

## Static and Dynamic Moments of the ${}^7\text{Li}$ Nucleus

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### Abstract

The data of Weller *et al.* (1985) on the tensor analysing powers for elastic and inelastic Coulomb scattering of aligned  ${}^7\text{Li}$  ions have been reanalysed in order to obtain information on the values of the four  ${}^7\text{Li}$  moments  $Q$ ,  $B(E2)\uparrow$ ,  $\tau_{11}$  and  $\tau_{12}$ . It is shown that a single set of values, chosen primarily to be consistent with the value of  $Q$  measured by molecular techniques and the values of  $B(E2)\uparrow$  and  $\tau_{12}$  required to fit unpolarised  ${}^7\text{Li}$  data, and also with the theoretical constraint  $\tau_{11} \approx -|\tau_{12}|$ , gives a good fit to the aligned  ${}^7\text{Li}$  data.

### 1. Introduction

In recent years, measured values of the quadrupole moment ( $Q$ ) of the  $\frac{3}{2}^-$  ground state of  ${}^7\text{Li}$  and the reduced E2 transition probability ( $B(E2)\uparrow$ ) to the  $\frac{1}{2}^-$  first excited state have been used to test models describing the low-lying states of  ${}^7\text{Li}$  and  ${}^7\text{Be}$ , or at least to select parameter values in the models (Mertelmeier and Hofmann 1986; Kajino 1986; Altmeyer *et al.* 1988; Buck and Merchant 1988). Such models have been used in the calculation of astrophysical  $S$ -factors for radiative-capture reactions involving these nuclei. Also Kajino *et al.* (1988) suggested a direct relationship between the  $S$ -factor for the reaction  ${}^4\text{He}(t, \gamma){}^7\text{Li}$  and the tensor moments ( $\tau_{11}$  and  $\tau_{12}$ ) of the nuclear polarisability involving the  ${}^7\text{Li}$  ground state and first excited state.

A significant discrepancy exists between recent values of  $Q$  measured by nuclear and molecular means. From fits to their measurements on Coulomb scattering of aligned  ${}^7\text{Li}$  ions, Weller *et al.* (1985) obtained  $Q = -3.70 \pm 0.08 \text{ e fm}^2$ , while analysis of data on the molecule LiH gave  $Q = -4.06 \text{ e fm}^2$  (Sundholm *et al.* 1984) and on LiF gave  $Q = -4.055 \pm 0.080 \text{ e fm}^2$  (Diercksen *et al.* 1988). There are similar but less striking discrepancies between the values obtained by different methods for  $B(E2)\uparrow$  and  $\tau_{12}$ . Weller *et al.* obtained  $B(E2)\uparrow = 8.3 \pm 0.5 \text{ e}^2 \text{ fm}^4$  and  $\tau_{12} = 0.23 \pm 0.06 \text{ fm}^3$ , while analysis of Coulomb excitation data obtained with unpolarised  ${}^7\text{Li}$  gave  $B(E2)\uparrow = 7.59 \pm 0.10 \text{ e}^2 \text{ fm}^4$  and  $\tau_{12} = 0.16 \pm 0.01 \text{ fm}^3$  (Vermeer *et al.* 1989). Weller *et al.* also gave  $\tau_{11} = 0.23 \pm 0.06 \text{ fm}^3$ ; this quantity is not obtainable from unpolarised  ${}^7\text{Li}$  data.

Kajino *et al.* (1988) criticised the values obtained by Weller *et al.* on the grounds that model calculations require  $\tau_{11}$  to be negative, whereas Weller *et al.* obtained a positive value.† Calculations based both on an  $\alpha+t$  cluster model (Mertelmeier and Hofmann 1986; Kajino *et al.* 1988) and on the shell model (Barker and Woods 1989) predict  $\tau_{11} \approx -|\tau_{12}|$ . The uncertainty given by Weller *et al.* would exclude any negative value of  $\tau_{11}$ . Thus there are problems with the values and uncertainties of the moments given by Weller *et al.* in comparison both with other experimental values and with model expectations.

Kajino *et al.* (1988) reanalysed the data of Weller *et al.* (1985), assuming that the parameters satisfied relations suggested by model calculations‡:

$$B(E2)\uparrow = (25/16\pi)Q^2(1 + \delta), \quad \delta = 0.085, \quad (1a)$$

$$\tau_{12} = \tau_{11}(1 + \eta), \quad \eta = 0.073. \quad (1b)$$

They also fixed  $Q$  at its molecular value,  $Q = -4.06 \text{ e fm}^2$ , giving  $B(E2)\uparrow = 8.90 \text{ e}^2 \text{ fm}^4$ , so that only one adjustable parameter remained. Then their best fit was obtained with  $\tau_{11} = -0.269 \text{ fm}^3$  and  $\tau_{12} = -0.289 \text{ fm}^3$ . All four of these values differ considerably from those of Weller *et al.* (1985). Also the values of  $B(E2)\uparrow$  and  $\tau_{12}$  are inconsistent with the values obtained from unpolarised  ${}^7\text{Li}$  data (Vermeer *et al.* 1989). The value of  $\delta$  in equation (1a) was obtained from  $\alpha+t$  cluster calculations (Kajino 1986); however, as discussed in Vermeer *et al.* (1989), there is a wide spread of  $\delta$  values obtained from other calculations, ranging from  $-0.092$  to  $0.29$ . Similarly, shell model calculations (Barker and Woods 1989) gave  $\eta = -0.08$ . Thus, use of the relations and values (1) is open to question.

Since neither of the previous analyses of the aligned  ${}^7\text{Li}$  data of Weller *et al.* (1985), by Weller *et al.* and by Kajino *et al.* (1988), seems to be entirely satisfactory, we here reanalyse the same data, to see if a fit is possible that is consistent with results from other data and with model expectations.

## 2. Fitting Procedure

The data of Weller *et al.* (1985) consist of angular distributions of the tensor analysing power for scattering of aligned  ${}^7\text{Li}$  ions by  ${}^{58}\text{Ni}$  at four sub-Coulomb energies and by  ${}^{120}\text{Sn}$  at three sub-Coulomb energies, both for elastic scattering,  $T_{20}^{\text{el}}$ , and for elastic plus inelastic scattering,  $T_{20}^{\text{el+in}}$ .

As in Weller *et al.* (1985) and Kajino *et al.* (1988), we use the program ECIS79 (Raynal 1972) to calculate the Coulomb scattering of the aligned  ${}^7\text{Li}$ , including only the ground and first excited states, for given values of the four parameters  $Q$ ,  $B(E2)\uparrow$ ,  $\tau_{11}$  and  $\tau_{12}$ . Calculated values of  $T_{20}^{\text{el}}$  and  $T_{20}^{\text{el+in}}$  depend on the tensor part of the polarisation potential, which in the adiabatic approximation

† In Egelhof *et al.* (1987), it is said that Weller *et al.*'s best fit gives  $\tau_{11} = -0.23 \text{ fm}^3$ . Our calculations, however, indicate that Weller *et al.* actually used  $\tau_{11} = +0.23 \text{ fm}^3$ , so that we agree with the criticism by Kajino *et al.*

‡ Although Kajino *et al.* (1988) implied that  $\tau_{12}$  must be negative, the sign of  $\tau_{12}$  is actually a matter of convention, as was discussed in Barker and Woods (1989). Kajino *et al.* used the minus sign in the expression (4b) below, consistent with their negative value of  $\tau_{12}$ .

has the radial dependence [Alder and Winther 1975; see equation (1) in Kajino *et al.* 1988]

$$\frac{4\pi}{5} \frac{Z_t e}{r^3} \langle I_i || M(E2) || I_f \rangle - \left( \frac{9\pi}{5} \right)^{\frac{1}{2}} \frac{Z_t^2 e^2}{r^4} \tau_{if}, \quad (2)$$

where

$$\tau_{if} = \frac{8}{9} \pi \left( \frac{10}{3} \right)^{\frac{1}{2}} \sum_n W(11I_i I_f, 2I_n) \langle I_i || M(E1) || I_n \rangle \langle I_n || M(E1) || I_f \rangle / (E_n - E_i). \quad (3)$$

Since  $Q = \frac{2}{5} \sqrt{\pi} \langle \frac{3}{2} || M(E2) || \frac{3}{2} \rangle$  and  $B(E2)^\dagger = \frac{1}{4} | \langle \frac{3}{2} || M(E2) || \frac{1}{2} \rangle |^2$ , the expression (2) becomes

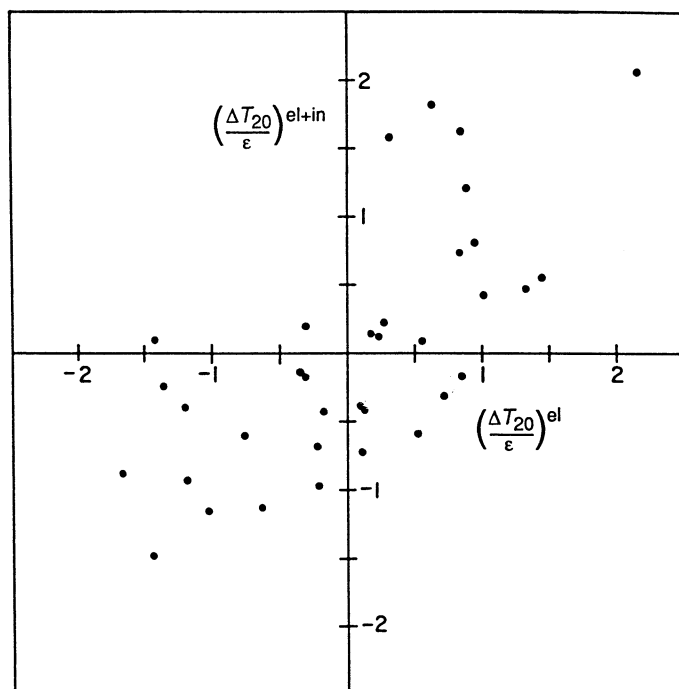
$$2\sqrt{\pi} \frac{Z_t e}{r^3} \left( Q - \frac{3}{2\sqrt{5}} \frac{Z_t e}{r} \tau_{11} \right) \quad (4a)$$

for the ground state diagonal matrix element, and

$$\frac{8\pi}{5} \frac{Z_t e}{r^3} \left( \pm \sqrt{B(E2)^\dagger} - \frac{3}{8} \sqrt{\frac{5}{\pi}} \frac{Z_t e}{r} \tau_{12} \right) \quad (4b)$$

for the transition matrix element coupling the ground state and the first excited state. The  $\pm$  ambiguity in expression (4b) originates from the sign of the off-diagonal matrix element  $\langle \frac{3}{2} || M(E2) || \frac{1}{2} \rangle$ , and this depends on the choice of the relative sign of the ground state and first excited state wave functions; we assume the plus sign in (4b), which is the convention adopted in Barker and Woods (1989) but opposite to that of Kajino *et al.* (1988). From (4a), it follows that there are strong correlations between the values of  $Q$  and  $\tau_{11}$ , and that the negative signs of both  $Q$  and  $\tau_{11}$ , as given by model calculations, imply destructive interference between the two contributions. Similarly from (4b),  $B(E2)^\dagger$  and  $\tau_{12}$  are strongly correlated, and the predicted positive sign of  $\tau_{12}$  implies destructive interference. Thus larger magnitudes of  $Q$  can be more or less compensated in fits by larger magnitudes of  $\tau_{11}$ , and larger values of  $B(E2)^\dagger$  by larger values of  $\tau_{12}$ .

Weller *et al.* (1985) pointed out that the quantity  $T_{20}^{\text{el}+\text{in}}$  is sensitive to the values of  $Q$  and  $\tau_{11}$ , but insensitive to  $B(E2)^\dagger$  and  $\tau_{12}$ , while  $T_{20}^{\text{el}} - T_{20}^{\text{el}+\text{in}}$  is sensitive to  $B(E2)^\dagger$  and  $\tau_{12}$ , but insensitive to  $Q$  and  $\tau_{11}$ . Weller *et al.* (1985) and Kajino *et al.* (1988) fitted values of  $T_{20}^{\text{el}+\text{in}}$  and also of  $T_{20}^{\text{el}}$ , which is sensitive to all four parameter values. In order to simplify the analysis, we fit values of  $T_{20}^{\text{el}+\text{in}}$  and of the inelastic cross section to the first excited state,  $\sigma^{\text{in}}$ , which is essentially proportional to  $T_{20}^{\text{el}} - T_{20}^{\text{el}+\text{in}}$ , as is demonstrated below. This decouples the four parameters into two pairs: values of  $Q$  and  $\tau_{11}$  are determined by fitting the  $T_{20}^{\text{el}+\text{in}}$  data, with  $B(E2)^\dagger$  and  $\tau_{12}$  taken to have reasonable fixed values, and values of  $B(E2)^\dagger$  and  $\tau_{12}$  are determined by



**Fig. 1.** Scatter diagram showing correlation between  $(\Delta T_{20}/\epsilon)^{\text{el}}$  and  $(\Delta T_{20}/\epsilon)^{\text{el+in}}$ , for the data of Weller *et al.* (1985), where  $\Delta T_{20} = T_{20}(\text{calc}) - T_{20}(\text{expt})$ .

fitting  $\sigma^{\text{in}}$ , where we again adopt reasonable values of  $Q$  and  $\tau_{11}$ . In order to determine  $\sigma^{\text{in}}$  we use the definition

$$T_{20}^{\text{el+in}} = \frac{\sigma^{\text{el}} T_{20}^{\text{el}} + \sigma^{\text{in}} T_{20}^{\text{in}}}{\sigma^{\text{el}} + \sigma^{\text{in}}} \quad (5)$$

and assume that

$$\sigma^{\text{R}} = \sigma^{\text{el}} + \sigma^{\text{in}}, \quad (6)$$

where  $\sigma^{\text{R}}$  is the Rutherford cross section. Equation (6) is sufficiently accurate at the present sub-Coulomb energies. Hence

$$\sigma^{\text{in}} = \sigma^{\text{R}} \frac{T_{20}^{\text{el}} - T_{20}^{\text{el+in}}}{T_{20}^{\text{el}} - T_{20}^{\text{in}}}. \quad (7)$$

The quantity  $T_{20}^{\text{in}}$  depends very little on the parameter values (and is independent of them in first order perturbation theory—Zupranski *et al.* 1979). Since, for the backward angles at which data are available,  $T_{20}^{\text{in}} = O(1)$ , while  $T_{20}^{\text{el}} = O(10^{-2})$ , it follows that the deduced values of  $\sigma^{\text{in}}$  depend on the experimental values essentially only through the combination  $T_{20}^{\text{el}} - T_{20}^{\text{el+in}}$ . The consequent sensitivity

of  $\sigma^{\text{in}}$  to  $B(E2)\uparrow$  and  $\tau_{12}$ , and insensitivity to  $Q$  and  $\tau_{11}$ , was also pointed out by Vermeer *et al.* (1984).

Because of correlations in the measured values of  $T_{20}^{\text{el}}$  and  $T_{20}^{\text{el+in}}$ , it is not obvious what uncertainties should be associated with the values of  $\sigma^{\text{in}}$ . These correlations are shown in Fig. 1, where values of  $[T_{20}^{\text{el+in}}(\text{calc}) - T_{20}^{\text{el+in}}(\text{expt})]/\epsilon^{\text{el+in}}$  are plotted against the corresponding quantity for elastic scattering for each of the 35 experimental configurations used by Weller *et al.* (1985) (5 angles, 7 target-energy combinations). Here  $T_{20}(\text{expt})$  and  $\epsilon$  are the experimental value and uncertainty as given by Weller *et al.*, and  $T_{20}(\text{calc})$  is the value calculated for the best-fit parameter set of Weller *et al.* Due to these correlations, the usual procedure of combining the uncertainties in  $T_{20}^{\text{el}}$  and  $T_{20}^{\text{el+in}}$  in quadrature would overestimate the uncertainties in  $\sigma^{\text{in}}$ , and this would be reflected in fits to  $\sigma^{\text{in}}$  giving  $\chi^2$  values that were too small. For the best-fit parameter set of Weller *et al.* (1985), the values of  $\chi^2$  for the fits to their original  $T_{20}^{\text{el}}$  and  $T_{20}^{\text{el+in}}$  data are respectively  $\chi_1^2 = 29.1$  and  $\chi_2^2 = 26.3$ , while the fit to  $\sigma^{\text{in}}$  with the uncertainties obtained as suggested above gives  $\chi_3^2 = 9.0$ . In order to obtain a value of  $\chi_3^2$  comparable with those of  $\chi_1^2$  and  $\chi_2^2$ , say  $\chi_3^2 = 27$ , we take the uncertainties in  $\sigma^{\text{in}}$  as  $\sqrt{1/3}$  times the values obtained by combining the uncertainties in  $T_{20}^{\text{el}}$  and  $T_{20}^{\text{el+in}}$  in quadrature. To the extent that the latter uncertainties are mainly statistical in nature, this procedure should not unduly bias the determination of the best values of  $B(E2)\uparrow$  and  $\tau_{12}$ .

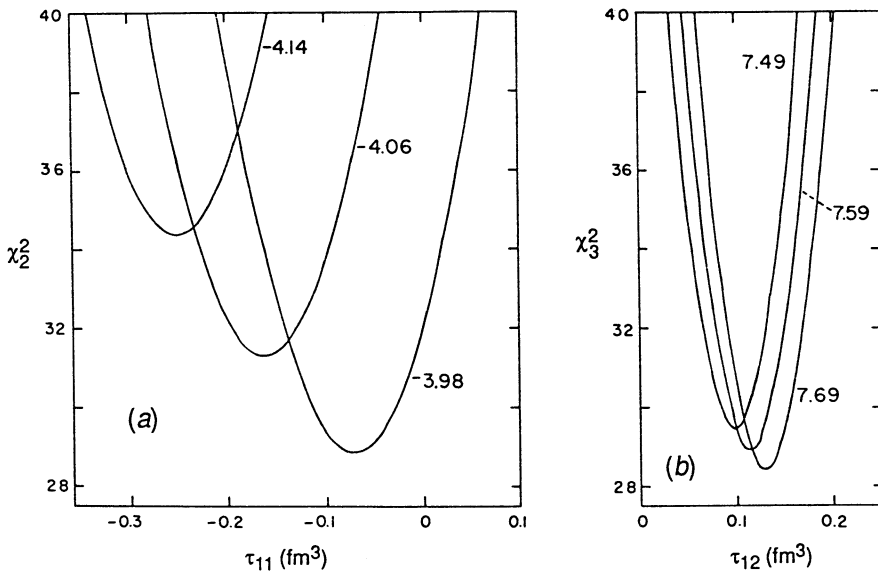
We calculate  $\chi_2^2$  for the  $T_{20}^{\text{el+in}}$  data on a grid of  $Q$  and  $\tau_{11}$  values, and  $\chi_3^2$  for the  $\sigma^{\text{in}}$  data on a grid of  $B(E2)\uparrow$  and  $\tau_{12}$  values. Because we are looking for acceptable fits with parameter values that are consistent with other information, we take the  $Q$  values to be in the range specified by the molecular value of  $Q$  and its uncertainty ( $-4.06 \pm 0.08 \text{ e fm}^2$ ), and the  $B(E2)\uparrow$  values to be similarly limited by the values of Vermeer *et al.* (1989) ( $7.59 \pm 0.10 \text{ e}^2 \text{ fm}^4$ ).

### 3. Results

Values of  $\chi_2^2$  for  $Q$  taken equal to the best molecular value  $-4.06 \text{ e fm}^2$  and to the two extreme molecular values  $-3.98$  and  $-4.14 \text{ e fm}^2$  are shown in Fig. 2a for a range of values of  $\tau_{11}$  [calculated with  $B(E2)\uparrow = 7.59 \text{ e}^2 \text{ fm}^4$  and  $\tau_{12} = 0.15 \text{ fm}^3$ , taken from the values (8) below]. Similarly Fig. 2b shows values of  $\chi_3^2$  for  $B(E2)\uparrow$  taken equal to the best value of Vermeer *et al.* (1989),  $B(E2)\uparrow = 7.59 \text{ e}^2 \text{ fm}^4$ , and to the two values differing from this by their given uncertainty of  $\pm 0.10 \text{ e}^2 \text{ fm}^4$ , for a range of values of  $\tau_{12}$  (calculated with  $Q = -4.06 \text{ e fm}^2$  and  $\tau_{11} = -0.15 \text{ fm}^3$ ).

The insensitivity of  $\chi_2^2$  to  $B(E2)\uparrow$  and  $\tau_{12}$ , and of  $\chi_3^2$  to  $Q$  and  $\tau_{11}$ , is demonstrated by the observation that over the ranges of  $B(E2)\uparrow$  and  $\tau_{12}$  shown in Fig. 2b,  $\chi_2^2$  varies by only 1%, and over the ranges of  $Q$  and  $\tau_{11}$  in Fig. 2a,  $\chi_3^2$  varies by less than 2%. This insensitivity justifies our decoupling procedure.

In Fig. 3, the data of Weller *et al.* (1985), consisting of values and uncertainties of  $T_{20}^{\text{el}}$  and  $T_{20}^{\text{el+in}}$ , are compared with values calculated for the parameter set (given in equations 8 below) that we take to be the most acceptable, both from fits to these data and from other considerations. Fig. 4 gives a similar comparison of  $\sigma^{\text{in}}$  values, where the experimental values and uncertainties are



**Fig. 2.** (a) Values of  $\chi_2^2$  as a function of  $\tau_{11}$  for the indicated values of  $Q$  (in  $e\text{fm}^2$ ), with  $B(E2)^\dagger$  and  $\tau_{12}$  fixed at the values (8). (b) Values of  $\chi_3^2$  as a function of  $\tau_{12}$  for the indicated values of  $B(E2)^\dagger$  (in  $e^2\text{fm}^4$ ), with  $Q$  and  $\tau_{11}$  fixed at the values (8).

derived from the data of Weller *et al.* as explained in Section 2. It is obvious that there is good agreement in both Figs 3 and 4.

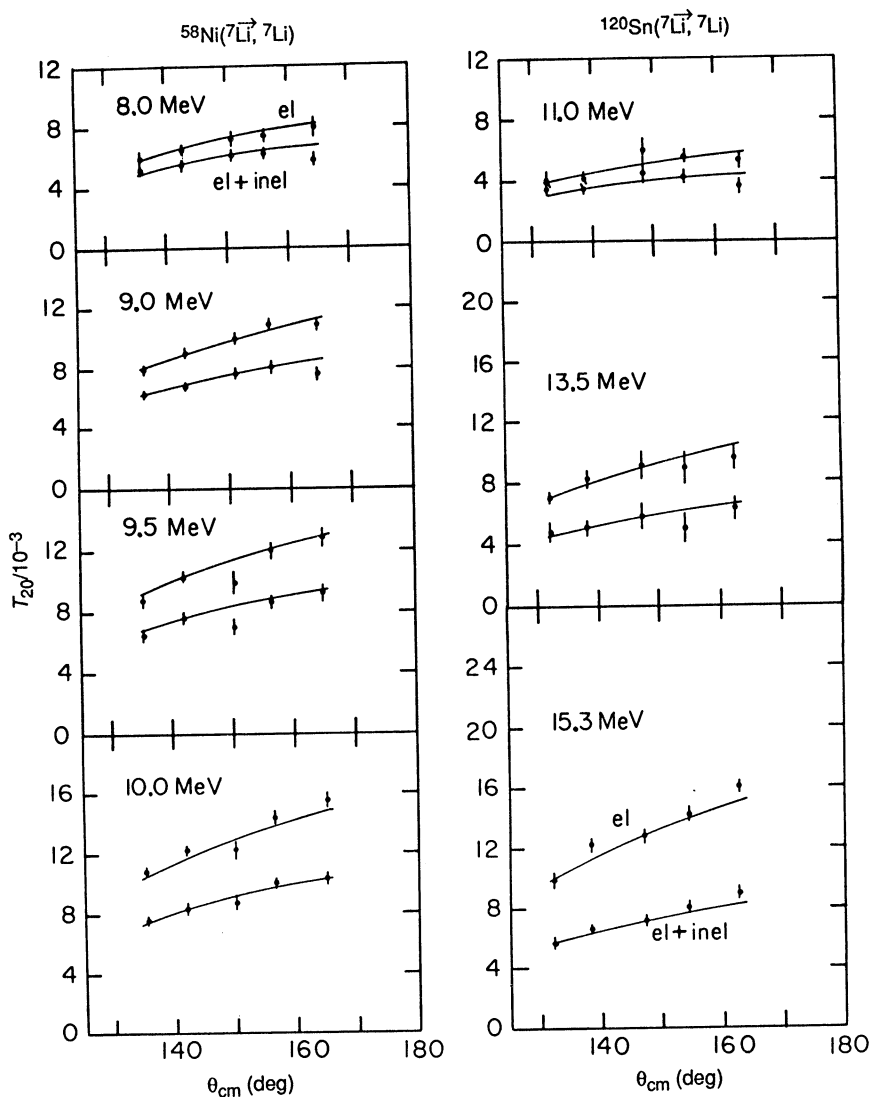
The best-fit parameter values of Weller *et al.* (1985) give  $\chi_1^2 = 29.1$ ,  $\chi_2^2 = 26.3$  and  $\chi_3^2 = 27.0$ , while those of Kajino *et al.* (1988) give  $\chi_1^2 = 34.4$ ,  $\chi_2^2 = 40.5$  and  $\chi_3^2 = 28.6$ .

#### 4. Discussion

For the molecular value of  $Q = -4.06\text{ efm}^2$ , our best fit to  $T_{20}^{\text{el+in}}$  gives  $\chi_2^2 = 31.3$ , with  $\tau_{11} = -0.16\text{ fm}^3$ . Within the 'allowed range' of molecular  $Q$  values, the smallest  $\chi_2^2$  is 28.8 for  $Q = -3.98\text{ efm}^2$  and  $\tau_{11} = -0.07\text{ fm}^3$ . Likewise, for the best value of  $B(E2)^\dagger = 7.59\text{ e}^2\text{fm}^4$  obtained from fits to unpolarised  ${}^7\text{Li}$  data (Vermeer *et al.* 1989), the best fit to  $\sigma^{\text{in}}$  gives  $\chi_3^2 = 28.9$ , with  $\tau_{12} = 0.11\text{ fm}^3$ , and within the 'allowed range' of Vermeer *et al.* the lowest  $\chi_3^2$  is 28.4 for  $B(E2)^\dagger = 7.69\text{ e}^2\text{fm}^4$  and  $\tau_{12} = 0.13\text{ fm}^3$ . After also taking into account the result of Vermeer *et al.* that  $\tau_{12} = 0.16 \pm 0.01\text{ fm}^3$ , and the expectation from model calculations (Mertelmeier and Hofmann 1986; Kajino *et al.* 1988; Barker and Woods 1989) that  $\tau_{11} \approx -|\tau_{12}|$ , we suggest as the most acceptable parameter values for fits to all the available data

$$\begin{aligned} Q &= -4.06\text{ efm}^2, & \tau_{11} &= -0.15\text{ fm}^3, \\ B(E2)^\dagger &= 7.59\text{ e}^2\text{fm}^4, & \tau_{12} &= 0.15\text{ fm}^3. \end{aligned} \quad (8)$$

They give  $\chi_1^2 = 32.3$ ,  $\chi_2^2 = 31.4$  and  $\chi_3^2 = 32.0$ . The values (8) are used for the calculated curves in Figs 3 and 4.

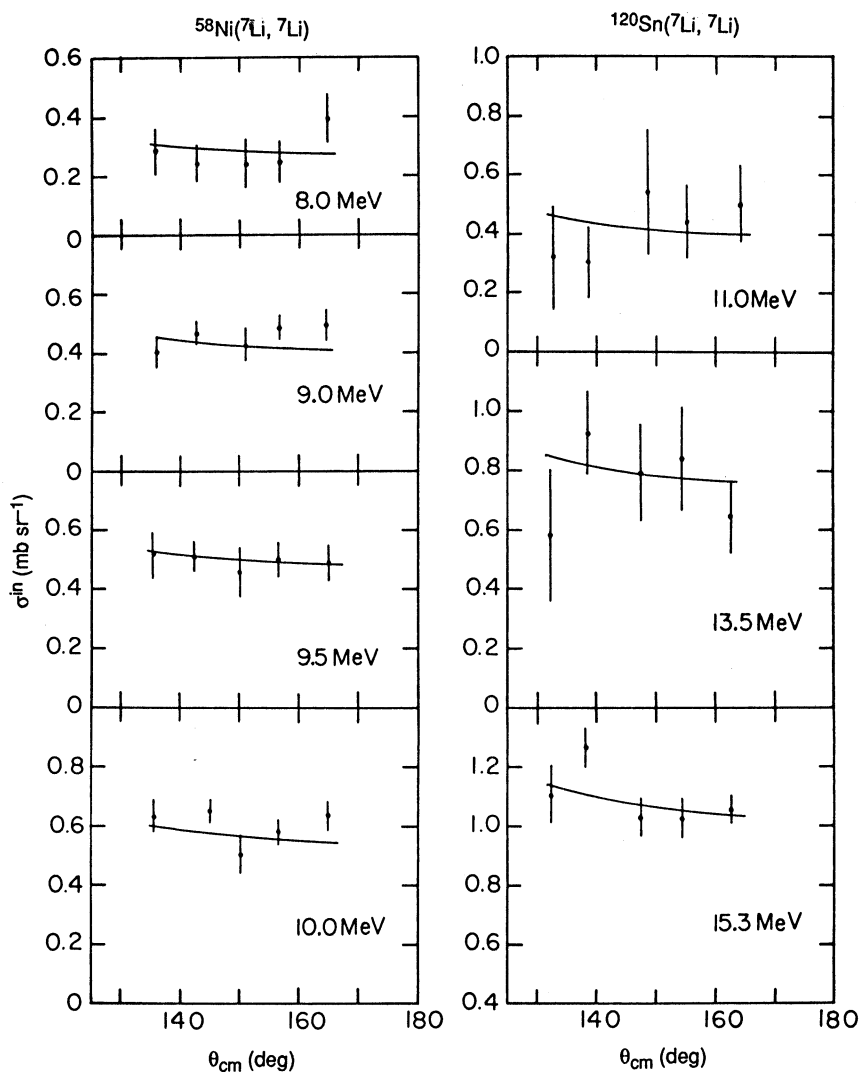


**Fig. 3.** Analysing powers  $T_{20}^{\text{el}}$  and  $T_{20}^{\text{el+in}}$  for the scattering of  ${}^7\text{Li}$  on  ${}^{58}\text{Ni}$  and  ${}^{120}\text{Sn}$  at various bombarding energies. The experimental values and error bars are from Weller *et al.* (1985). The curves are calculated using the parameter set (8).

The parameter values and uncertainties that Weller *et al.* (1985) obtained from fitting their aligned  ${}^7\text{Li}$  data are

$$\begin{aligned}
 Q &= -3.70 \pm 0.08 \, e \, \text{fm}^2, & \tau_{11} &= 0.23 \pm 0.06 \, \text{fm}^3, \\
 B(E2)^\dagger &= 8.3 \pm 0.5 \, e^2 \, \text{fm}^4, & \tau_{12} &= 0.23 \pm 0.06 \, \text{fm}^3.
 \end{aligned} \tag{9}$$

The parameter values (8) lie outside the ranges given in (9) (note in particular the different signs of  $\tau_{11}$ ). Weller *et al.* obtained their uncertainties from the

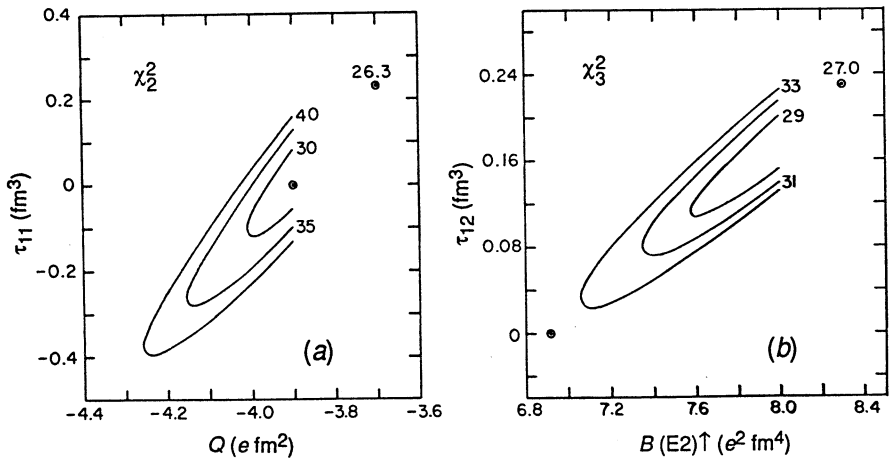


**Fig. 4.** Inelastic cross section  $\sigma^{\text{in}}$  for the scattering of  ${}^7\text{Li}$  on  ${}^{58}\text{Ni}$  and  ${}^{120}\text{Sn}$  at various bombarding energies. The experimental values and error bars are deduced from the measured values of Weller *et al.* as explained in Section 2. The curves are calculated using the parameter set (8).

error matrix or equivalently from the  $\chi^2+1$  rule (see Egelhof *et al.* 1987).<sup>†</sup> This procedure is valid, however, only if all the measured quantities are

<sup>†</sup> There is an inconsistency in Fig. 3 of Egelhof *et al.* (1987). From the  $1\sigma$  contour shown in the lower part of the figure, the uncertainties in  $\tau_{11}$  and  $\tau_{12}$  should each be about  $\pm 0.12 \text{ fm}^3$ , rather than the values  $\pm 0.06 \text{ fm}^3$  indicated by the upper part of the figure; it is the latter values that are given by Weller *et al.* (1985).

uncorrelated (Bevington 1969), and this is certainly not the case for the aligned  ${}^7\text{Li}$  data, as is evident from Fig. 1. Thus the parameter values (8) should not be ruled out on the grounds that they are incompatible with the values and uncertainties given in (9). In fact, the  $\chi^2$  value for the fit to the complete data of Weller *et al.* with the parameter values (8) is  $\chi^2 \equiv \chi_1^2 + \chi_2^2 = 63.7$  (66 degrees of freedom), which suggests that the fit is acceptable. It seems that the uncertainties given by Weller *et al.* for their parameter values are too small.

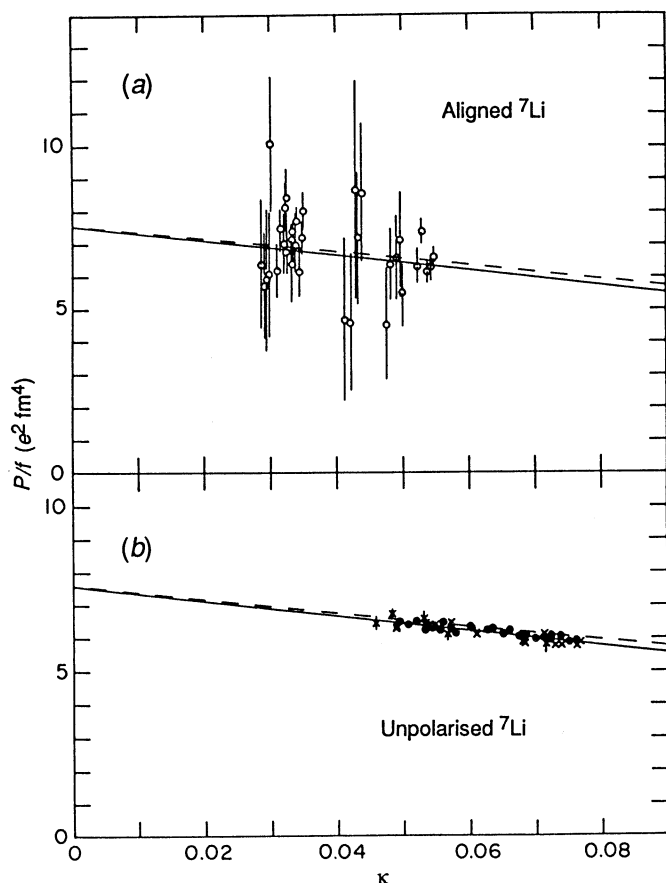


**Fig. 5.** (a) Contour diagram of  $\chi^2_2$  as a function of  $Q$  and  $\tau_{11}$ . (b) Contour diagram of  $\chi^2_3$  as a function of  $B(E2)^\ddagger$  and  $\tau_{12}$ . For both (a) and (b), the points correspond to the best-fit parameter set of Weller *et al.* (1985) and to their best fit with  $\tau_{11} = \tau_{12} = 0$ . The values of  $\chi^2_2$  and  $\chi^2_3$  are as indicated.

The strong correlation between the values of  $Q$  and  $\tau_{11}$ , and between  $B(E2)^\ddagger$  and  $\tau_{12}$ , as a result of the forms of the expressions (4), is illustrated in Fig. 5, which shows contours of constant  $\chi^2_2$  and  $\chi^2_3$  in the  $Q$ - $\tau_{11}$  and  $B(E2)^\ddagger$ - $\tau_{12}$  planes respectively. Also plotted are points corresponding to the best-fit parameter set of Weller *et al.* (1985) and to their best-fit values of  $Q$  and  $B(E2)^\ddagger$  when  $\tau_{11}$  and  $\tau_{12}$  are restricted to be zero. In Fig. 5a, the major axis of the contours and the points of Weller *et al.* are well represented by the linear relation

$$Q - 0.89\tau_{11} \text{ e fm}^{-1} = -3.9 \text{ e fm}^2. \quad (10)$$

However, unless the value of one or other of  $Q$  and  $\tau_{11}$  is restricted by some other means, the value of neither  $Q$  nor  $\tau_{11}$  will be well determined by fitting the aligned  ${}^7\text{Li}$  data. Similar remarks apply to  $B(E2)^\ddagger$  and  $\tau_{12}$ . The strong correlation between  $B(E2)^\ddagger$  and  $\tau_{12}$  was mentioned by both Weller *et al.* (1985) and Vermeer *et al.* (1989), but previous contour diagrams similar to Fig. 5 connected only the weakly-correlated quantities  $Q$  and  $k$ , where  $k$  is proportional to  $\tau_{12}$  (Vermeer *et al.* 1984), and  $\tau_{11}$  and  $\tau_{12}$  (Egelhof *et al.* 1987).



**Fig. 6.** Plot of  $P/f$  as a function of the quantity  $\kappa$ , which occurs in the relation (11). (a) Experimental points derived from the aligned  $^7\text{Li}$  data of Weller *et al.* (1985). (b) Experimental points from the unpolarised  $^7\text{Li}$  data of Vermeer *et al.* (1984) (closed circles) and of Bamberger *et al.* (1972) (crosses), taken from Fig. 1 of Vermeer *et al.* (1989). Where no error bars are shown, the uncertainties are less than or equal to the size of the data points. For both (a) and (b), the solid line is the best fit of Vermeer *et al.* (1989) to the unpolarised  $^7\text{Li}$  data, and the dashed line corresponds to the parameter set (8).

In order to indicate the relative precision possible in the values of  $B(E2)^\dagger$  and  $\tau_{12}$  determined from fits to the aligned  $^7\text{Li}$  data and to unpolarised  $^7\text{Li}$  data, we plot the data as in Fig. 1 of Vermeer *et al.* (1989). This makes use of the approximate relation

$$P \equiv \sigma^{\text{in}}/(\sigma^{\text{el}} + \sigma^{\text{in}}) = fB(E2)^\dagger(1 - \kappa k), \quad (11)$$

where  $k = 50 \cdot 3 \tau_{12} / \sqrt{B(E2)^\dagger} \text{ efm}^{-1}$ . The quantities  $f$  and  $\kappa$  are functions of the experimental conditions (energy, angle, etc.) and are calculated from the Winther-de Boer (1966) program, with the molecular value of  $Q$ . We have found

that this program gives values of  $P$  agreeing with the results of ECIS79 well within the required limits of accuracy. In Fig. 6, values of  $P/f$  are plotted as a function of  $\kappa$ , both for the unpolarised  ${}^7\text{Li}$  data as in Vermeer *et al.* (1989) and as derived above from the aligned  ${}^7\text{Li}$  data of Weller *et al.* (1985). In each part of Fig. 6, the solid line is the best fit of Vermeer *et al.* to the unpolarised  ${}^7\text{Li}$  data and the dashed line corresponds to the parameter values (8). In each case the intercept on the  $y$ -axis is equal to the value of  $B(E2)\uparrow$  and the slope is  $-kB(E2)\uparrow$ . It is seen that the aligned  ${}^7\text{Li}$  data are consistent with the unpolarised  ${}^7\text{Li}$  data, and should have little influence in the determination of  $B(E2)\uparrow$  and  $\tau_{12}$ .

We suggest that the uncertainties in the parameter values (8) for  $Q$ ,  $B(E2)\uparrow$  and  $\tau_{12}$  should be taken from previous work, i.e.  $\pm 0.08 \text{ e fm}^2$ ,  $\pm 0.10 \text{ e}^2 \text{ fm}^4$  and  $\pm 0.01 \text{ fm}^3$  respectively. It is difficult to estimate the uncertainty in  $\tau_{11}$ ; from analysis of the aligned  ${}^7\text{Li}$  data alone there could be a large uncertainty (about  $\pm 0.1 \text{ fm}^3$ ) in the value of  $\tau_{11}$  due to the strong correlation with the value of  $Q$ , but the theoretical constraint  $\tau_{11} \approx -|\tau_{12}|$  leads us to suggest a smaller uncertainty of  $\pm 0.05 \text{ fm}^3$ .

The values of  $\tau_{11}$  and  $\tau_{12}$  that we recommend are very different from those of Kajino *et al.* (1988). For this reason, and also because we are not convinced by their arguments relating the tensor moments to the astrophysical  $S$ -factor (see also the comment in Barker and Woods 1989), we think that the value of the zero-energy  $S$ -factor for the  ${}^4\text{He}(t, \gamma){}^7\text{Li}$  reaction derived by Kajino *et al.* should be treated with caution.

After this paper was essentially completed, related work by Grawert and Derner (1989) became available. From their analysis of the same aligned  ${}^7\text{Li}$  data of Weller *et al.* (1985), they also found strong correlations between the values of  $Q$  and  $\tau_{11}$ , and between  $B(E2)\uparrow$  and  $\tau_{12}$ ; however, their use of semiclassical scattering theory introduces significant errors in the calculation of  $T_{20}$ .

## 5. Summary

We have reanalysed the data of Weller *et al.* (1985) on the tensor analysing powers for Coulomb scattering of aligned  ${}^7\text{Li}$  ions, in order to obtain information on the values of the four  ${}^7\text{Li}$  moments  $Q$ ,  $B(E2)\uparrow$ ,  $\tau_{11}$  and  $\tau_{12}$ . We use a fitting procedure that effectively decouples the parameters into two pairs. It is found that a single set of parameter values, chosen primarily to be consistent with the value of  $Q$  measured by molecular techniques and the values of  $B(E2)\uparrow$  and  $\tau_{12}$  required to fit unpolarised  ${}^7\text{Li}$  data, and also with the theoretical constraint  $\tau_{11} \approx -|\tau_{12}|$ , gives a good fit to the aligned  ${}^7\text{Li}$  data. Our recommended values and uncertainties are:

$$\begin{aligned} Q &= -4.06 \pm 0.08 \text{ e fm}^2, & \tau_{11} &= -0.15 \pm 0.05 \text{ fm}^3, \\ B(E2)\uparrow &= 7.59 \pm 0.10 \text{ e}^2 \text{ fm}^4, & \tau_{12} &= 0.15 \pm 0.01 \text{ fm}^3. \end{aligned} \quad (12)$$

It is suggested that previous analyses of the same data that led to very different results either severely underestimated the uncertainties in their parameter values (Weller *et al.* 1985) or assumed inappropriate relations between the moments (Kajino *et al.* 1988).

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