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Large Scale Structure of the Universe: Theoretical Problems*

E. Saar

Tartu Astrophysical Observatory,
Toravere, 202444 Estonia, USSR.

Abstract

Implications of the observed large scale structure on the physics of the early universe are described. A short review of Soviet work on the subject is given, and the present status of the fractal model of the large scale structure is discussed.

1. Introduction

The properties of the present-day large scale structure are important and interesting *per se*, without any further implications. Astronomy has retained yet a strong 'geographical' approach to learn, map out and name the universe around us. In this respect the study of large scale structure serves to extend the borders of our maps. A perfect example of this approach is a recent atlas of the local structure by Tully and Fisher (1987).

However, there is a physicist in every astronomer, too, who tells us that observational data are never respectable enough by themselves, and that we have to find out the meaning of it all. For the observed large scale structure there are two sides to this hidden knowledge—the way the structure formed and the nature of the initial seeds it formed from. There exist, as usual, a number of different theories for both problems.

In order to understand the ways the observations may serve to decide between those theories, let us review the current theoretical picture of the formation of the large scale structure first. This also gives me an opportunity to give a short review of the present Soviet work on the subject, with particular emphasis on the fractal model.

2. Initial Seeds for Structure

The currently popular inflation scenario has, among other things, given us recipes to predict the initial perturbations responsible for the existence of structure in the universe (see e.g. the review by Linde 1984). Different possibilities here are based on different physical pictures of the elementary particles and fields at extremely high energies, the number of which seems to be comparable to the total number of theoreticians and is about one

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thousand at least! In cosmology the number of presently popular pictures is, fortunately, much less. For our purposes they may be divided into two distinct classes—gaussian and non-gaussian initial perturbations.

The gaussian perturbations represent the conventional idea of chaos as a gaussian stationary random field (see Adler 1981). This field can be represented as a sum of plane waves of different wavelength with random (homogeneously distributed) phases and amplitudes, which are chosen from a gaussian distribution. To completely describe these fields we need to fix only one function describing the dependence of the dispersion of the amplitude distribution on the wavelength (the spectrum of the field).

The spectrum most used today is called the Zeldovich spectrum, which supposes scale-free perturbations of metric (gravitational potential in fact). Depending on the nature of the particles that form the main (dark) component of gravitating matter, the perturbations evolve in different ways, and their spectra at the epoch when the formation of structure starts may be totally different. The two possibilities people usually choose from at present are the hot dark matter spectrum and the much more popular cold dark matter spectrum. I shall describe those in more detail later on, when reviewing the scenarios of the formation of structure.

Other, more recent and unconventional hypotheses are usually clumped together under the label of non-gaussian perturbations. One can call any field non-gaussian that does not conform to the conditions listed above, but most of the theories of the inflation stage seem to give us fields that have phase dependencies built in (Allen *et al.* 1987). Built-in phase dependencies are, of course, initial patterns, which may well be frozen into the present structure. An extreme example of the patterns possible is the proposal by Kofman and Linde (1987) to build completely empty voids into the matter distribution already from the beginning.

Another interesting example is the well-known cosmic string scenario (Zeldovich 1980; Vilenkin 1981). The cosmic strings form no continuous field in space, but they may serve as seeds for later galaxy formation. Moreover, in this case we really do not need initial perturbations for the bulk of the gravitating matter.

The last two examples belong to another picture of chaos—the fractal picture. Recent work in dynamical systems shows that this may really be the generic picture of chaos. Here we have no continuity of our fields in space, but we have built-in patterns and self-similarity of structures instead. In order to learn which type of chaos occurred in our past we must follow the evolution of initial perturbations up to the present-day structure.

3. Dynamical Evolution of Structure

It is easy enough to follow the evolution of the initial perturbations into fully formed structure if they are small (the so-called linear stage). This can be done, of course, only for perturbations that *are* initially small, and not for the strings, which are nonlinear from the very beginning. Things get more complicated and interesting when structure becomes nonlinear. These stages can be presently studied by numerical simulations only.

The first type of numerical simulation we shall review concerns models which are not very popular nowadays—those with hot dark matter (HDM). This model became high fashion in about 1980 with the measurement of the neutrino mass. The distinctive feature of the model is a sharp cut-off in the spectrum of the perturbations at a large wavelength, corresponding to a typical mass of a rich cluster of galaxies. The evolution of structure here is a top–down process, the large-scale skeleton forming first and then fragmenting into galaxies. There are no perturbations at smaller scales.

These models represented the observed structure well enough up to the moment when the inflationary universe became the 'true religion.' This universe has too large a matter density for the HDM model—the large scale structure evolves too fast and is too heavy and too hot to form galaxies. The present way out of these difficulties is to use unstable particles for the HDM, which decay just after the large scale features have formed and before galaxies begin to form. The last series of these models by Doroshkevich *et al.* (1988) is definitely state-of-art and fits nicely all the observational data. It means that the top–down scenario is, in fact, alive and well. Of course, this model is definitely not fractal, and some fractal features may be introduced into it only in the process of the fragmentation of structure into smaller objects.

Another conventional model is the cold dark matter (CDM) model, which has no real cutoff in the spectrum, and leads to the formation of well-developed structure both at small and large scales. The formation of structure in this model goes bottom–up, with less massive objects forming first and then merging together. This is the current orthodox religion as far as simulations go. There are many groups active in this field, but for our review I shall mention A. Klypin's work (see Klypin *et al.* 1989) and that of M. Gramann from our observatory (see Gramann 1988). Gramann included the cosmological constant in her models to get rid of the self-destruction of structure, which is also a problem for CDM models.

Now CDM models, starting from a normal gaussian field, may easily lead to an observed fractal large scale structure. This is due to a peculiar perturbation spectrum they have, which extends to very high frequencies. It is known that if we cut such a field at a certain level, the isosurface of the field we obtain is fractal (Adler 1981). Just this sort of cut is implied by current biasing scenarios of galaxy formation.

Both types of simulations described above follow the dynamical evolution of a piece of the universe from the end of the linear stage (small perturbations) to the present day and beyond. This process takes a long time both in reality and in actual simulations, and simulations lead usually to accumulation of numerical errors, which seriously reduce the dynamical and spatial range of the results. Now, recently there has appeared a new technique to predict the pattern of the large scale structure at any moment without following the evolution at all (Kofman and Shandarin 1988). This approach was labelled 'the adhesion model' by its authors, meaning that dynamical processes inside the structures are ignored, and particles usually passing each other are meant to stick together. This technique is still young, but certainly promising.

In contrast to the previously described models, the string models demand a numerical approach from the very beginning. It seems that those models, beginning from a fractal start, remain fractal. Bennett and Bouchet (1988) demonstrated that the string network itself has a fractal structure, and Scherrer *et al.* (1989) showed that hot dark matter with string loop seeds leads to a structure resembling that from the cold dark matter and close to observations. This sort of structure is probably fractal too. This subject is not yet popular in the USSR.

In summary, if the observed large-scale structure is fractal, it is rather difficult to say at once what the initial conditions were like. As we saw, it is probably possible to get a fractal structure both from a fractal initial state or from a continuous random field with a specific (CDM-type) spectrum. This means that we must study the fractal properties of the structure in more detail. But before doing this we must get the answer to the most important question—is the structure really fractal?

4. Evidence for and against a Fractal Structure

I have spoken of fractals so far without giving any definitions or examples of fractals. (I believe that this is not really necessary in Australia—anybody who has been in Sydney or seen its map knows well the beautiful fractal of the waterways which, first, resembles some superclusters we see and, secondly, has all the same problems of upper and lower fractal ranges found in the supercluster pattern.) Evidence for a fractal large scale structure in the sky comes from several sources, as described by Einasto (1990, present issue p. 145). There are, first, tight correlations between the correlation radius and the depth of a sample, and a similar dependence between the median void size and the sample size. There are also two more direct results—the power-law pair distribution function derived by Coleman *et al.* (1988) and the mass–distance behaviour in our neighbourhood studied by Klypin *et al.* (1989).

The main facts used against the fractal picture are two (Peebles 1988). Those are the Poisson scaling behaviour of the two-dimensional correlation radius with the sample depth and the number count–apparent magnitude relation, which tells us that space is homogeneously populated by galaxies.

Thus, the arguments for are essentially 'three-dimensional' and are based on data from our nearby neighbourhood (the CfA-I sample up to 8000 $\mathrm{km\,s^{-1}}$ mainly). The arguments against are 'two-dimensional' and are based on samples extending to much larger distances. Let us look at the nearby region first.

The distribution of galaxies in the region responsible for most recent statistical results is shown in Fig. 1. It is a cube aligned along the axes of the supergalactic coordinates, with a cube side of 8000 $\mathrm{km\,s^{-1}}\,H^{-1}$ (H is your own version of the Hubble constant). You look into this cube down from high supergalactic Z. Our Galaxy lies in the centre of the bottom face of the cube. The formation seen in the lower part of the figure is our Local Supercluster, while the Coma Supergalaxy hangs from the upper face of the cube, and they are connected with a solid bridge. This bridge spans a huge void between the two superclusters. The most striking feature of this picture is its regularity. Apart from some small clouds of galaxies it is essentially dominated by the main features listed above.

Now, all our statistical samples are usually cones with a tip in our location, with axis aligned almost along the supergalactic Y-axis (directed upwards in the figure), and extending 50° from the axis. These cones lie entirely inside our cube, and when we increase the sample depth, they encompass more and more of the void between the two superclusters. It can be shown that the correlation amplitude (the value of the correlation function at zero separation) is inversely proportional to the filling factor of the sample (Saar 1989). Thus the correlation radius (or amplitude) dependence on the sample depth really does not need fractals, although it can be explained by a fractal distribution of galaxies.

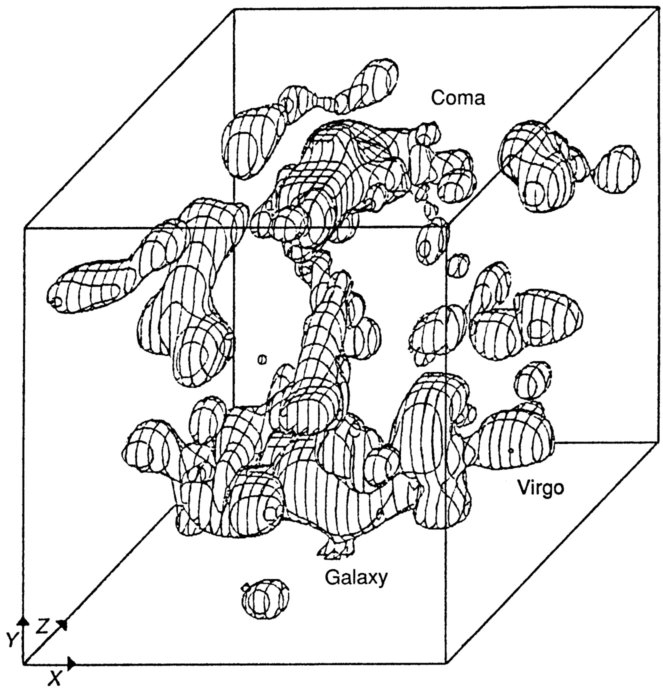

Fig. 1. The distribution of galaxies in our neighbourhood.

Anyway, the mathematical model behind the correlation picture—the stationary random field—cannot describe the observed situation at all. The distribution of galaxies in Fig. 1 is far from a stationary random field. It means that if the fractal picture does not work, we must look for something radically different.

The two results on the slope of the pair distribution and mass–radius relation, although formally supporting fractals, are not very decisive either. Both are very 'galacto-centric', meaning that the results for large-distance behaviour concern only one region of reference, that of our closest vicinity, and it is impossible to say how well they describe the whole sample. So we have at hand only one really good argument for fractals—that concerning the behaviour of the void sizes.

Now, ironically, there exists one more argument of a negative kind that in fact works to support the fractal picture. This is the fact that fractal features are very difficult to measure for the kind of severely undersampled samples we have. Fractal characteristics are usually determined looking for power-law dependencies between spatial scales and densities, separations, etc. Now, in our best complete samples we have less than one thousand galaxies, meaning that the range of scales that can be studied is less than 1:10. And even there both the discreteness effects (too few points per unit spatial scale) and the edge effects are severe. As an example I show in Fig. 2 the results of a practical determination of the fractal dimension for a Poisson distribution of 10 000 points in a square (solid circles). The curve shows the correction for the discreteness effects (undersampling), and the large-scale deviation comes from the edge effects. So, although the density of points is much larger here than in observed samples, we do not even reach the real dimension of two.

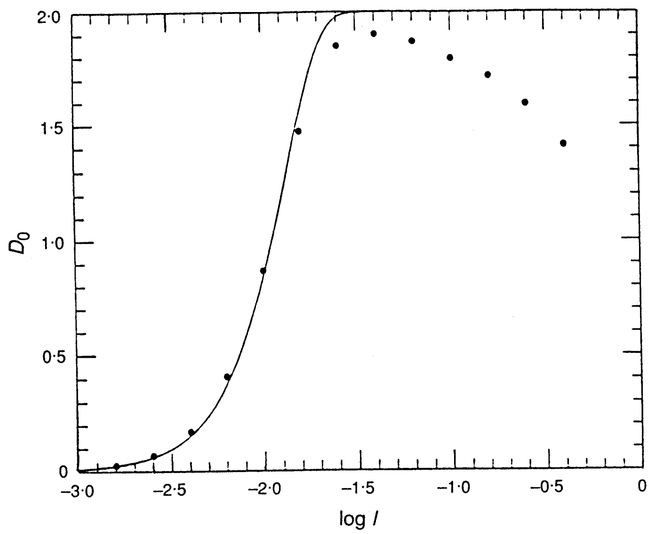

Fig. 2. Dependence of the fractal (box-counting) dimension D_0 on the scale l (measured in units of the size of the region) for a Poisson point distribution in a square. The curve shows the effect of undersampling (too few points in a cell of size l).

A similar effect dominates the observed sample in Fig. 3 (the CfA-I cone up to 8000 km s^{-1}). Here we see good fractal behaviour only in the case of a biassed CDM simulation, and observational data do not seem to support fractal behaviour at all (no fixed dimension). Correcting this for discreteness effects gives us the bold line with $D \approx 1 \cdot 8$. This result has not been corrected for edge effects as yet.

What about the deep (2D) results? They are, firstly, not very clearly established. Most of the correlation scaling results do not have proper error estimates, coming from earlier times than the present error estimation methods, and the number count results depend on the luminosity function used. And, of course, the galaxy distribution is not Poissonian at all, as these results try to tell us. This means that they are extremely insensitive to details of structure of sizes up to 50 Mpc (the sizes of the observed voids).

Anyway, it is safe to assume that there must exist an upper limit to the size of fractal regions, and really deep samples must be homogeneous. As

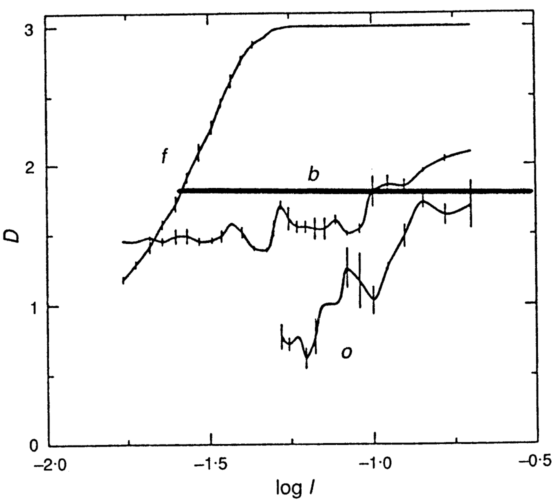

Fig. 3. Fractal dimension versus scale size for three samples: curve *f* gives the results for a full CDM simulation (all particles); *b* for a biassed version of the same simulation; and *o* for a complete sample of galaxies in the CfA-I cone up to 8000 km s^{-1}. The bold line shows the effect of an undersampling correction on the observational results.

emphasised by Mandelbrot (1989), there are many ways of crossover from a fractal to a homogeneous density distribution. Judging by the present 3D samples we have, the crossover region cannot begin nearer than 50–60 Mpc. And probably the way of crossover will help us to choose between different models and initial conditions. Surely the crossovers for string models and for a biassed CDM model are entirely different.

5. Conclusions

As we have seen, the situation at present is unclear. The correlation picture is not fit to describe the nearby structure, but we do not know yet if fractals are better. To learn this we must, firstly, get data for deeper samples to study the crossover scales and, secondly, get more data for nearby regions, too, to sample better the structure details. Of course, in order to improve the spatial resolution ten times we must measure the redshifts for a thousand times more galaxies—we will certainly run out of galaxies before this can be done. I do hope that there are enough galaxies left to measure in the southern sky!

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