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Predictability of the 12-month Running Averaged Sunspot Number in the Presence of an 8-month Quasi-periodicity on a Solar Cycle

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Abstract

The method of McNish and Lincoln (1949) for the prediction of the 12-month running averaged sunspot number R_{12} is supposed to generate useful estimates of R_{12} for periods up to one year ahead. However, it has been noted that for prediction periods beyond about 8 months, the variance of the prediction of R_{12} ($\sigma^2_{R_{12}}$) approaches the variance of the 'average' solar cycle. Moreover, the variance $\sigma^2_{R_{12}}$ for prediction periods greater than about 8 months is also of the order of the variance ($\sigma^2_{R_1}$) of the observed monthly-mean sunspot data R_1. Since the observed sunspot data R_1 is used to estimate R_{12}, the variance of R_1 may be used to attach statistical significance to the predictions of R_{12}. Thus, the sunspot number R_{12} cannot be usefully predicted more than 8 months ahead, because the variance of the prediction becomes too large (i.e. $\sigma^2_{R_{12}} \gtrsim \sigma^2_{R_1}$). However, a quasi-periodicity of about 8 months in R_1 is observed during the decay phase of solar cycle 21. It is shown in this paper that the variance $\sigma^2_{R_1}$ ought to be doubled in the presence of the 8-month quasi-periodicity of the sunspot cycle. Further, by taking account of this quasi-periodicity, it is possible to make useful predictions of R_{12} up to a year (and more) ahead. An application of the R_{12} predictions is in forecasting the ionospheric F2 layer critical frequencies a few months ahead.

1. Introduction

The CCIR R_{12} predictor method makes use of a linear increase of monthly-median f_0F_2 [denoted by $(f_0F_2)_\mathrm{m}$] with 12-month running averaged sunspot number R_{12} for predicting $(f_0F_2)_\mathrm{m}$ 6 months in advance (CCIR 1983). The method of McNish and Lincoln (1949) may be used to predict 12-month running averaged sunspot number R_{12}. In this method, the average cycle curve $\bar{R}_{12}$ is fitted by a least-squares fitting procedure to the 'latest' values of R_{12} observed up to 6 months previously. The number of data points used in the curve fitting procedure is determined by the proximity of the 'latest' observed values of R_{12} from the minimum of the cycle. The minima of the 'latest' observed R_{12} and $\bar{R}_{12}$ curves are superimposed. Now it is known that various solar cycles vary markedly in (sunspot number) magnitude and periodicity (length). However, the fitting procedure does not allow for scaling of the R_{12} curve in either magnitude or length. The extrapolations of the 'smoothed' sunspot number R_{12} may be made up to a few months (perhaps one year) in advance (from the last observed value of R_{12}: Bogart 1982; Holland and Vaughan

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1984; Koeckelenbergh 1986; Kuklin 1986). (In the terminology of the present study 'smoothed' sunspot numbers refer to running averaged or polynomial fitted values of the sunspot numbers in a given solar cycle, while an 'average' sunspot cycles refers to a mean cycle averaged over a number of past solar cycles.) Observed monthly-mean sunspot (R_1) data are used to estimate R_{12}. It is observed that the variance of the prediction of R_{12} (denoted by $\sigma^2_{R_{12}}$) for periods about one year ahead becomes greater than the variance of the monthly-mean sunspot data $\sigma^2_{R_1}$. Thus, the variance of the monthly-mean sunspot numbers $\sigma^2_{R_1}$ could be adopted as an upper bound to check the statistical usefulness of R_{12} predictions.

The monthly-mean values of sunspot number show a number of characteristic fluctuations on an 11-year solar cycle. The dominant ones are those having periodicities of about 6 and 12 months, particularly on the decay portion of a sunspot cycle (see Dodson and Hedeman 1972; Kuklin 1976, 1986; Bogart 1982; Koeckelenbergh 1986).

The present paper is directed towards determining the effects of an 8-month quasi-periodicity (observed on solar cycle 21) on the monthly predictability of R_{12}. Due to the presence of the 8-month periodicity, the ordinarily computed value of the variance $\sigma^2_{R_1}$ is shown to be under-estimated. In the presence of this quasi-periodicity, an evaluation of $\sigma^2_{R_1}$ would require that (a) polynomials be fitted separately to different portions of a sunspot cycle and (b) autocorrelation analyses be performed on the deviations of the monthly-mean sunspot values from the fitted curves. Such a technique has been effectively applied in ionospheric predictions of f_0F_2 both on hourly and monthly scales (Pasricha *et al.* 1987, 1988). The form of the autocorrelation function reveals a (mean) 8-month quasi-periodicity in the 'autocorrelated' monthly-mean sunspot data. The roughly exponential fall-off of the autocorrelation function signifies some degree of 'persistence' in the monthly-mean sunspot values. Finally, the corrected (enhanced) value of the variance $\sigma^2_{R_1}$ (in the presence of the 8-month periodicity) may be obtained by following the method of Bartels (1935). Now it is known that for prediction periods greater than about 8 months $\sigma^2_{R_{12}}$ approaches $\sigma^2_{R_1}$. An increase in the value of $\sigma^2_{R_1}$ (in the presence of the 8-month periodicity) would suggest that R_{12} may be predicted for periods up to 12 months (and more) with a better statistical reliability. The variance of the prediction of R_{12} ($\sigma^2_{R_{12}}$) is used in the CCIR R_{12} predictor method to compute the variance of the prediction of the monthly-median $(f_0F_2)_m$.

2. Variance of the Prediction of Monthly-median f_0F_2 in Terms of $\sigma^2_{R_{12}}$

The uncertainty in the estimation of $(f_0F_2)_m$ through its regression on R_{12} is given by the sum of the variance estimates of (a) the prediction of R_{12}, (b) the determination of the coefficients of the regression curves, and (c) the scatter of $(f_0F_2)_m$ values about the regression curves. The three variance estimates may be denoted by $\sigma^2_{R,F}$, σ^2_p and σ^2_f respectively. For instance, typical values of $\sigma^2_{R,F}$, σ^2_p and σ^2_f at a low-latitude station for the daytime period of the equinoxes are $0\cdot6$, $0\cdot1$ and $1\cdot2$ MHz^2 respectively (Pasricha *et al.* 1988). The corresponding night-time values are $1\cdot2$, $0\cdot1$ and $1\cdot2$ MHz^2 respectively. The variance of the estimation of $(f_0F2)_m$ in terms of the variance of the prediction of R_{12} may be given as $\sigma^2_{R,F} = B^2\sigma^2_{R_{12}}$, where $\sigma^2_{R_{12}}$ is the variance of the prediction of R_{12} and B is a coefficient in the linear regression of $(f_0F2)_m$ on R_{12}. Here $\sigma^2_{R_{12}}$ is the mean variance obtained from a number of previous cycles of monthly-mean sunspot number R_1 through a curve fitting procedure (see Section 6). The variance $\sigma^2_{R_{12}}$ empirically determines the accuracy of the prediction method as a function of the extent of the extrapolation of R_{12} a few months ahead. The extent of the variability in predicted R_{12} ($\sigma^2_{R_{12}}$) is determined by the requirements of the ionospheric forecasting of $(f_0F2)_m$. From the representative values of $\sigma^2_{R,F}$ and σ^2_f [of the scatter of $(f_0F2)_m$] quoted above, it is apparent that $\sigma^2_{R,F} \lesssim \sigma^2_f$. Thus the elevation of $\sigma^2_{R,F}$ through $\sigma^2_{R_{12}}$ (and hence $\sigma^2_{R_1}$) seems quite justified.

3. Variance of an Autocorrelated Time Series

In an autocorrelated time series the individual data points are not independent of one another. The variance of non-random deviations (from averages) was first computed by Bartels (see Chapman and Bartels 1940; Forbush *et al.* 1983). The variance of a quasi-periodic time series after having averaged over a period T (denoted by σ^2_T) is related to the variance of the original time series σ^2 through the relation

$$\frac{\sigma^2_T}{\sigma^2} = \frac{T_0}{T}, \tag{1}$$

where T_0 is a characteristic time at which individual data points become uncorrelated. For a gaussian process T_0 may be shown to be

$$T_0 \approx \sqrt{\pi}\,\tau_0, \tag{2}$$

where τ_0 is a suitable decorrelation period. The time T_0 may be taken as a rough measure of the 'persistence' in the data. Data points spaced T_0 apart are therefore uncorrelated. The corrected variance σ^2_c (in the presence of a periodicity in a time series) is related to the variance σ^2_i (of the time series obtained by ordinary statistical methods) through the relation

$$\sigma^2_c = T_0\,\sigma^2_i, \tag{3}$$

when T_0 is measured in units of the sampling interval (a month for the present study). The variance σ^2_i in the presence of a quasi-periodicity is reduced because the number of data points in a time series is reduced by a factor of T_0. Consequently, the variance σ^2_i needs to be enhanced by a factor of T_0.

The gaussian statistical model may be assumed to govern the distribution of the deviations of the monthly-mean sunspot numbers (from the least-squares fitted 'average' polynomial) with time. The 'average' polynomial is obtained after averaging a number of such polynomials over different solar cycles. The variance of the 'average' polynomial is given by relation (1). The corresponding variance σ^2_T is, however, not as small as predicted by relation (1). This is due to a large variability in constituent solar cycles used to obtain the 'average' solar cycle. It also explains why one has to make corrections in the 'average' solar cycle to match the trend in the 'latest' observed solar cycle. The 'smoothed' sunspot values, pertaining to a single polynomial fit to a given solar cycle, would amount to 'no-averaging'. Hence, the variance of the smoothed sunspot values (i.e. a polynomial fit to a given solar cycle) is simply given by σ^2 (in relation 1). However, due to the quasi-periodic nature of the 8-month fluctuations on a solar cycle σ^2 (=$\sigma^2_{R_1}$ obtained by ordinary methods) must be enchanced by a factor of T_0 (relation 3).

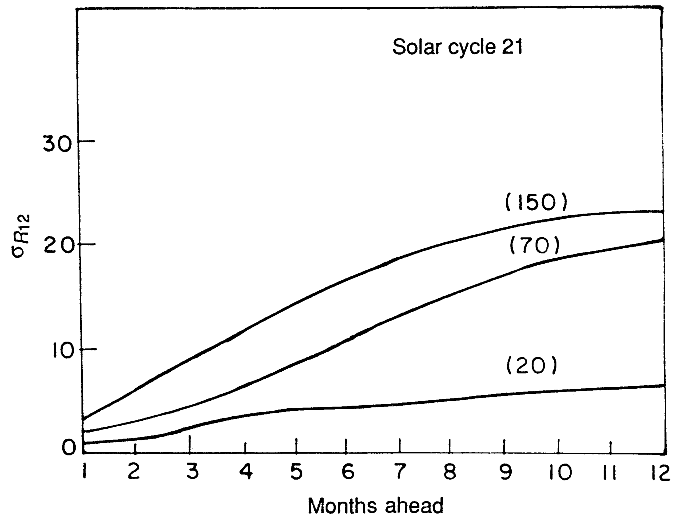

Fig. 1. The r.m.s. error in the prediction of R_{12} of 20, 70 and 150 sunspot numbers for various months ahead. These prediction errors have been obtained from Solar Geophysical Data for solar cycle 21.

4. Statistically Significant Prediction of R_{12} and Its Dependence on $\sigma^2_{R_1}$

A statistically significant prediction of R_{12} requires that the variances in the sunspot data be known in the following cases:

(*a*) variance associated with an 'average' solar cycle of $\bar{R}_{12}$;

(*b*) variance of the monthly-mean sunspot data about a 'smoothed' R_{12} solar cycle (denoted by $\sigma^2_{R_1}$); and

(*c*) variance of the prediction of R_{12} a few months ahead (denoted by $\sigma^2_{R_{12}}$).

The statistical uncertainty in the average solar cycle [case (a)] varies from 7 to 22 r.m.s. sunspot numbers (SSN; see McNish and Lincoln 1949). The (mean) variance $\sigma^2_{R_1}$ in the low, intermediate (growth and decay) and peak portions of a solar cycle [case (b)] are 6^2, 16^2 and 22^2 respectively. The variances of the predictions of R_{12} in case (c) of 20, 70 and 150 sunspot numbers, up to one year ahead, are obtained from Solar Geophysical Data (see Fig. 1). From Fig. 1 it may be concluded that

(i) variance $\sigma^2_{R_{12}}$ is less than $\sigma^2_{R_1}$ for prediction periods less than about 8 months;

(ii) variance $\sigma^2_{R_{12}}$ is of the order of $\sigma^2_{R_1}$ for prediction periods between 8 months and one year; and

(iii) variance $\sigma^2_{R_{12}}$ rapidly approaches (and becomes greater than) the variance of the 'average' solar cycle [in case (a)] for prediction periods greater than about one year.

The rather large variance in R_{12} predictions in (iii) is due to the larger uncertainty associated with the prediction procedure. The prediction procedure also makes $\sigma^2_{R_{12}}$ in (i) less than $\sigma^2_{R_1}$. [Otherwise the variance $\sigma^2_{R_{12}}$ would be simply the variance of the smoothed R_{12} curve of case (b) which is of the order of $\sigma^2_{R_1}$.] The physical justification of the prediction procedure is the common belief that the ionosphere does not respond (in a predictable manner) to the monthly variability in solar activity, but does show the effects of gross cycle behaviour which is measured by the smoothed sunspot number R_{12}. A variance condition may thus be imposed on the useful prediction of R_{12}: $\sigma^2_{R_{12}} \lesssim \sigma^2_{R_1}$. A useful prediction period for R_{12} would be about 8 months in accordance with (i). As implied by (ii), predictions of R_{12} for periods between 8 and 12 months would not be statistically meaningful.

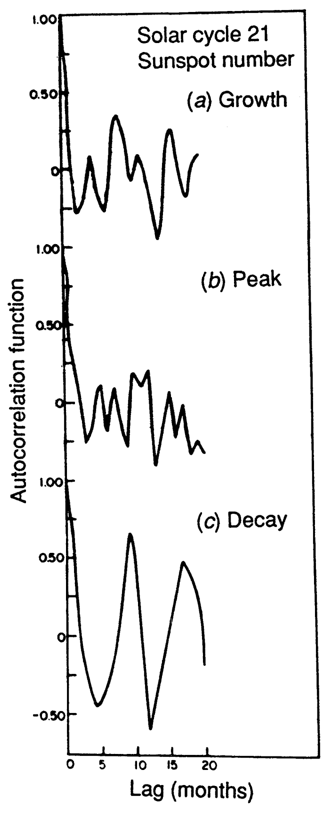

Fig. 2. Sample autocorrelation functions of the monthly-mean sunspot data over different portions of solar cycle 21. The decay portion shows an 8-month quasi-periodicity.

5. Database

Monthly-mean values of the sunspot number over four solar cycles (1944–84) have been analysed to compute the variance in the sunspot data. The three portions of low, intermediate and peak sunspot cycles have been fitted by a parabolic, cubic and parabolic polynomials respectively. The polynomial trends to each of the portions of the sunspot cycle are subsequently subtracted from the individual monthly-mean sunspot values. Autocorrelation functions are computed for such deviations for each of the 12 samples (four each of growth, peak and decay portions), comprising 32 data points, by the inverse Fast Fourier Transform algorithm. The autocorrelation functions for the decay portions of each of the 18th, 19th and 21st solar cycles revealed an 8-month quasi-periodicity. The quasi-periodicity in the decay portion of the 20th solar cycle was 10 months. In addition, the growth and peak portions of the 18th solar cycle also showed a 5-month quasi-periodicity.

6. Variance $\sigma^2_{R_1}$ in Presence of 8-month Quasi-periodicity Observed on the 21st Solar Cycle

A 12-month running average R_{12} of the observed monthly-mean sunspot values R_1 tends to eliminate fluctuations with periods of ~12 months in the data. The cycle of the R_{12} values may still contain (albeit smoothed) any quasi-periodic variations present in the monthly-mean values of R_1. One might use 12-month (running averaged) sunspot numbers as the basis for computing the variance of monthly-mean sunspot numbers. However, it may be shown that the deviations of the observed monthly-mean sunspot numbers (from the 12-month smoothed values) depict spurious periodicities at ~12/3·6 (3·3 months) and yet another at 12/1·5 (8 months), using the mathematical relations of Owens (1978). Thus in order to compute $\sigma^2_{R_1}$ it is necessary to fit a polynomial to the 12-month smoothed data, and later to remove it from the monthly-mean data. Equivalently, a polynomial may be fitted to the monthly-mean data. In the present study, autocorrelation analyses are performed on the deviations of the monthly-mean sunspot numbers from the least-squares fitted polynomials (separately to different portions of a given solar cycle). Sample autocorrelation functions for the growth, peak and decay portions of the 21st solar cycle are given in Fig. 2. The autocorrelation function of the decay portion shows a (mean) 8-month periodicity in the data. The variance of the monthly-mean sunspot numbers in this sample decay portion, in the presence of the 8-month periodicity, may be obtained as follows. It may be assumed that the initial fall-off of the autocorrelation function is given by a gaussian function, $\rho(\tau) \sim \exp(\tau^2/\tau_0^2)$. A statistically significant level of the decorrelation of $\rho(\tau)$ is the 2σ(r.m.s.) point. For a gaussian process, the temporal width τ_0 such that $\rho(\tau) \approx 0 \cdot 15$ refers to the 2σ point. The decorrelation period τ_0 for the sample decay portion is about one month. (The estimate of persistence from monthly sunspot data ought to be 'shaky' though as the first lagged point in an autocorrelation function is at one month.) Thus the (corrected) variance of the monthly-mean sunspot numbers, in the decay portion, may be given by (using relations 2 and 3, and with the computed value of $\sigma_i^2 = 18^2$)

$$\sigma_{R_1}^2 \approx \sqrt{\pi}\, 18^2 \approx 24^2.$$

The ratio of this corrected value of $\sigma_{R_1}^2$ to its value obtained by ordinary statistical methods is equal to $24^2/18^2 \approx 1 \cdot 8$. Thus, due to the presence of an 8-month periodicity in the intermediate portion of a solar cycle, its variance may become roughly the same as that of the peak portion.

Since sunspots last about one solar rotation, i.e. about one month, the existence of persistence of one month in sunspot data has an obvious physical basis. However, there is evidence for the existence of recurrent groups of sunspots for periods up to one year. It has been explained in terms of the persistence of some 'active' longitudes on the solar disc (see Bogart 1982). It is also believed that the persistence of solar activity (as seen through sunspot number) is likely to be longer during low activity compared with high activity cycles (Koeckelenbergh 1986). Also, the recurrent sunspots persist to a greater extent during the decay phase of a solar cycle, compared with the initial rise. The changes in the IMF sector structures, during enhanced solar activity, probably indicate the predictability of solar activity up to about 3 months (Kuklin 1986).

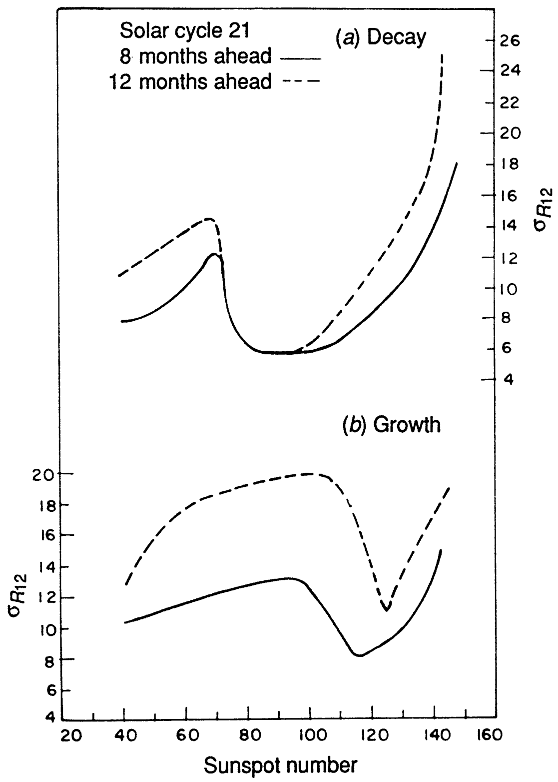

Fig. 3. The r.m.s. errors of the prediction of R_{12} in the growth and decay phases of solar cycle 21 (obtained from Solar Geophysical Data).

7. Predictability of R_{12} in the Presence of 8-month Quasi-periodicity

The r.m.s. errors of prediction of R_{12} ($\sigma_{R_{12}}$) in both the grown and decay phases of the 21st solar cycle are obtained from Solar Geophysical Data. These are presented in Fig. 3 for prediction periods 8 and 12 months ahead. The r.m.s. error in the monthly-mean sunspot data (σ_{R_1}) in the growth phase is 18 SSN. In Fig. 3*b*, $\sigma_{R_{12}}$ for the prediction period of 8 months is less than σ_{R_1}, but for the prediction period of 12 months is $\sim\sigma_{R_1}$. Hence the sunspot number R_{12} cannot be usefully predicted 12 months ahead during the growth phase of the solar cycle. In the decay phase (Fig. 3*a*), the r.m.s. errors for the prediction periods of 8 and 12 months are less than (the corrected) σ_{R_1} of 24 SSN. Now it is well known that the decay phase of the solar cycle can be predicted better than the growth phase, due to the fact that the peak of the solar cycle is known prior to the prediction of the decay phase. It may be easily seen that the presence of a residual periodicity in R_{12} (due to a periodicity in R_1) further aids better predictions of R_{12} for prediction periods of about one year. It also explains why the r.m.s. errors $\sigma_{R_{12}}$ for prediction periods of 8 and 12 months are approximately the same (Fig. 3*a*).

8. Conclusions

The smoothed sunspot numbers R_{12} are predicted up to a few months (perhaps one year) ahead by different regression procedures. An 'average' solar cycle, obtained from a number of solar cycles, forms the basis for the prediction. This 'average' solar cycle has to be matched to the given solar cycle in order to give useful R_{12} predictions. A prediction procedure tends to reduce the variance $\sigma^2_{R_{12}}$ of the predictions of R_{12} for periods up to about 8 months. For larger prediction periods, the variance $\sigma^2_{R_{12}}$ exceeds the variance of the 'average' solar cycle. Moreover, the variance of monthly-mean sunspot numbers $\sigma^2_{R_1}$ is also of the order of $\sigma^2_{R_{12}}$ for prediction periods greater than about 8 months. Thus, the predictions of R_{12} up to one year in advance probably lose any statistical significance. The variance $\sigma^2_{R_1}$ may be adopted as an upper bound for a statistically significant prediction of R_{12}. The computation of $\sigma^2_{R_1}$ requires that polynomials be fitted separately to different portions of a solar cycle. Now, it is known that characteristic sunspot fluctuations may occur over a sunspot cycle. When computing the variance $\sigma^2_{R_1}$ of monthly-mean sunspot numbers, a correction must be made to account for this periodicity. A sample calculation made in the decay portion of the 21st solar cycle shows that the variance $\sigma^2_{R_1}$ almost doubles. The variance of the prediction of R_{12} obtained from Solar Geophysical Data is then found to be less than $\sigma^2_{R_1}$. The useful prediction period is thus extended to one year or more. The CCIR R_{12} method makes use of these predictions of R_{12} for forecasting monthly-median frequencies of the ionospheric F-region six months in advance (CCIR 1983).

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