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Formation of the Structure of the Universe: Observational Aspects*

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Abstract

A review of recent developments in the study of the structure of the universe is given. We focus on two problems: the fractal description of the universe, and on observational constraints on the bias in galaxy formation.

1. Introduction

During dynamical evolution certain properties of stellar systems are conserved. Thus, by studying the present structure of systems conclusions can be made on the formation story and the evolutionary history of these systems. This idea was used for the first time by Ernst Öpik to determine the age of the universe and stars (Öpik 1933, 1938). Later these ideas have been frequently used by many astronomers to reconstruct the history of particular stellar systems. This approach has found wide acceptance after the famous paper by Eggen *et al.* (1962) on the early evolution of the Galaxy.

Dynamical evolution of stellar systems is slower the larger their scale. The largest astronomical systems observed so far in the universe are superclusters of galaxies and their complexes. These systems evolve apparently so slowly that what we see in the sky is only a slightly modified picture of the beginning of the formation of structure of the universe. To study the formation history of the large scale structure of the universe in more detail astronomers use the following strategy:

- make assumptions about the properties of the dynamically dark matter and calculate the initial spectrum of density fluctuations,
- simulate the dynamical evolution of the universe,
- compare results of model calculations with observations,
- improve the model, correct the initial assumptions, and repeat the procedure.

The crucial point in this reasoning chain is the comparison of models with observations. Already early studies demonstrated that galaxies are not distributed randomly. Some galaxies are located in clusters, others form a

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more or less randomly populated field. This two component picture of galaxy distribution is strengthened by morphological evidence: clustered galaxies are mostly ellipticals and S0s, whereas field galaxies are dominated by spirals and irregulars.

The first flaw in this simple paradigm was uncovered in the mid-70s. Almost simultaneously several groups reported that field galaxies form rather regular structures—filaments of different richness and length (Jõeveer and Einasto 1978; Tarenghi *et al.* 1978; Tifft and Gregory 1978; Tully and Fisher 1978). Filaments join clusters of galaxies into a multi-branched connected network, and the space between filaments is almost devoid of any visible matter. The space filled with galaxy systems comprises a small fraction of the total volume of the universe. The new picture of the universe has been called cellular or filamentary.

The next step in the change of our paradigms on the structure of the universe came in the mid-80s when a certain hierarchy in the distribution of galaxy systems was discovered. The idea of hierarchical clustering is not new, but goes back to Charlier (1908) and de Vaucouleurs (1970). Recent data gave new life to this picture. To describe hierarchical clustering fractal formalism has often been used (Mandelbrot 1982, 1987).

Another recent development is related to galaxy formation. In numerical simulations dynamical evolution of all gravitating particles is followed. These particles constitute the invisible dark matter. From astronomical observations we can see only visible galaxies. This would be no problem if galaxies followed the distribution of all the gravitating matter. New data emphasise that this is not the case and that galaxy formation is a biased process.

In this review we shall discuss recent progress in the study of the structure and formation of the universe from the observational point of view. We address fractal properties of the universe and biasing in galaxy formation. We also discuss the 'cosmography' of the largest observed voids. The review ends with a list of some unresolved problems and conclusions.

Throughout the review we shall use the Hubble constant $H_0=100\ h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

2. Fractal Description of the Structure of the Universe

(a) Why?

The statistical measure of the clustering of galaxies is the correlation function. This is a preferred statistic used in most papers since it is directly related to the spectrum of initial perturbations of the density and to other quantities of the large scale distribution of galaxies. It was thought that observed samples of galaxies can be considered as fair samples and that by calculating the correlation function we can determine the basic parameters essential and sufficient to describe the whole universe (Peebles 1980).

Recent studies have shown that reality is more complicated. First of all the classical correlation analysis itself gave unexpected results. The correlation length r_0 [defined as the value of the argument at which the correlation function has the value $\xi(r_0) = 1$] for galaxies is $r_0 \approx 5\ h^{-1}$ Mpc (Davis and Peebles 1983), whereas for rich clusters of galaxies it is $r_0 \approx 25\ h^{-1}$ Mpc (Klypin and Kopylov 1983; Bahcall and Soneira 1983).

To explain this property of the correlation function Kaiser (1984) and Szalay and Schramm (1985) suggested that clusters are high-density regions of the same underlying galaxy distribution. From a purely geometric point of view the observed phenomenon had a simple explanation. Jones and Jones (1985) and Einasto, Klypin and Saar (1986, hereafter denoted EKS) demonstrated that the increase of the correlation length with sample size is due to differing filling factors: galaxy samples cover a smaller volume in and around the Local Supercluster and have a higher filling factor than cluster sample which extend over several superclusters with huge voids between them. Thus the correlation function describes *two* different physical properties of galaxy distribution, the clustering and the presence of voids between clusters.

Mandelbrot (1982), Lachieze-Rey (1986), Calzetti *et al.* (1987) and Ruffini *et al.* (1988) suggested that these properties of the correlation function are expected if the universe has fractal properties. Moreover, Mandelbrot (1987) emphasised that in an infinite fractal universe the correlation is meaningless in its present form and other functions are needed to describe the structure.

(b) Self-similarity of Voids; Correlation Analysis

If the universe really has a fractal structure this can have serious consequences for our understanding of physical processes of structure formation as well as for statistical tools to describe the structure. Thus it is desirable to look to the problem from different points of view.

One of the key properties of the fractal structure is the self-similarity of the structure on various scales. Einasto, Einasto, and Gramann (1989, hereafter EEG) extended the EKS study and considered the self-similarity of voids in galaxy and cluster distributions. As a basic source of galaxy redshifts the compilation by Huchra (1988) was used, and cluster redshifts were collected from various sources. Observed redshifts were corrected for the solar motion and Virgocentric flow, and in clusters of galaxies velocities with respect to the cluster centroid were compressed to avoid the 'finger of God' effect. After these corrections velocities can be used as distance indicators. Subsamples of galaxies were chosen from the whole dataset which correspond to complete redshifts surveys up to a certain apparent magnitude or galaxy diameter. All subsamples have a constant absolute magnitude limit over the whole subsample volume.

For comparison EEG used two sets of models. The first one was based on numerical simulations of structure formation in a cold dark matter (CDM) dominated universe with the nonzero cosmological constant by Gramann (1988). The second set is an artificial model of randomly spaced clusters. The internal structure of the clusters was generated in accordance with real clusters so that the resulting correlation function represents well the observed correlation function. The CDM model was calculated for a cube of sidelength $L = 40\ h^{-2}$ Mpc; the number of data points in the model after the exclusion of points from low density regions (see Section 3 for biased galaxy formation) was $N \approx 20\,000$. The random cluster model has a sidelength of $L = 40$ Mpc and $N \approx 5000$.

The basic real and model samples were divided into subsamples of various cube size. For all subsamples the correlation function was calculated and the correlation length derived. Results of these calculations are plotted in Figs 1*b*, 2*b*, and 3*b*. We see that for real and CDM model samples the basic result by EKS is confirmed: the correlation length continuously increases with sample size. In the random cluster model the correlation length is practically constant.

To check for a possible luminosity effect both EKS and EEG derived correlation functions for identical sample volumes but different absolute magnitude intervals. These calculations demonstrate that (except for the brightest galaxies which are concentrated towards clusters) galaxies of different luminosity have within a fixed volume practically identical correlation properties.

(c) *Self-similarity of Voids; Diameters of Voids*

The next step in studying the self-similarity was to calculate the diameters of the largest voids in subsamples of different size. For this purpose two methods were used—the beam and empty sphere methods. In the first case beams were put into the sample volume and distances between two consecutive density maxima were derived to represent diameters of voids along the beams. In the second case void centres were derived as points with maximum distances to nearest galaxies in different directions; for these centres the distances to nearest galaxies yield the void radii. Both methods show that the mean diameter of voids for samples of different sidelength increases for the real and CDM model, but is practically constant for the random cluster model. Results are plotted in Figs 1*a*, 2*a* and 3*a*.

After this study was finished Einasto *et al.* (1990) derived for all samples and subsamples the void probability function $P(V)$, introduced by White (1979) and defined as the probability of finding no galaxies in a volume of size V, randomly chosen in the sample. This function can be represented by all of the n-point correlation functions (see White 1979). This function has an information content different from the correlation function (White 1979; Maurogordato and Lachieze-Rey 1987; Fry *et al.* 1989). In the present context it was used to study the self-similarity of voids. Like all distribution functions the void probability function also defines a scale. We shall use the median void radius R_m, which is defined as the value of the argument where the function has the value $P(R_m) = 0 \cdot 5$. Results of these calculations are shown in Fig. 4.

We see that mean void diameters as well as median void radii increase with the sample size. For galaxy samples this increase is almost linear; for CDM model samples at large sample size the slope of the D_m and R_m versus L relation is lower than for the real case. This is a natural property of simulated model samples since large scale perturbations are suppressed in model calculations and we expect to reach a homegeneous distribution of particles. In contrast to real and CDM model samples random cluster samples have practically constant void diameters.

This behaviour of samples is expected. In CDM models the fractal structure is built in (if a random gaussian field is truncated at a certain threshold density level, the resulting dividing surface between filled and empty space

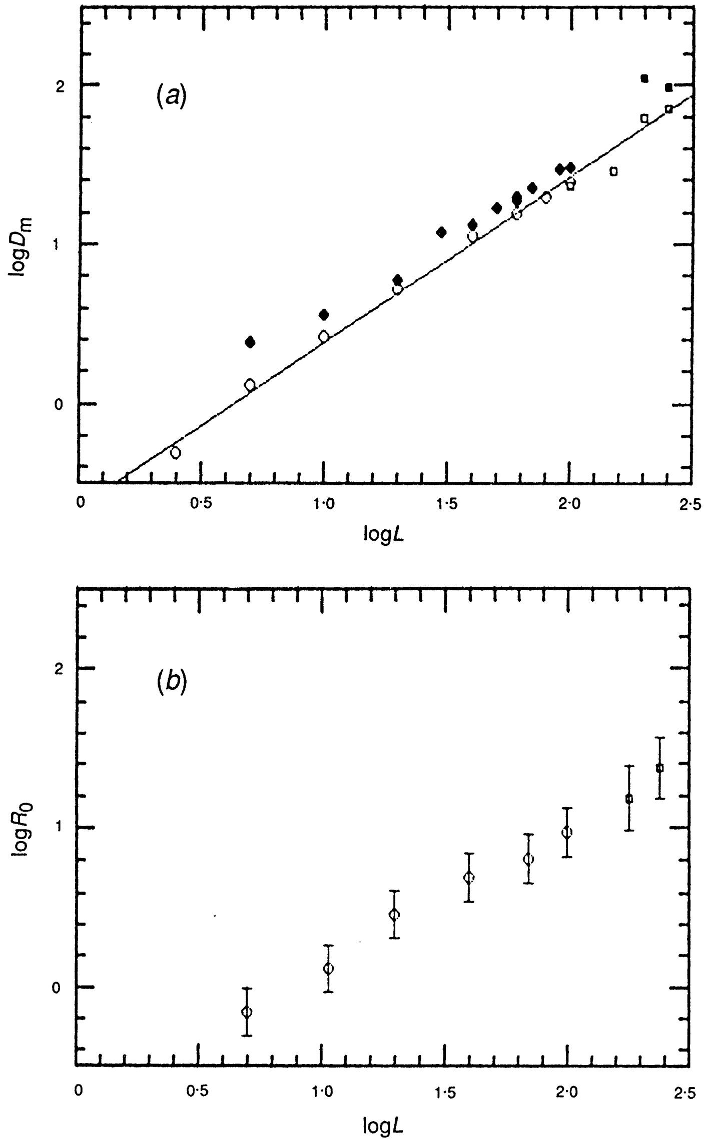

Fig. 1. (*a*) Mean void diameter D_m plotted as a function of the sample size L, both expressed in h^{-1} Mpc. Designations are as follows: circles denote galaxy samples, squares cluster samples. Open symbols represent results obtained with the beam method, solid symbols correspond to data obtained with the empty sphere method. The regression line was determined from galaxy samples and its slope is $1 \cdot 04 \pm 0 \cdot 05$. (*b*) Correlation length R_0 versus sample size L for observed galaxy and cluster samples, denoted by open circles and squares, respectively. The rms errors were estimated using the scatter of individual determinations for various samples. The overall mean relative error was attributed to all R_0 values. [From Einasto *et al.* (1989).]

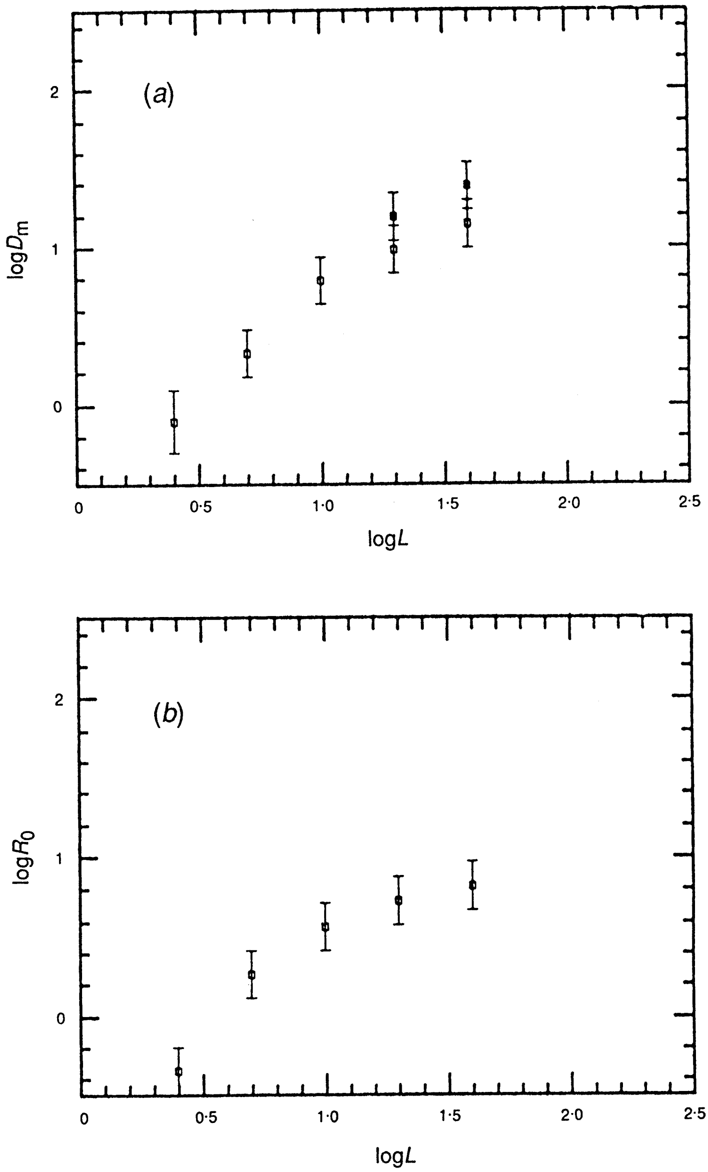

Fig. 2. (*a*) Mean void diameter D_m versus sample size L (in units of h^{-1} Mpc) for the CDM model. Normal galaxy samples are denoted by open squares and galaxy samples in high-density regions corresponding to clusters of galaxies are marked with solid squares. Error bars correspond to external errors, estimated from the scatter of individual determinations. (*b*) Correlation length R_0 versus sample size L for the CDM model. Error bars were found as for mean void diameters. [From Einasto *et al.* (1989).]

is a fractal), while in the random cluster model it is not. Thus the void diameter as well as the correlation analysis can be considered as indicators for the presence of fractal structure.

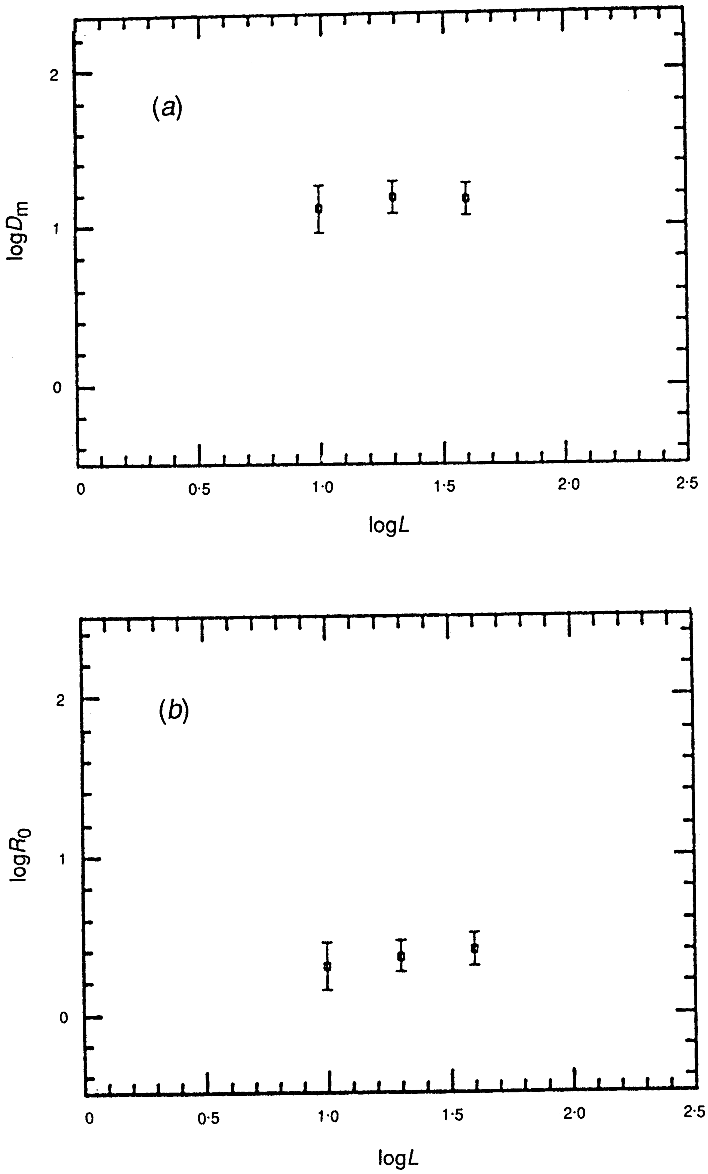

Fig. 3. (*a*) Mean diameter of voids D_m versus sample length L for randomly spaced cluster samples. Errors were estimated as for Fig. 2. (*b*) Mean correlation length R_0 versus sample size L for randomly spaced clusters. [From Einasto *et al.* (1989).]

As in all natural objects the fractal description cannot be extended to very small or very large scales. Available data show that the fractal structure as determined from the self-similarity of voids is really observed in a certain scale interval. In samples of size $L \leq 2\,h^{-1}$ Mpc voids are not defined—these samples contain only one density enhancement if any and this is insufficient to define a void (the other side of the void must also be observed to derive its diameter). Void diameters can be determined for samples

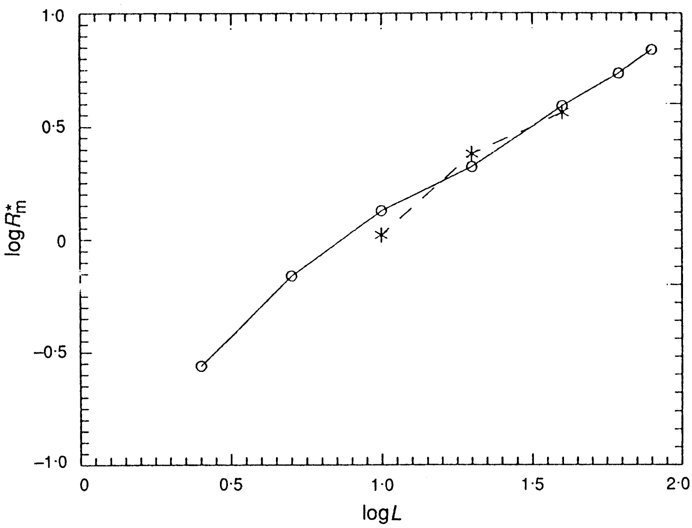

Fig. 4. Median radius of voids R^*_m plotted as a function of the sample size L, both expressed in h^{-1} Mpc. The median radii of voids are reduced to a constant number of particles in samples, $N = 1000$. Real samples are denoted by circles, simulated samples by asterisks.

of size $L \geq 2 \cdot 5\, h^{-1}$ Mpc; this can be considered as the lower limit of the self-similarity and also the fractal nature of the structure.

The upper limit of the fractal nature is of special importance since it determines the scale at which we can consider the universe homogeneous. Presently available data are not deep enough to show the levelling off of the D_m versus L relation.

(d) Structure of Largest Voids

The largest voids observed and described by Jõeveer *et al.* (1978), Kirshner *et al.* (1981), de Lapparent *et al.* (1986), Fairall (1988) and others have diameters of the order 50 h^{-1} Mpc. In relation to fractal structure and the theory of galaxy formation it is important to know whether voids are really empty of all types of galaxies or filled in with dwarf or active galaxies as expected for some theories of galaxy formation (Dekel and Silk 1986).

Detailed studies by Lipovetsky (1987), Thuan *et al.* (1987), Pierre *et al.* (1988), Iovino *et al.* (1988), Einasto (1988, 1989), Eder *et al.* (1989) among others demonstrate, however, that dwarf and active galaxies populate regions identical to bright galaxies and voids are really empty. Most isolated galaxies do not form a distinct population; they are just outlying members of larger galaxy systems.

Of particular interest are voids defined by clusters, superclusters and other objects forming very rarefied populations in the universe. Fig. 1*a* shows that voids defined by rich Abel clusters have mean diameters of the order 100 h^{-1} Mpc, i.e. twice the diameter defined by galaxies. This phenomenon was noticed first by Jõeveer *et al.* (1978) and studied in more detail by Zeldovich *et al.* (1982) and Einasto and Miller (1983). They noted that all nearby rich clusters within a sphere of 70 h^{-1} Mpc form a relatively thin sheet in space which is inclined by 20° to the supergalatic plane. Rich clusters at a distance interval from 75 to 150 h^{-1} Mpc lie at much higher supergalactic latitudes. These objects define two huge cells, the Northern Local Cell and the Southern Local Cell, both having diameters $\approx 100\ h^{-1}$ Mpc. Rich clusters form the skeleton of the cell walls; the interior does not contain any rich clusters.

Recent studies have confirmed the reality of these results. The recently discovered Great Attractor also lies in the sheet of nearby rich clusters (Lynden-Bell *et al.* 1988). Here we observe also a strong concentration of galaxies (Tully 1986, 1987), radio galaxies (Shaver and Pierre 1990; Shaver 1990) and X-ray clusters (Lahav *et al.* (1989). More distant radio galaxies and X-ray clusters are located at higher supergalatic latitudes.

What is important for the fractal properties as well as for the theory of galaxy formation is that the interiors of both local cells are not empty but contain galaxy filaments of various length. Several such filaments are prominent in front of the Hercules supercluster (Tago *et al.* 1986). We note a striking difference in properties of galaxy filaments *within large voids* and *in superclusters*: superclusters are defined by objects forming deep potential wells (rich clusters, X-ray clusters, radio galaxies), here galaxy filaments are rich; in voids filaments are poor, there are no deep potential wells here to form rich clusters, radio galaxies etc.

These data emphasise that the hierarchy of systems of galaxies extends to rather large scales. From a survey of extremely rich clusters Kopylov *et al.* (1988) have found voids of diameter 200 h^{-1} Mpc. The sample is, however, rather small and the reality of such large voids needs confirmation with better data. But voids of diameter 100 h^{-1} Mpc are routinely seen in cluster samples. Thus the hierarchy extends at least to this scale.

Another problem is whether these large voids can be considered as the extension of the fractal structure at respective scales. As noted above the voids defined by rich clusters and other objects forming deep potential wells are not empty, they contain filaments of lower richness. But for a fractal structure the voids should be completely empty. Probably the fractal description breaks down here. If so we must conclude that the scale length $\approx 100\ h^{-1}$ Mpc is close to the upper limit of the fractal structure. To confirm this conclusion deeper galaxy samples covering a sufficiently large area on the sky are needed.

(e) Fractal Dimensions of Galaxy Systems

Another problem connected with the fractal description of structure is the fractal or effective dimension of galaxy systems. This problem has been addressed by Jones *et al.* (1988) and Klypin *et al.* (1989). In the first paper the fractal dimension was derived for the real sample of galaxies between the Virgo and Coma superclusters as well as for the CDM model by Gramann. For both the real and model samples an effective dimension $D \approx 2$ was found. Also it was found that the distribution of points can be expressed by a multifractal.

Klypin *et al.* studied the Virgo supercluster and the inter-supercluster region between the Virgo and Coma superclusters separately. They actually derived not the fractal dimension but the effective dimension of the density law. For the supercluster region they got $D \approx 2$, and for the inter-supercluster region $D \approx 1 \cdot 3$. The relationship of these dimensions with the actual fractal dimension is still unclear.

3. Observational Evidence on Biased Galaxy Formation

An important property of the distribution of galaxies is the absence of visible objects between galaxy filaments and superclusters. Numerical simulations indicate the formation of a connected network of filaments. However, these simulations also demonstrate the presence of a striking difference between theory and observations: in simulation there exists a more or less homogeneous population of non-clustered test particles, which has no counterpart in the observed distribution.

This discrepancy was discussed first by Jõeveer *et al.* (1978). They found a filling factor of $0 \cdot 01$ for matter associated with galaxies and emphasised that it is 'very unlikely that the process of galaxy and supercluster formation was effective enough to evacuate completely such large volumes as cell interiors'. Analytical calculations by Saar (1979) demonstrated that gravitation, the only force responsible for the formation of large scale structure, works very slowly, and there must be primeval particles in voids, seen in numerical simulations as the population of field particles.

Einasto and Einasto (1987), Einasto (1988, 1989) and Eder *et al.* (1989) studied the distribution of galaxies of various luminosity and morphological type at different isolation levels. They demonstrated that isolated galaxies form extended tails of galaxy systems. There exists no population of really isolated galaxies more or less randomly located in voids. We must conclude that galaxies form only in a high-density environment, and in low density regions matter remains in its primeval form. This conclusion was made by Zeldovich *et al.* (1982) using data available in the early 80s.

The absence of galaxies in voids emphasises that galaxy formation is a threshold phenomenon: at low density no formation takes place at all, at high density galaxies of all types can form [galaxies of different luminosity are not segregated, see Eder *et al.* (1989) and Einasto (1989)]. The threshold density level can be estimated by comparing numerical simulations with observations. This has been done by Gramann (1989) using simulations by Gramann (1988) and Efstathiou *et al.* (1985). She has constructed a series of subsamples at varying threshold density levels and compared the percolation parameter B (Einasto *et al.* 1984) of respective subsamples of test particles with the percolation parameter of real samples. At low threshold density only particles in the lowest density regions of voids are removed from the sample and the percolation parameter is in good agreement with the observed value. At a certain threshold density level bridges between clusters are destroyed and the percolation parameter changes rapidly, as seen from Fig. 5. This test demonstrates that a high threshold density contradicts observations. At the present epoch and the critical threshold density at least a half of all particles must be located in high density regions. These estimates put a lower limit on the fraction of particles in systems; the actual fraction of particles in systems can be somewhat higher. An integration of equations describing the dynamical evolution of matter in low and high density regions suggests that at the

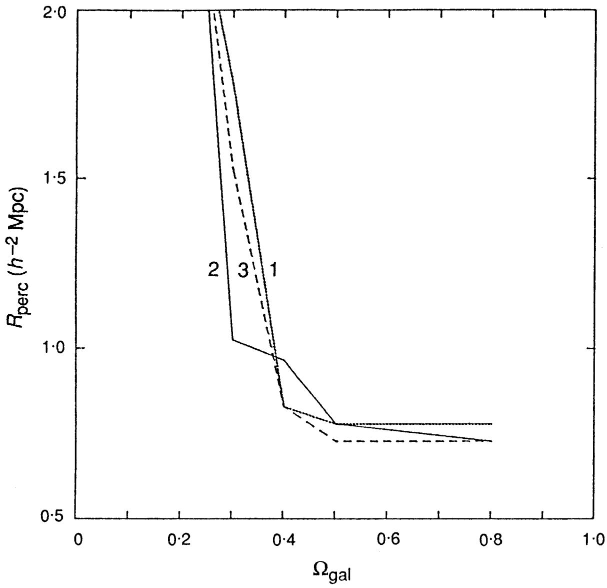

Fig. 5. Percolation radius R_{perc} versus density associated with galaxy systems Ω_{gal} for the CDM model with critical mass density by Efstathiou *et al.* (1985) at epoch $\alpha = 1 \cdot 4$ for three realisations of the model.

present epoch about two-thirds of matter is in systems and one-third in voids (Saar 1979).

The observed amount of matter associated with galaxies (galaxies themselves and dark coronas surrounding galaxies and galaxy systems) is approximately $0 \cdot 2$ of the critical cosmological density (Trimble 1987). This density estimate and the above cited estimate of the fraction of matter associated with galaxy systems can pose a difficulty to models with critical density. One possibility to avoid this difficulty is to use models with a positive cosmological constant. In such models the total amount of matter can be adjusted to satisfy the observed density of matter associated with galaxies.

4. Problems

If the results reported above are confirmed we will have to change our paradigms concerning the structure of the universe in a rather radical way. First of all, statistics like the correlation function have rather limited descriptive character and their use as fundamental descriptors of the universe is questionable. The whole statistical apparatus built up to describe the universe perhaps needs revision. Before making this revision we must ask how reliable are the results.

EKS and EEG have estimated internal and external errors of void diameters and correlation radii. These estimates demonstrate that the observed trend of the increase of the length scales with the sample size L is determined rather well (error bars are indicated in Figs 1, 2 and 3). The same conclusion concerns the connectivity of galaxy systems studied by Gramann (1989).

There remains a more serious question. Observed data sets are rather limited and we do not know if the region we live in is statistically representative or not. Thus our result concerning the fractal nature of the distribution needs confirmation from larger and deeper galaxy samples. It is not completely excluded that some properties of the observed distribution mimic the presence of a fractal structure whereas there are other aspects of the structure which cannot be represented by a fractal formalism. In other words we do not know how useful the fractal description is and what it means in terms of galaxy formation. However, it is clear that a clarification of these questions is important.

5. Conclusions

The principal conclusion from the studies discussed above is the following: presently available galaxy samples cannot be considered as fair samples of the universe. Thus, care is needed in interpreting various statistical results in physical terms: their validity to describe the structure of the whole universe is still uncertain.

The second basic conclusion concerns the use of statistics to describe the structure of the universe. If the structure is really fractal then the correlation function in its present form does not have the meaning which is attributed to it: a univeral tool to derive the principal properties of the universe. It remains open which statistics should be used to give a better description of the universe.

The percolation study raises a serious question about the validity of the closed model of the universe.

The reliability of present data is however insufficient to draw final conclusions on the structure of the universe. First of all more complete data on the real universe are needed for further progress. Also better statistical tools to describe the structure are needed. And finally a detailed study of physical processes related to galaxy formation is required.

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