }}$ were calculated using matrix elements ultimately determined in the present work. Energies deemed to be unsafe were $36-$ and $37-\mathrm{MeV}{ }^{12} \mathrm{C}$ and $50-\mathrm{MeV}{ }^{16} \mathrm{O}$ for ${ }^{142} \mathrm{Ce}$, and $13-\mathrm{MeV}{ }^{4} \mathrm{He}$ for


Fig. 6. Safe-energy plots for excitation of the $2_{1}^{+}$states of ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ by ${ }^{4} \mathrm{He},{ }^{12} \mathrm{C}$ and ${ }^{16} \mathrm{O}$.
${ }^{144} \mathrm{Nd}$; the corresponding data were not included in the Coulomb-excitation analysis used to determine matrix elements. We have assumed that energies found to be safe for the $2_{1}^{+}$state would also be safe for the $2_{2}^{+}, 2_{3}^{+}, 3_{1}^{-}$and $4_{1}^{+}$ states. Although this is not necessarily true in principle (Spear et al. 1978), it is a reasonable assumption at the level of precision corresponding to the statistical quality of the data obtained for the higher states.

Values of $P_{\text {exp }}$ obtained at bombarding energies found to be free from nuclear interference were fitted using theoretical excitation probabilities calculated with the Winther-de Boer (1966) semi-classical Coulomb excitation code. For both ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$, the $2_{1}^{+}, 4_{1}^{+}, 2_{2}^{+}, 3_{1}^{-}$and $2_{3}^{+}$excited states were included in the analysis. For ${ }^{142} \mathrm{Ce}$, virtually no information existed on pertinent matrix elements; therefore sufficient data were obtained to determine the following quantities:

$$
\begin{gathered}
B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right), \quad Q\left(2_{1}^{+}\right), \quad B\left(\mathrm{E} 2 ; 4_{1}^{+} \rightarrow 2_{1}^{+}\right), \quad B\left(\mathrm{E} 2 ; 2_{2}^{+} \rightarrow 2_{1}^{+}\right), \quad B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{2}^{+}\right), \\
B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{3}^{+}\right), \quad B\left(\mathrm{E} ; 0_{1}^{+} \rightarrow 3_{1}^{-}\right) \quad \text { and } \quad B\left(\mathrm{E} 4 ; 0_{1}^{+} \rightarrow 4_{1}^{+}\right) .
\end{gathered}
$$

For ${ }^{144} \mathrm{Nd}$, it was assumed on the basis of published information (Metzger 1969; Fahlander et al. 1980; Snelling and Hamilton 1983) that the matrix elements

$$
\left\langle 0_{1}^{+}\|M(\mathrm{E} 2)\| 2_{2}^{+}\right\rangle, \quad\left\langle 2_{1}^{+}\|M(\mathrm{E} 2)\| 2_{2}^{+}\right\rangle, \quad\left\langle 0_{1}^{+}\|M(\mathrm{E} 2)\| 2_{3}^{+}\right\rangle \quad \text { and } \quad\left\langle 2_{1}^{+}\|M(\mathrm{E} 2)\| 2_{3}^{+}\right\rangle
$$

Table 3. Transition strengths and $Q\left(2_{1}^{+}\right)$values obtained in the present work

|  | ${ }^{142} \mathrm{Ce}$ | ${ }^{144} \mathrm{Nd}$ |
| :---: | :---: | :---: |
| $Q\left(2_{1}^{+}\right)(e b)$ | $-0.16(5)^{\text {A }}$ | $-0 \cdot 15(6)^{\text {A,B }}$ |
| $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)\left(e^{2} \mathrm{~b}^{2}\right)$ | $-0.479(4)^{\text {A }}$ | $-0.491(4)^{\text {A,B }}$ |
| $B\left(\mathrm{E} 2 ; 4_{1}^{+} \rightarrow 2_{1}^{+}\right)\left(e^{2} \mathrm{~b}^{2}\right)$ | $0 \cdot 117(10)$ | 0.100(9) |
| $B\left(\mathrm{E} 2 ; 2_{2}^{+} \rightarrow 2_{1}^{+}\right)\left(e^{2} \mathrm{~b}^{2}\right)$ | $0 \cdot 162(37)$ |  |
| $B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{2}^{+}\right)\left(e^{2} \mathrm{~b}^{2}\right)$ | <0.008 |  |
| $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{3}^{+}\right)\left(e^{2} \mathrm{~b}^{2}\right)$ | 0.070(11) |  |
| $B\left(\mathrm{E} 2 ; 2_{3}^{+} \rightarrow 2_{1}^{+}\right)\left(e^{2} \mathrm{~b}^{2}\right)$ | $0.033(11)^{\text {C }}$ |  |
| $B\left(\mathrm{M1} ; 2_{3}^{+} \rightarrow 2_{1}^{+}\right)\left(\mu_{N}^{2}\right)$ | $0 \cdot 26(5)^{\text {C }}$ |  |
| $B\left(E 3 ; 0_{1}^{+} \rightarrow 3_{1}^{-}\right)\left(e^{2} \mathrm{~b}^{3}\right)$ | 0.202(13) | 0-263(10) |
| $B\left(E 4 ; 0_{1}^{+} \rightarrow 4_{1}^{+}\right)\left(e^{2} \mathrm{~b}^{4}\right)$ | <0.036 | 0-263(10) |

A These values assume $P_{4}\left(2_{3}^{+}\right)>0$; see Table 4 for alternative values.
${ }^{\mathrm{B}}$ These values assume $P_{4}\left(2_{2}^{+}\right)<0$; see Table 4 for alternative values.
${ }^{C}$ Calculated using measured value of $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{3}^{+}\right)$and the branching and mixing ratios listed by Peker (1984).

Table 4. Values of $Q\left(2_{1}^{+}\right)$and $B\left(E 2 ; \mathbf{0}_{\mathbf{1}}^{+} \rightarrow \mathbf{2}_{1}^{+}\right)$obtained for various possible combinations of the signs of $\boldsymbol{P}_{\mathbf{4}}\left(\mathbf{2}_{3}^{+}\right)$and $\boldsymbol{P}_{\mathbf{4}}\left(\mathbf{2}_{2}^{+}\right)$
It is assumed that $P_{4}\left(2_{2}^{+}\right)=0$ for ${ }^{142} \mathrm{Ce}$

| $\begin{gathered} \text { Sign } \\ P_{4}\left(2_{3}^{+}\right) \end{gathered}$ | $\begin{gathered} \text { Sign } \\ P_{4}\left(2_{2}^{+}\right) \end{gathered}$ | ${ }^{142} \mathrm{Ce}$ |  | ${ }^{144} \mathrm{Nd}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\underset{\underset{\left(\mathrm{E} 2 ; \mathrm{O}^{+} \rightarrow 2^{2}\right.}{\left(\mathrm{e}^{2}\right)}}{ }$ | $\underset{(e b)}{Q\left(2_{1}^{+}\right)}$ | $\begin{gathered} B\left(\mathrm{E} 2 ; 0_{0}^{+} \rightarrow 2_{1}^{+}\right) \\ \left(e^{2} \mathrm{~b}^{2}\right) \end{gathered}$ | $\underset{(e b)}{Q\left(2_{1}^{+}\right)}$ |
| + | - | 0.479(4) | -0.16(5) | 0.491(4) | -0.15(6) |
| - | - | 0.482(4) | -0.37(5) | 0-492(4) | -0.28(6) |
| + | + | $0.479(4)$ | -0.16(5) | 0.491(4) | -0.05(6) |
| - | + | 0.482(4) | -0.37(5) | 0-492(4) | -0.19(6) |

Table 5. Effects of various small corrections and uncertainties in the determination of $Q\left(2_{1}^{+}\right)$and $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)$for ${ }^{142} \mathrm{Ce}$

The values are very similar for ${ }^{144} \mathrm{Nd}$

| Effect | $\Delta B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)\left(e^{2} \mathrm{~b}^{2}\right)$ | $\Delta Q\left(2_{1}^{+}\right)(e \mathrm{~b})$ |
| :--- | :---: | :---: |
| Electron screening | -0.004 | +0.004 |
| Vacuum polarisation | +0.007 | -0.011 |
| Nuclear polarisation | -0.0003 | -0.032 |
| Quantal correction | +0.002 | -0.022 |
| GDR correction | -0.0001 | +0.048 |
| Uncertainties in 2 | $\pm 0.0004$ | $\pm 0.025$ |
| Uncertainty in beam energy | $\pm 0.0024$ | $\pm 0.003$ |

had the magnitudes $0.055(6), 0.69(10), 0.25(3)$ and $0.34(11)$ eb, respectively. The usual small corrections (Fewell et al. 1979) were applied for electron screening, vacuum polarisation, nuclear polarisation, effects of target thickness, El interference from the giant-dipole resonance (GDR), and use of the semiclassical approximation ('quantal correction').

The results obtained are summarised in Table 3. The value obtained for $Q\left(2_{1}^{+}\right)$and, to a lesser extent, for $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)$, is sensitive to interference
terms from higher states. The sign of the interference term involving the state $J_{n}^{\pi}$ depends on the sign of $P_{4}\left(J_{n}^{\pi}\right)$, where $P_{4}\left(U_{n}^{\pi}\right)$ is defined by

$$
\begin{equation*}
P_{4}\left(J_{n}^{\pi}\right)=\left\langle 0_{1}^{+}\|M(\mathrm{E} 2)\| 2_{1}^{+}\right\rangle\left\langle 2_{1}^{+}\|M(\mathrm{E} 2)\| 2_{1}^{+}\right\rangle\left\langle 0_{1}^{+}\|M(\mathrm{E} 2)\| J_{n}^{\pi}\right\rangle\left\langle 2_{1}^{+}\|M(\mathrm{E} 2)\| J_{n}^{\pi}\right\rangle . \tag{4}
\end{equation*}
$$

For most nuclei the $2_{2}^{+}$state produces the major interference effect. However, in the present case $\left\langle 0_{1}^{+}\|M(E 2)\| 2_{2}^{+}\right\rangle$is very small and the largest effect comes from the $2_{3}^{+}$state. Results obtained for alternative signs of $P_{4}\left(2_{2}^{+}\right)$and $P_{4}\left(2_{3}^{+}\right)$ are given in Table 4. It is expected on rather general theoretical grounds (Kumar 1969) that $P_{4}\left(2_{2}^{+}\right)<0$; the discussion presented later in this paper will be based on this assumption, but would not be significantly affected if $P_{4}\left(2_{2}^{+}\right)$ were taken to be positive. A list of the effects of various small corrections and uncertainties in the determination of $Q\left(2_{1}^{+}\right)$and $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)$is given in Table 5.

In order to visualise the influence of each set of data on the determination of $Q\left(2_{1}^{+}\right)$and $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)$, an approximate expression for the excitation probability $P$ of the form

$$
\begin{equation*}
P=f B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)\left\{1+\rho Q\left(2_{1}^{+}\right)\right\} \tag{5}
\end{equation*}
$$

is useful. The quantities $\rho$ (the sensitivity parameter) and $f$ are calculated from the Winther-de Boer program. Fig. 7 shows plots of $P_{\exp } / f$ as a function of $\rho$. The fits to the data are represented by straight lines with intercepts on the vertical axis equal to $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)$and slopes of $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right) Q\left(2_{1}^{+}\right)$.

## 4. Comparison of Present Results with Previous Work

(a) Quadrupole Moments $Q\left(2_{1}^{+}\right)$
${ }^{142} \mathrm{Ce}$. For ${ }^{142} \mathrm{Ce}$ we obtain $Q\left(2_{1}^{+}\right)=-0 \cdot 16(5) \mathrm{eb}$, assuming $P_{4}\left(2_{2}^{+}\right)=0$ and $P_{4}\left(2_{3}^{+}\right)>0$. The only previous experimental value is $-0 \cdot 12(9) \mathrm{eb}$, obtained by Engler (1970) using Coulomb excitation by ${ }^{16} \mathrm{O}$ projectiles. The present result is superior because of better statistical accuracy and a better knowledge of higher state matrix elements and various small corrections. Engler took into account only the $2_{2}^{+}$level when considering the effects of higher states, and used calculated values for $\left\langle 2_{1}^{+}\|M(E 2)\| 2_{2}^{+}\right\rangle$and $\left\langle 0_{1}^{+}\|M(E 2)\| 2_{2}^{+}\right\rangle$which differ substantially from the values measured in the present work.
${ }^{144} \mathrm{Nd}$. For ${ }^{144} \mathrm{Nd}$ we obtain $Q\left(2_{1}^{+}\right)=-0 \cdot 15(6) \mathrm{eb}$, assuming $P_{4}\left(2_{2}^{+}\right)<0$ and $P_{4}\left(2_{3}^{+}\right)>0$. Crowley et al. (1971) obtained $Q\left(2_{1}^{+}\right)=-0 \cdot 39(21)$ eb from Coulomb excitation with ${ }^{16} \mathrm{O}$ projectiles. However, when results of $\gamma$ ray yield measurements were combined with their particle data, they obtained $-0.07(15) \mathrm{eb}$. Furthermore, they were unable, because of insufficient experimental information, to allow for interference from the $2_{2}^{+}$and $2_{3}^{+}$states.
(b) Values of $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)$
${ }^{142} \mathrm{Ce}$. The present value of $B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)=0 \cdot 479(4) e^{2} \mathrm{~b}^{2}$ is in satisfactory agreement with previous values listed by Raman et al. (1987). The data are plotted in chronological order of publication in Fig. 8.
${ }^{144} \mathrm{Nd}$. Our value of $B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)=0.491(4) e^{2} \mathrm{~b}^{2}$ is in significant disagreement with the value $0.58(1) e^{2} \mathrm{~b}^{2}$ reported recently by Ahmad et al. (1988)


Fig. 7. Plots of $P_{\exp } / f$ against the sensitivity parameter $\rho$ : (a) ${ }^{144} \mathrm{Nd}$ with $Q\left(2_{1}^{+}\right)=-0 \cdot 15(6) \mathrm{eb}$ and $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)=0 \cdot 491(4) e^{2} \mathrm{~b}^{2}$ and $(b){ }^{142} \mathrm{Ce}$ with $Q\left(2_{1}^{+}\right)=-0 \cdot 16(5)$ eb and $B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)=$ $0.479(4) e^{2} \mathrm{~b}^{2}$. It is assumed that $P_{4}\left(2_{3}^{+}\right)>0$ for both nuclei, and that $P_{4}\left(2_{2}^{+}\right)<0$ for ${ }^{144} \mathrm{Nd}$.
from Coulomb excitation with ${ }^{4} \mathrm{He}$ projectiles. It is, however, in excellent agreement with the four most recent determinations prior to that of Ahmad et al. (Eccleshall et al. 1966; Burginyon et al. 1967; Crowley et al. 1971; Fahlander et al. 1980); the weighted mean of these four is $0.499(17) e^{2} \mathrm{~b}^{2}$. All the data listed by Raman et al. (1987) are plotted, together with the present value, in Fig. 8.

## (c) Values of $B\left(E 2 ; 4_{1}^{+} \rightarrow 2_{1}^{+}\right)$

We are not aware of any previous measurement of this quantity for ${ }^{142} \mathrm{Ce}$. For ${ }^{144} \mathrm{Nd}$ our value of $0 \cdot 100(9) e^{2} \mathrm{~b}^{2}$ is in good agreement with the value of $0 \cdot 86(16) e^{2} b^{2}$ reported by Fahlander et al. (1980) from Coulomb-excitation measurements.


Fig. 8. Previous values of $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)$for ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ plotted in chronological order of publication, together with that of the present work (PW). The results shown for ${ }^{142} \mathrm{Ce}$ are from Coulomb excitation (Andreev et al. 1961; Eccleshall et al. 1966; Engler 1970; present work) and from inelastic electron scattering (Pitthan 1973). Those for ${ }^{144} \mathrm{Nd}$, all from Coulomb excitation, are from Lemberg (1960), Nathan and Popov (1960), Ecceshall et al. (1966), Burginyon et al. (1967), Crowley et al. (1971), Fahlander et al. (1980), and Ahmad et al. (1988).

Table 6. Comparison of experimental values of transition strengths, quadrupole moments $Q\left(2_{1}^{+}\right)$, and mixing ratios $\delta\left(2_{3}^{+} \rightarrow 2_{1}^{+}\right)$for ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ with the predictions of IBM-2 calculations
Unless indicated otherwise, experimental values are from the present work (Table 3)

|  | Theory <br> IBM-2 $\mathrm{U}(5)$ | ${ }^{142} \mathrm{Ce}$ | Experiment |
| :--- | :---: | :---: | :---: |
| $Q\left(2_{1}^{+}\right)(e b)$ | $-0 \cdot 11$ | $-0 \cdot 16(5)$ | $-0 \cdot 15(6)$ |
| $B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)\left(e^{2} \mathrm{~b}^{2}\right)$ | $0 \cdot 52$ | $0 \cdot 479(4)$ | $0 \cdot 491(4)$ |
| $B\left(\mathrm{E} 2 ; 4_{1}^{+} \rightarrow 2_{1}^{+}\right)\left(e^{2} \mathrm{~b}^{2}\right)$ | $0 \cdot 17$ | $0 \cdot 117(10)$ | $0 \cdot 100(9)$ |
| $B\left(\mathrm{E} 2 ; 2_{2}^{+} \rightarrow 2_{1}^{+}\right)\left(e^{2} \mathrm{~b}^{2}\right)$ | $0 \cdot 17$ | $0 \cdot 16(4)$ | $0 \cdot 095(30)^{\mathrm{B}}$ |
| $B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{2}^{+}\right)\left(e^{2} \mathrm{~b}^{2}\right)$ | 0 | $<0 \cdot 008$ | $0 \cdot 0030(6)^{\mathrm{B}}$ |
| $B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{3}^{+}\right)\left(e^{2} \mathrm{~b}^{2}\right)$ | $0 \cdot 058$ | $0 \cdot 070(11)$ | $0 \cdot 065(16)^{\mathrm{C}} \mathrm{C}$ |
| $B\left(\mathrm{E} 2 ; 2_{3}^{+} \rightarrow 2_{1}^{+}\right)\left(e^{2} \mathrm{~b}^{2}\right)$ | $0 \cdot 016$ | $0 \cdot 033(11)$ | $0 \cdot 023(15)^{\mathrm{C}, \mathrm{D}}$ |
| $B\left(\mathrm{M} 1 ; 2_{3}^{+} \rightarrow 2_{1}^{+}\right)\left(\mu_{\mathrm{N}}^{2}\right)$ | $0 \cdot 23$ | $0 \cdot 26(5)$ | $0 \cdot 15(4)^{\mathrm{C}, \mathrm{D}}$ |
| $\delta\left(2_{3}^{+} \rightarrow 2_{1}^{+}\right)$ | $+0 \cdot 30$ | $+0 \cdot 41(7)^{\mathrm{E}}$ | $+0 \cdot 31(11)^{\mathrm{D}}$ |

[^0](d) Values of $\mathrm{B}\left(\mathrm{E3} ; \mathrm{O}_{1}^{+} \rightarrow 3_{1}^{-}\right)$

For both ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ the only previous data are from the Coulombexcitation experiment of Hansen and Nathan (1963). It is well known that the results of that experiment are unreliable because of problems with nuclear interference (Spear 1989). The present values correspond to E3 transition strengths of $24 \cdot 1(16)$ and $30 \cdot 5(12)$ W.u. for ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ respectively. These enhancements are similar to those of other nuclei in this mass region (Spear 1989).

## 5. Discussion

## (a) Comparison of Results with IBM-2 Predictions

Table 6 presents a comparison between experimental results for ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ and predictions based on the calculations of Hamilton et al. (1984) made using the $\mathrm{U}(5)$ limit of IBM-2 with boson effective charges $e_{\nu}=0.24 \mathrm{eb}$ and $e_{\pi}=0.12 \mathrm{eb}$, structure parameters $\chi_{\nu}=-1.3$ and $\chi_{\pi}=0$, and boson numbers $N_{\nu}=1, N_{\pi}=4$. This parameter set gives identical predictions for ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ because it uses $N_{\pi}=4$ for both nuclei. The best value to use for $N_{\pi}$ is uncertain because of the possible $Z=64$ shell closure (see e.g. Casten 1985; Wolf and Casten 1987). The calculations assume that the $2_{1}^{+}$level is a pure one d-boson symmetric state, the $2_{2}^{+}$and $4_{1}^{+}$levels are pure two d-boson symmetric states, and the $2_{3}^{+}$level is a pure one d-boson mixed-symmetry state.

The overall agreement between theory and experiment is excellent; the experimental values for the two nuclei are very similar, and the magnitudes agree very well with the theoretical predictions. Of particular significance is the agreement for the proposed mixed-symmetry state $\left(2_{3}^{+}\right)$. For both nuclei the results fulfil the requirements for a mixed-symmetry state: (i) There is a large M1 strength to the lowest fully symmetric state $\left[B\left(M 1 ; 2_{3}^{+} \rightarrow 2_{1}^{+}\right)=0 \cdot 15(3)\right.$ W.u. and $0 \cdot 08(2) \mathrm{W} . \mathrm{u}$. for ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ respectively]; these values are considerably larger than typical M1 strengths for nuclei in this mass region (Endt 1981). (ii) The values of $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{3}^{+}\right)$are moderately large $[3 \cdot 2(5)$ and $2 \cdot 9(7)$ W.u. respectively] and agree with Iachello's estimate of about 3 W.u.

Detailed IBM-2 calculations by Robinson et al. (1988) suggested that the properties of a mixed-symmetry state in ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ are shared between the $2_{2}^{+}$and $2_{3}^{+}$levels, i.e. that both levels are highly mixed combinations of one d-boson mixed-symmetry and two d-boson symmetric configurations. The present results do not support this view: in both nuclei the properties of the $2_{3}^{+}$level are exactly those expected for a pure one d-boson mixed symmetry state, and the $2_{2}^{+}$level has the properties of a two d-boson symmetric state [the very small value of $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{2}^{+}\right)$precludes a substantial mixed-symmetry component in the $2_{2}^{+}$level]. The situation in ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ appears to be in strong contrast with that in two other cases of suggested symmetry mixing in nuclei outside the well-deformed regions of the periodic table: for both the light nucleus ${ }^{56} \mathrm{Fe}$ (Eid et al. 1986) and the O(6) nucleus ${ }^{134} \mathrm{Ba}$ (Molnár et al. 1988), it seems that the properties of a theoretical mixed-symmetry state would have to be shared between two or more experimental levels.

The most controversial aspect of the parameter set used by Hamilton et al. (1984) is the positive value for ( $e_{v}-e_{\pi}$ ), obtained from an analysis of


Fig. 9. A plot of the quantity $\left[N B\left(E 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right) / N_{\pi}^{2}\right]^{1 / 2}$ against $N_{\nu} / N_{\pi}$ for some nuclei in the region of present interest, where $N_{\nu}$ and $N_{\pi}$ are the neutron and proton boson numbers respectively, and $N=N_{\nu}+N_{\pi}$. The straight line is a least-squares fit to the data. Unless specifically indicated, the experimental errors are negligible compared with the size of the data points. This plot is an updated version of Fig. 1 of Hamilton et al. (1984); values obtained in the present work have been used for ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$, and an additional data point has been added (for ${ }^{142} \mathrm{Ba}$ ). Values for ${ }^{138} \mathrm{Ba},{ }^{142} \mathrm{Ba}$, ${ }^{140} \mathrm{Ce},{ }^{144} \mathrm{Nd}$ and ${ }^{146} \mathrm{Nd}$ are taken from the compilation of Raman et al. (1987).
$B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)$values for various vibrational nuclei in the mass region $A=138-46$, using the formula

$$
\begin{equation*}
\left\{N B\left(\mathrm{E} 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right) / N_{\pi}^{2}\right\}^{1 / 2}=e_{\pi}+\left(e_{\nu} N_{\nu} / N_{\pi}\right), \tag{6}
\end{equation*}
$$

where $N=N_{\nu}+N_{\pi}$. A plot of $\left\{N B\left(E 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right) / N_{\pi}^{2}\right\}^{1 / 2}$ against $N_{\nu} / N_{\pi}$ is shown in Fig. 9, which is an updated version of Fig. 1 of Hamilton et al. The straight line is a least-squares fit to the data, corresponding to $e_{\pi}=0.12 \mathrm{eb}$ and $e_{\nu}=0.23 \mathrm{eb}$. These values are in excellent agreement with those obtained by Hamilton et al. ( 0.12 and 0.24 eb respectively). This approach has been criticised by Scholten et al. (1986) on the grounds that there may be a strong mass dependence of the effective charges; the application of equation (6) assumed constant effective charges. Puddu et al. (1980) chose $e_{\nu}=e_{\pi}=0 \cdot 12 \mathrm{eb}$ in their study of $\mathrm{Xe}, \mathrm{Ba}$ and Ce isotopes with $N<82$. Similarly, Robinson et al. (1988) reported that the choice of $e_{\nu}=e_{\pi}=0.12 \mathrm{eb}$ gave the optimum reproduction of experimental $B(E 2)$ values in ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$. However, it is evident from equation (1) that equal values for $e_{v}$ and $e_{\pi}$ imply zero E2 strength between the ground state and the lowest $2^{+}$mixed-symmetry state, which is inconsistent with the observation that the strength of this transition
is about 3 W.u. From equations (1) and (2) of Hamilton et al. (1984), it is seen that $\left(e_{\nu}-e_{\pi}\right)>0$ fixes $P_{4}\left(2_{3}^{+}\right)>0$, and theoretical predictions are obtained which are in excellent agreement with experiment (Table 6). In particular, the experimental values of $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{3}^{+}\right)$imply (equation 1) values for $\left|e_{\nu}-e_{\pi}\right|$ of $0 \cdot 132(10)$ and $0 \cdot 127(16) \mathrm{eb}$ for ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ respectively, in excellent agreement with the value of $\left(e_{\nu}-e_{\pi}\right)=0 \cdot 12 \mathrm{eb}$ obtained by Hamilton et al.

Table 7. IBM-2 parameters required to fit the data of Tables 4 and 6 for ( $e_{v}-e_{\pi}$ ) $>0$, i.e. $P_{4}\left(2_{3}^{+}\right)>0$, and for $\left(e_{v}-e_{\pi}\right)<0$, i.e. $P_{4}\left(2_{3}^{+}\right)<0$

|  | ${ }^{142} \mathrm{Ce}$ |  | ${ }^{144} \mathrm{Nd}$ |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $P_{4}\left(2_{3}^{+}\right)>0$ | $P_{4}\left(2_{3}^{+}\right)<0$ | $P_{4}\left(2_{3}^{+}\right)>0$ | $P_{4}\left(2_{3}^{+}\right)<0$ |
| $e_{\nu}(e \mathrm{~b})$ | $0 \cdot 244(8)$ | $0 \cdot 033(8)$ | $0 \cdot 243(12)$ | $0 \cdot 038(12)$ |
| $e_{\pi}(e b)$ | $0 \cdot 112(2)$ | $0 \cdot 165(2)$ | $0 \cdot 115(4)$ | $0 \cdot 166(4)$ |
| $X_{\nu}$ | $-1 \cdot 9(3)$ | $-18(5)$ | $-1 \cdot 6(6)$ | $-12(6)$ |
| $X_{\pi}$ | $-0 \cdot 03(30)$ | $-0 \cdot 77(20)$ | $-0 \cdot 1(5)$ | $-0 \cdot 54(29)$ |

It is possible to test the sign of ( $e_{\nu}-e_{\pi}$ ) from its indirect effect on the value of $Q\left(2_{1}^{+}\right)$. Referring again to equations (1) and (2) of Hamilton et al., we have $\left\langle 0_{1}^{+}\|M(E 2)\| 2_{1}^{+}\right\rangle=(5 / N)^{1 / 2}\left(e_{\pi} N_{\pi}+e_{\nu} N_{\nu}\right)$ which must be positive, $\left\langle 2_{1}^{+}\|M(E 2)\| 2_{1}^{+}\right\rangle$must be negative since the experimental value of $Q\left(2_{1}^{+}\right)$is negative, and $\left\langle 2_{1}^{+}\|M(E 2)\| 2_{3}^{+}\right\rangle$must be negative since the experimental value of the mixing ratio $\delta\left(2_{3}^{+} \rightarrow 2_{1}^{+}\right)$is positive (Table 6) and an analysis of $g$-factors in this mass region by Sambataro et al. (1984) concluded that $g_{\mathrm{p}} \sim 1$ and $g_{\mathrm{n}} \sim 0$, so that $\left\langle 2_{1}^{+}\|M(\mathrm{M} 1)\| 2_{3}^{+}\right\rangle$is negative. Therefore, since $\left\langle 0_{1}^{+}\|M(\mathrm{E} 2)\| 2_{3}^{+}\right\rangle=\left(5 N_{\nu} N_{\pi} / N\right)^{1 / 2}\left(e_{\nu}-e_{\pi}\right)$, a negative value for ( $e_{\nu}-e_{\pi}$ ) would result in $P_{4}\left(2_{3}^{+}\right)<0$. This would give $Q\left(2_{1}^{+}\right)=-0 \cdot 37(5)$ and $-0 \cdot 28(6)$ for ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ respectively (Table 4). These values, when combined with other data (Table 6), would require an unacceptable set of IBM-2 parameters. This is shown in Table 7, which gives values of parameters obtained when the data of Table 6 are used in the expressions for $\left\langle 0_{1}^{+}\|M(\mathrm{E} 2)\| 2_{1}^{+}\right\rangle,\left\langle 2_{1}^{+}\|M(\mathrm{E} 2)\| 2_{1}^{+}\right\rangle$, $\left\langle 0_{1}^{+}\|M(E 2)\| 2_{3}^{+}\right\rangle$and $\left\langle 2_{1}^{+}\|M(E 2)\| 2_{3}^{+}\right\rangle$given by Hamilton et al. (1984), assuming the values of $Q\left(2_{1}^{+}\right)$given in Table 4 for the alternative signs of $P_{4}\left(2_{3}^{+}\right)$[it is assumed that $P_{4}\left(2_{2}^{+}\right)<0$ but the alternative choice of $P_{4}\left(2_{2}^{+}\right)>0$ would make no significant difference to the argument]. Since both ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ lie at the start of a neutron shell and in the middle of a proton shell, it would be expected (Puddu et al. 1980) that $\chi_{\nu} \sim-1$ and $\chi_{\pi} \sim 0$. It is seen that for both nuclei the choice of $P_{4}\left(2_{3}^{+}\right)<0$, arising from the assumption that $\left(e_{v}-e_{\pi}\right)<0$, gives unrealistically large magnitudes for $\chi_{v}$. On the other hand, the choice of $P_{4}\left(2_{3}^{+}\right)>0$, i.e. $\left(e_{v}-e_{\pi}\right)>0$, gives values for both $\chi_{\nu}$ and $\chi_{\pi}$ which are in accord with expectation. Even if $P_{4}\left(2_{2}^{+}\right)$were chosen to be positive for ${ }^{144} \mathrm{Nd}$ (it is zero for ${ }^{142} \mathrm{Ce}$ ), a fit to the data including the corresponding value of $Q\left(2_{1}^{+}\right)=-0 \cdot 19(6) e b$ (Table 4) would give $\chi_{v}=-11(6)$. Thus, the measured values of $Q\left(2_{1}^{+}\right)$strongly support the positive sign adopted for ( $e_{\nu}-e_{\pi}$ ) by Hamilton et al.

It is conceivable that solutions with $e_{\pi}$ greater than $e_{\nu}$ could be found by using different values of $N_{\nu}$ or $N_{\pi}$, or by using the full IBM-2 Hamiltonian instead of the strict $U(5)$ limit. However, the relatively simple approach of Hamilton et al. gives an excellent description of these nuclei.

In commenting on their surprising conclusion that $e_{\nu}>e_{\pi}$, Hamilton et al. made two points: (a) the effective-charge parameters involve a (length) ${ }^{2}$ factor and the neutrons are filling higher shells than the protons, and (b) a large effective neutron charge would be expected in this region just before the onset of deformation at $N=88$. It is interesting to note that the shell-model calculations of Heyde and Sau (1986) indicated that in the $N=84$ mass region it is possible to obtain values of $e_{\nu}$ greater than $e_{\pi}$ for $N_{\nu}=1$ and $N_{\pi} z 4$ (see Fig. 9 of their paper).

## (b) Comparison of Results with Other Calculations

The main emphasis of this paper is the comparison of measured electromagnetic matrix elements in ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ with IBM-2 predictions, with particular reference to mixed-symmetry states. However, some of these matrix elements have also been calculated with other nuclear models. A brief discussion of these calculations is given in this section.

Vanden Berghe (1975) applied the two-particle core-coupling model to the $N=84$ isotones. Using an effective neutron charge $e_{\mathrm{n}}^{\text {eff }}=0.5 e$, he obtained $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)=0.319 e^{2} \mathrm{~b}^{2}$, which is much smaller than the experimental values (Table 6). Using $e_{\mathrm{n}}^{\text {eff }}=e$, he obtained $0.428 e^{2} \mathrm{~b}^{2}$, which is in much better agreement with experiment. The calculated values for $Q\left(2_{1}^{+}\right)$were -0.23 eb ( $e_{\mathrm{n}}^{\text {eff }}=0.5 e$ ) and $-0.32 \mathrm{eb}\left(e_{\mathrm{n}}^{\text {eff }}=e\right)$, which fall within the range of experimental values covered by the alternative signs for $P_{4}\left(2_{3}^{+}\right)$(see Table 4). However, the calculation is very sensitive to the relative amounts of $2 f_{7 / 2}$ and $3 p_{3 / 2}$ configurations assumed for the two extra-core neutrons; an earlier calculation by Heyde and Brussaard (1967) assumed a pure $2 \mathrm{f}_{7 / 2}$ configuration and gave the wrong sign for $Q\left(2_{1}^{+}\right)$.
Table 8. Comparison between experimental values of transition strengths (from Table 6) and values calculated by Faessler and Nojarov (1986) using an extension of the vibrational model with large ( L ) and small ( S ) bases of single-particle states

|  |  | $B\left(\mathrm{Ml} ; 2_{3}^{+} \rightarrow 2_{1}^{+}\right)$ <br> $\left(10^{-3} \mu_{\mathrm{N}}^{2}\right)$ | $B\left(\mathrm{E} 2 ; 2_{3}^{+} \rightarrow 2_{1}^{+}\right)$ <br> $\left(10^{-4} e^{2} \mathrm{~b}^{2}\right)$ | $B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{3}^{+}\right)$ <br> $\left(e^{2} \mathrm{~b}^{2}\right)$ |
| :--- | :--- | :---: | :---: | :---: |
| ${ }^{142} \mathrm{Ce}$ | Calc. (L) | 11 | $0 \cdot 46$ | $0 \cdot 14$ |
|  | Calc. (S) | 108 | 44 | $0 \cdot 16$ |
| 144 Nd | Expt | $260(50)$ | $330(110)$ | $0 \cdot 070(11)$ |
|  | Calc. (L) | 9 | $0 \cdot 51$ | $0 \cdot 15$ |
|  | Calc. (S) | 128 | 38 | $0 \cdot 19$ |
|  | Expt | $150(40)$ | $230(150)$ | $0 \cdot 065(16)$ |

Table 9. Comparison between $B(E 2)$ values ( $e^{2} b^{2}$ ) calculated for ${ }^{144} \mathrm{Nd} b y$ Gupta (1988), using the pairing-plus-quadrupole model, and experimental values (from Table 6)

| Quantity | Theory <br> (Gupta 1988) | Experiment <br> (Table 6) |
| :--- | :---: | :---: |
| $B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)$ | $0 \cdot 71$ | $0 \cdot 491(4)$ |
| $B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{2}^{+}\right)$ | 0.010 | $0 \cdot 0030(6)$ |
| $B\left(\mathrm{E} 2 ; 0_{1}^{+} \rightarrow 2_{3}^{+}\right)$ | 0.0004 | $0 \cdot 065(16)$ |
| $B\left(\mathrm{E} 2 ; 2_{2}^{+} \rightarrow 2_{1}^{+}\right)$ | $0 \cdot 140$ | $0 \cdot 095(30)$ |
| $B\left(\mathrm{E} 2 ; 4_{1}^{+} \rightarrow 2_{1}^{+}\right)$ | $0 \cdot .27$ | $0 \cdot 100(9)$ |

Faessler and Nojarov (1986) interpreted the $2^{+}$mixed-symmetry state as an isovector quadrupole vibration (involving separate quadrupole vibrations of the protons and neutrons). Their calculated values of transition strengths are compared with experiment in Table 8. The 'large basis' calculations include all proton states from the oscillator shells $\mathcal{N} \leq 4$ and all neutron states with $\mathcal{N} \leq 5$; the 'small basis' calculations are restricted to states of the $\mathcal{N}=4$ proton shell and the $\mathcal{N}=5$ neutron shell. The calculated values of $B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{3}^{+}\right)$are 2 to 3 times larger than experiment, which may be due to the neglect of mixing with the giant isovector quadrupole resonance. Calculated values for $B\left(\mathrm{E} 2 ; 2_{3}^{+} \rightarrow 2_{1}^{+}\right)$and $B\left(\mathrm{M} 1 ; 2_{3}^{+} \rightarrow 2_{1}^{+}\right)$are smaller than experiment, some by very large amounts; however, the small-basis values are much closer to experiment than are those calculated with the large basis.

Gupta (1988) performed extensive calculations for ${ }^{144} \mathrm{Nd}$ using the pairing-plus-quadrupole model. He obtained $Q\left(2_{1}^{+}\right)=-0.38 \mathrm{eb}$, which is in good agreement with the experimental value for $P_{4}\left(2_{3}^{+}\right)<0$, but not with that for $P_{4}\left(2_{3}^{+}\right)>0$ (see Table 4). His calculated $B(E 2)$ values are compared with experiment in Table 9; the agreement is not good.

## 6. Summary

Quadrupole moments $Q\left(2_{1}^{+}\right)$and various other electromagnetic matrix elements have been measured for the $N=84$ nuclei ${ }^{142} \mathrm{Ce}$ and ${ }^{144} \mathrm{Nd}$ using Coulomb excitation. When taken in conjunction with other experimental information, the results show that these two nuclei have very similar properties, and that these properties are in excellent agreement with predictions made by Hamilton et al. (1984) using the $U(5)$ limit of IBM-2. The results support the parameter set that they use, including the controversial assumption of a larger effective charge for neutron bosons than for proton bosons. The observed properties of the $2_{3}^{+}$state in both nuclei agree with those expected for a pure one d-boson mixed-symmetry state.

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