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Delayed Inflation of a Closed Cosmological Model Driven by the False Vacuum in Five Dimensions

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Abstract

An inflationary cosmology in five dimensions is discussed that has the unique feature that if initial conditions are tuned appropriately with the values of the unstable maximum of the Higgs potential the universe is trapped in a Lemaitre-type equilibrium point after which it expands in an exponential manner. Such a quasi-static intermediate point may be responsible for the homogeneity of the universe at late times.

1. Five-dimensional Cosmology and Inflation

The insight on the part of Guth (1981) inspired by the work of Coleman (1977) that the solution to the horizon problem can be found by exponential expansion in the false vacuum has ushered in a new era of cosmological thought. In addition to the horizon problem, inflationary cosmology also offers a solution to the problem of the flatness of the universe and the absence of primordial monopoles from the present universe. The problems with inflation are centred in the mechanism of particle creation and galaxy formation. Silk and Turner (1986) have recently noted that inflation cannot resolve the problem of structure on small scales (galaxies) and large scales (voids and superclusters) with the same spectrum of density fluctuation in the background of a single period of inflation; they have suggested two periods of inflation to resolve the question of large scale versus small scale structure. In quite another direction, Demianski (1986) has shown that inflation is admitted in a closed universe with a large amount of primordial entropy. In an effort to understand the relation of inflation to the topological structure of the background space, in this note we briefly study a five-dimensional cosmology in a closed three space driven by the false vacuum.

The important point behind this investigation is that inflation does occur in a closed background and if the initial conditions are tuned to the vacuum expectation value of the Higgs field at the metastable maximum, the universe experiences a Lemaitre-type of delay with perhaps time enough to homogenise any matter present and wipe out any initial density fluctuations that might have seeded prior to this. The result is that the conflict between the large scale and small scale structure might be dispensed with. Though this is speculative, the quasi-equilibrium position predicted by our theory will certainly leave its trace in the following eras of galaxy formation.

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2. Closed Cosmological Model in Five Dimensions Driven by False Vacuum

In two previous papers (Wolf 1986, 1989) we have discussed five-dimensional models both for Einstein gravity and for a theory admitting creation. We did not, however, discuss the details of a closed expansion in the false vacuum but only commented on the two cases of asymptotic inflation and recontraction. Here we address briefly the problem of a quasi-equilibrium point and start by writing the Einstein equations in five dimensions:

$$R_{\mu\nu} = -k_5(T_{\mu\nu} - \tfrac{1}{3} T g_{\mu\nu}); \qquad k_5 = \frac{8\pi G_5}{c^4}. \tag{1}$$

For the Ricci components we have for the maximally symmetric metric

$$g_{\mu\nu} = \begin{bmatrix} -1 & & \\ & R_3^2 \tilde{g}_{ij} & \\ & & a^2 \end{bmatrix}, \tag{2}$$

where $\mu, \nu = 0, 1, 2, 3, 5$; R_3 and a are three space and five-dimensional scale factors; $i,j = 1,2,3$; and K_3 is the three space curvature. Further,

$$R_{00} = \frac{3\ddot{R}_3}{R_3} + \frac{\ddot{a}}{a}, \tag{3}$$

$$R_{ij} = -\left\{\frac{2K_3}{R_3^2} + 2\left(\frac{\dot{R}_3}{R_3}\right)^2 + \frac{\ddot{R}_3}{R_3} + \frac{\dot{a}\dot{R}_3}{aR_3}\right\} g_{ij}, \tag{4}$$

$$R_{55} = -\left(\frac{\ddot{a}}{a} + \frac{3\dot{R}_3\,\dot{a}}{R_3\,a}\right) g_{55}. \tag{5}$$

For the energy–momentum components we have

$$T_{\mu\nu} = -V_0\, g_{\mu\nu}, \quad T = -5V_0,$$

$$T_{00} = V_0, \quad T_{ij} = -V_0\, g_{ij}, \quad T_{55} = -V_0\, g_{55}.$$

Equation (1) gives

$$\frac{3\ddot{R}_3}{R_3} + \frac{\ddot{a}}{a} = -k_5(-\tfrac{2}{3}V_0), \tag{6}$$

$$-\left\{\frac{2K_3}{R_3^2} + 2\left(\frac{\dot{R}_3}{R_3}\right)^2 + \frac{\ddot{R}_3}{R_3} + \frac{\dot{a}\dot{R}_3}{aR_3}\right\} = -k_5(\tfrac{2}{3}V_0), \tag{7}$$

$$-\left(\frac{\ddot{a}}{a} + \frac{3\dot{R}_3\,\dot{a}}{R_3\,a}\right) = -k_5(\tfrac{2}{3}V_0). \tag{8}$$

From equations (6) and (7) we have

$$\frac{6K_3}{R_3^2} + 6\left(\frac{\dot{R}_3}{R_3}\right)^2 + 6\,\frac{\ddot{R}_3}{R_3} + 3\,\frac{\dot{a}\dot{R}_3}{aR_3} + \frac{\ddot{a}}{a} = \frac{8k_5\, V_0}{3}, \tag{9}$$

and combining with equation (8) we have

$$\frac{6K_3}{R_3^2}+6\left(\frac{\dot{R}_3}{R_3}\right)^2+6\,\frac{\ddot{R}_3}{R_3}=2k_5\,V_0\,. \tag{10}$$

We study the case $K_3=1$, calling $\dot{R}_3=P=\dot{R}$ and $R_3=R$, so that

$$6+6P^2+6PR\,\frac{\mathrm{d}P}{\mathrm{d}R}=2k_5\,V_0\,R^2 \quad \text{or} \quad 1+P^2+PR\,\frac{\mathrm{d}P}{\mathrm{d}R}=\tfrac{1}{3}k_5\,V_0\,R^2\,.$$

Calling $PR=V$ and integrating we find that

$$(R\dot{R})^2=\frac{k_5\,V_0}{6}\,R^4-R^2+C\,, \tag{11}$$

where C can be found from R and $\dot{R}$ at $t=0$.

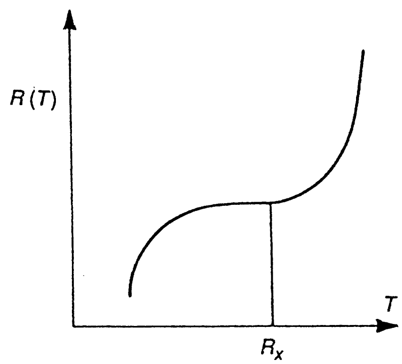

Fig. 1. Expansion of the universe showing the quasi-static equilibrium point.

The integral of equation (11) is

$$\int_{R_0}^{R}\frac{R\,\mathrm{d}R}{(\frac{1}{6}k_5\,V_0\,R^4-R^2+C)^{\frac{1}{2}}}=t\,. \tag{12}$$

If $kV_0R^4>6R^2-6C$, then the solution is asymptotically inflationary to confirm the Demianski (1986) result in four dimensions. However, if $\dot{R}=0$ at

$$\frac{\mathrm{d}}{\mathrm{d}R}\left(\frac{k_5\,V_0\,R^2}{6}-1+\frac{C}{R^2}\right)^{\frac{1}{2}}=0\,,$$

then the universe will experience a quasi-static equilibrium point, as in Fig. 1, or

$$\frac{k_5\,V_0\,R_x^4}{6}-R_x^2+C=0\,; \qquad \frac{k_5\,V_0\,R_x}{3}-\frac{2C}{R_x^3}=0\,,$$

$$R_x=\left(\frac{6C}{k_5\,V_0}\right)^{\frac{1}{4}}=\left\{\frac{3}{k_5\,V_0}\pm\left(\frac{9}{k_5^2\,V_0^2}-\frac{6C}{k_5\,V_0}\right)^{\frac{1}{2}}\right\}^{\frac{1}{2}}, \tag{13}$$

giving $C = 3/2K_5 V_0$ and $R_x = (3/K_5 V_0)^{\frac{1}{2}}$ for both the plus and minus signs in (13). We see that the quasi-static equilibrium point results when we choose C or $\dot{R}$, R at $t = 0$ to be the solution above. Equation (13) is the essential result that we intended to derive and a similar situation is being studied in other cosmological models. After the quasi-equilibrium point the universe will then expand in the usual exponential manner. We also note from (12) that if $\dot{R} = 0$ and $(d/dR)\dot{R} \neq 0$, then the universe will contract and not inflate. The solution for a also gives an asymptotic inflationary universe from (8) for R_3 inflating, and this represents the principal weakness of the model. We may, however, envisage a phase wherein the five-dimensional scale factor experiences a temporary expansion in the fifth dimension prior to the creation of matter, after which the dynamics of the matter fields will force a contraction of the fifth dimension. We would also like to point out that the above behaviour is unique to the five-dimensional cosmology; it does not occur in the four-dimensional cosmology driven by the false vacuum for any topology, and does not occur in any dimensional cosmology higher than five with the driving force being solely the false vacuum. This is because of additional terms in the equations that disallow the Lemaitre-type equilibrium point for any initial conditions.

In closing, the introduction of positive curvature in combination with specific initial conditions allows for a quasi-static equilibrium point for expansion in the false vacuum in five dimensions which would allow the early universe to homogenise before the inception of a following inflationary stage.

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