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Annual Reconstruction of the Solar Cycle from Atmospheric ^{14}C Variations

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Abstract

Initially, the rise and fall components of the 11-year solar sunspot cycle are approximated by separate least-squares polynomials for four cycle classifications, which are determined by the magnitude of the average of the annual sunspot numbers per cycle. Following, a method is formulated to generate detailed reconstruction of the annual variation of a solar cycle based on this cycle average, and the results obtained for cycles −4 through to 21 are compared with the annual Zurich values. This procedure is then employed to establish annual sunspot numbers using published average cycle values obtained from atmospheric carbon 14 variations, which have been derived from the chemical analysis of tree ring sections. The reconstructed sequences are correlated with the observed cycle values and with tree ring width index chronologies which exhibit a significant 11-year periodicity. It is anticipated that the long carbon 14 records and parallel dendrochronological data could be employed to obtain a more detailed portrayal of previous periods of strong solar activity than that given by current estimates based on historical records.

1. Introduction

Accurate sunspot statistics cover only about 21 solar cycles, the period from 1749, with records of less reliability going back to 1700 which are essentially based on revised values of Wolf's (1868) estimates. Earlier values of annual sunspot numbers over 1610–99 have been constructed from intermittent sunspot records, and for the period 1500–1609 they have been compiled from historical records of the number of days per year on which aurorae were sighted (Schove 1983). Moreover, Schove (1955) has also assembled a table, giving estimated years of solar maximum and their magnitude since approximately 500BC, likewise based on accounts of naked eye sunspot sightings and aurorae. His tabulation has been compiled subject to certain assumptions relating to time limits between successive maxima and with the stipulation that there are nine maxima in 100 years. It was concluded that besides the 11-year cycle a 78-year cycle in phase is evident, being equivalent to seven solar cycles, and a cycle of about 200 years may also exist in auroral activity. Furthermore, the estimated annual sunspot numbers over the period of the Maunder minimum indicate a continuation of solar activity, albeit at a reduced level, in contrast with the estimates given by Eddy (1976) which indicate that there were long periods of zero sunspots.

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It is now possible to add supplementary detail to these historically based early estimates of solar activity from several areas of current research. In particular, the carbon 14 record can be utilised to provide an approximation to the magnitude of solar activity and tree ring width index chronologies used to adjust the phase of the averaged 11-year cycles. Such procedures of course require validation by application to the known data post-1700.

Some recent spectral investigations of tree ring index chronologies show a significant 11-year periodicity imbedded in the ring widths (Mori 1981; Murphy and Whetton 1989) with a phase delay relative to the solar cycle of about three years, based on cross spectral analysis. In addition lagged auto-correlation analysis of this tree ring data exhibits a strong positive correlation at 11-year intervals, and cross correlation with annual sunspot data since 1700 confirms a phase delay of about three years. The latter results also portray a periodicity between the two time series of about 11 years. Such chronologies, although generally based on noisy data, can provide proxy evidence over several centuries for solar variability, which in turn can be used to substantiate data compiled from other sources.

Relative variations in atmospheric ^{14}C levels were derived by Stuiver and Quay (1980) from chemical analysis of tree ring sections extending over a period of about 900 years. Through the use of a carbon reservoir model they calculated ^{14}C production rate changes, relative to the long term average value, attributable to solar activity. Approximate average sunspot numbers per cycle (W_M) were thus derived from an empirical relationship to establish the magnitude of solar variation over the period AD1000 to 1900. Three long intermissions of solar activity are evident, and based on criteria incorporated in the model they correspond to periods of near absence of sunspots. Other and considerably longer ^{14}C records have been compiled which have yielded several dominant cycles in addition to the 11-year cycle (Sonett and Finney 1989).

The overall objective of this paper is to formulate a method to generate a detailed reconstruction of the solar cycle from ^{14}C records which give W_M values. Initially, it is necessary to prototype the annual variation of the solar cycle by determining separate polynomials, depending upon the magnitude of W_M, to represent the rise and fall components. Next a method of solar cycle reconstruction based on W_M values from the Zurich sequence of sunspot numbers is developed and the resulting R_M values compared against the actual values. The established procedure is then applied to reconstruct annual sunspot numbers incorporating the W_M values from the ^{14}C model of Stuiver and Quay (1981). Both reconstructed sequences are correlated with the observed cycle values and all with a tree ring width index series. The viability of adopting the model to establish the details of earlier periods of strong solar activity, as delineated by the ^{14}C results, is considered. It is noted that correlations with dendrochronological data over the same period could provide a check, as an alternative to historical auroral sightings, on the occurrences of maximum epochs.

Eddy (1977) has given a diagrammatic summary of the time span of available data on past solar variability and it is immediately evident, and somewhat disappointing, that a lot of important observations have only been made systematically over a few solar cycles. In contrast, the carbon 14 record, which has been extended back to 5000BC, represents the best and possibly the only mode of historical reconstruction of solar variability as manifested by the solar cycle. This, of course, is based on the assumption that there is an interdependence between the ^{14}C production rate and sunspot number; the results based on the model employed by Stuiver and Quay (1980) offer substantive evidence of this association.

2. Solar Cycle Variation

Ever since its discovery, many attempts have been made to formulate the variation, in time, of the 11-year solar sunspot cycle with the primary objective of predicting the occurrence of the epochs of cycle maximum and minimum along with the magnitude of the cycle maximum, R_M. The records of yearly Zurich sunspot numbers show, since 1700, considerable variation in both R_M and the length of the '11-year' cycle. Waldmeier (1976) suggested that the profile of the annual values over a cycle is essentially governed by the value of the maximum sunspot number with the approximate dependence between R_M and T_R, the time in years from cycle minimum to maximum, given by

$$\log R_M = 2 \cdot 73 - 0 \cdot 18 T_R .$$

Generally we have $T_R \approx 3$ for high R_M cycles, 4 for medium and 5–6 for low R_M cycles, which produces an asymmetry in the curves for high and medium cycles. A variety of multi-parameter analytical formulations has been proposed to approximate the variation or relative sunspot numbers over each cycle (Vitinskii 1962). Although it is necessary to establish a set of these constants for each individual cycle (Stewart and Panofsky 1938), it is also possible to obtain a representative curve for the very high, high, medium and low cycles based on averaging.

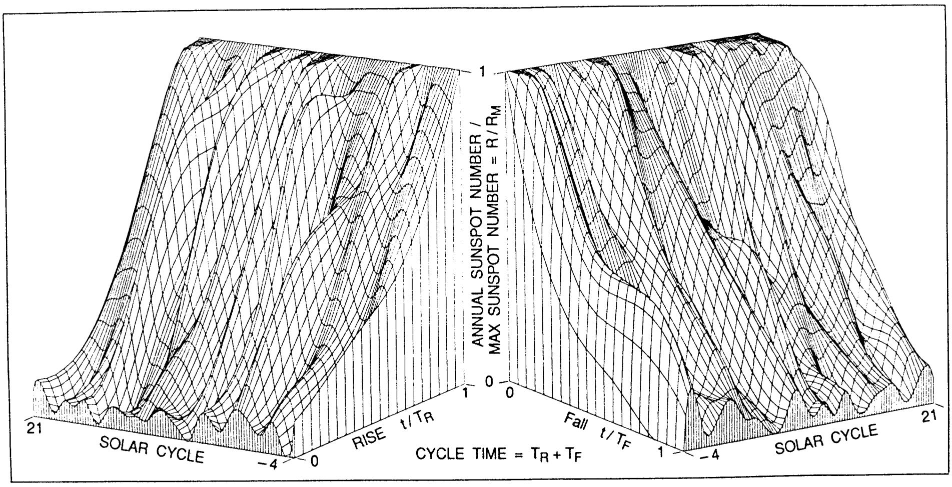

Fig. 1. Representation of the variation of the rise and fall components of cycles −4 to 21 when the annual sunspot numbers for each cycle are scaled by the associated R_M for the cycle. The rise time is scaled by T_R, in years, for each cycle and the fall time is scaled by T_F for each cycle. In both cases the scaled time t is such that $0 \leq t \leq 1$ and $0 \leq R/R_M \leq 1$.

The two surfaces plotted in Fig. 1 represent, on a comparative basis, the variation from minimum to maximum, and separately from maximum to minimum, of the annual relative sunspot numbers in solar cycles −4 through to 21. They are essentially based on data obtained from the record of the yearly means of Zurich numbers over 1700–1957, given by Chernosky and Hagan (1958), and on values from the following years published in the *Journal of Geophysical Research*. In this representation, the annual values of R from minimum to maximum in each cycle have been normalised by R_M for the cycle and the time of rise in years, T_R, scaled to one. The number of values of R/R_M, over the time interval [0, 1] will depend upon the nature of the cycle, which is essentially determined by the magnitude of R_M, and equals $T_R + 1$. Cubic spline interpolation was next employed, when necessary, to establish a set of ordinates over the interval (0, 1) at the scaled time values of 0·2, 0·4, 0·6 and 0·8. The normalised values at each end of the interval are unchanged by the procedure and are given by R_m/R_M and 1, with $R_m = 0$ in some cycles. The smoothed surface, portraying the rise from minimum to maximum over the 26 recorded cycles, has been drawn up from this data. A similar approach has been employed to determine the data used for the fall from maximum to minimum surface. In this case the minimum value of the next solar cycle is utilised and the number of points before interpolation is given by $T_F + 1$, where T_F is the time of fall.

Table 1. Sunspot data and classification for cycles −4 to 21

W_M range	R_M range	Type	Type T_R (yr)	Cycle (number)	Total	Actual average R_M	T_R
≥60	106 → 190	V	3	2, 3, 4, 8, 18, 19, 20, 21	8	142	3·1
≥50, <60	96 → 139	H	4	−2, 9, 10, 11, 17	5	117	4·0
≥40, <50	78 → 106	M	5	−1, 1, 15, 16	4	94	4·8
<40	46 → 85	L	6	−4, −3, 0, 5, 6, 7, 12, 13, 14	9	64	5·8

When visualised on this basis it appears that the rise and fall components of any solar cycle can be represented, within reasonable accuracy, by an appropriately determined mean curve. However, the diagrams given by Waldmeier (1968) for $R_M = 60$ to 150, illustrating the relationship of the sunspot curves to the year of the previous minimum and the year of the maximum, indicate that the value of R_M is the significant factor governing the shape of the cycle profile. Hence it would appear appropriate to determine separate representative cycle curves based on ranges of R_M or W_M, the average of the annual sunspot relative numbers in the cycle. A classification of the 26 cycles into four categories is given in Table 1 where $T_R = 3$ yr for very high W_M (V) to $T_R = 6$ yr for low W_M (L), along with high (H) and medium (M) groups. The choice of W_M is dictated by the published form of the sunspot number data calculated from the ^{14}C production record, which has been averaged over a 10-year interval.

Least-squares analysis has been employed to determine the form of polynomial curves, of degrees 1 to 4 which are constrained to pass through the point (1, 1), to characterise the increase in yearly sunspot numbers from minimum to maximum for each of the V, H, M and L cycle types. Although the scaled data are rather scattered, it can be perceived that an adequate representation is given by a polynomial dependence and that alternative formulations, such as a constrained exponential variation, would be unlikely to reduce the mean-square error. These polynomials take the form

$$R/R_M = 1 + \sum_{i=1}^{n} a_i(1-t)^i; \qquad n = 1,2,3,4, \quad 0 \le t \le 1,$$

and on the basis of mean-square error, the optimum curves for the four types (V → L) are given by polynomials of degree 4. Larger mean-square errors result when least-squares polynomials are fitted to the total data set of 26 cycles, which tends to validate the four group classification.

An equivalent analysis has been applied to the fall component of the solar cycle, in this case all the least-squares curves pass through (0, 1) and are given by

$$R/R_M = 1 + \sum_{i=1}^{n} a_i t^i; \qquad n = 1,2,3,4, \quad 0 \le t \le 1.$$

3. Solar Cycle Reconstruction

Initially, the W_M values for solar cycles −4 to 21 will be utilised to model the actual solar cycle, on a year to year basis from 1700 to 1988, employing the rise and fall profiles already established. A fundamental cycle length of 11 years will be adopted, but slightly modified, to give nine complete cycles every one hundred years in the reconstructed model. The necessary input data specify: year of initial cycle minimum (AD1700 for cycle −4), cycle number, length of cycle in years (11 or 12), type of cycle and corresponding value of W_M. The value of T_R is implied and given in Table 1.

Selecting quartics, the rising curve for an M cycle, for example, is given by

$$Y_R(t) = 1\cdot0000 - 1\cdot9146(1-t) + 5\cdot7187(1-t)^2 - 9\cdot7700(1-t)^3 + 5\cdot0241(1-t)^4,$$

the falling curve has the form

$$Y_F(t) = 1\cdot0000 - 0\cdot6954t - 2\cdot6286t^2 + 4\cdot3916t^3 - 1\cdot9788t^4,$$

and the sum

$$S = \left(\sum_{i=0}^{5} Y_R(i/5) + \sum_{i=1}^{5} Y_F(i/6)\right)/11,$$

representing the scaled average, is evaluated.

The scaling factor for the cycle is now established from

$$\alpha = W_M/S,$$

and the yearly reconstructed values for sunspot numbers from

$$\alpha Y_R(i/5), \qquad i = 0, 1, \dots 5$$

for the rising branch, with R_M given by $\alpha Y_R(1)$, and from

$$\alpha Y_F(i/6), \qquad i = 1, 2, \dots 5$$

for the falling branch of the 11-year cycle.

The value of S, which is fixed for each cycle type, actually determines the ratio W_M/R_M. Only a small variation is found between the numerical values for the cycle classifications, which can be explained by the remarkably high linear correlation $r = 0 \cdot 961$ ($P < 0 \cdot 0001$), which exists between R_M and W_M for the 26 solar cycles over 1700–1986. A linear regression is given by

$$R_M = 8 \cdot 03 + 1 \cdot 89 W_M,$$

which in turn can be approximated by

$$R_M = 2 \cdot 05 W_M,$$

for values of W_M within the actual range for all cycles. This ratio is consistent with the S values determined for the cycles, which are given below. A modification of Waldmeir's (1976) relationship between R_M and T_R now establishes the approximation

$$\log W_M = 2 \cdot 42 - 0 \cdot 18 T_R.$$

Table 2. Reconstructed sunspot numbers for cycle −1

Year	1733	34	35	36	37	38	39	40	41	42	43
Actual R	6	17	33	64	77	106	96	67	39	20	16
Reconstructed R	6	18	45	65	78	100	83	62	42	28	18

Repetition of the above technique now allows the yearly sunspot number to be approximated over the years 1700–1988 on the basis of known W_M, with

$$S_L = 0 \cdot 455, \quad S_M = 0 \cdot 495, \quad S_H = 0 \cdot 457, \quad S_V = 0 \cdot 506.$$

To illustrate the outcome, the actual values for cycle −1, with minimum in year 1733, maximum in 1738, $R_M = 106$, $W_M = 49 \cdot 7$ and classified as M, are compared with the reconstructed values in Table 2.

Fig. 2 exhibits the reconstructed annual mean sunspot number over cycles −4 to 21 along with the actual variation. A further measure of this correspondence is established by the relative magnitudes of the lagged linear cross-correlation

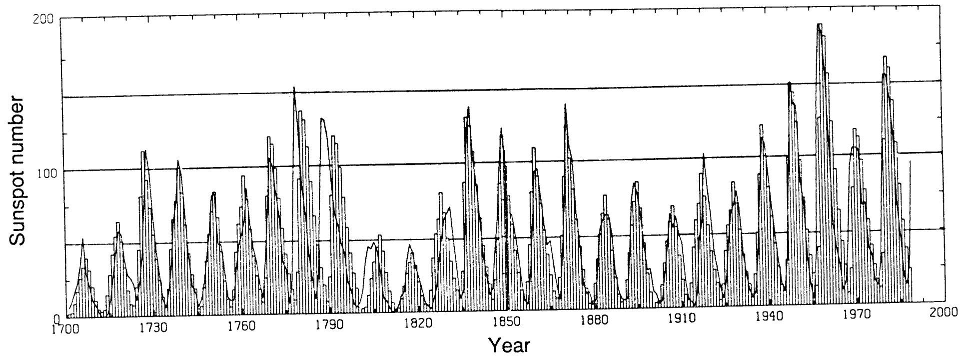

Fig. 2. Model reconstruction of the annual variation of solar cycles −4 to 21 (vertical bars) and the actual variation (curve).

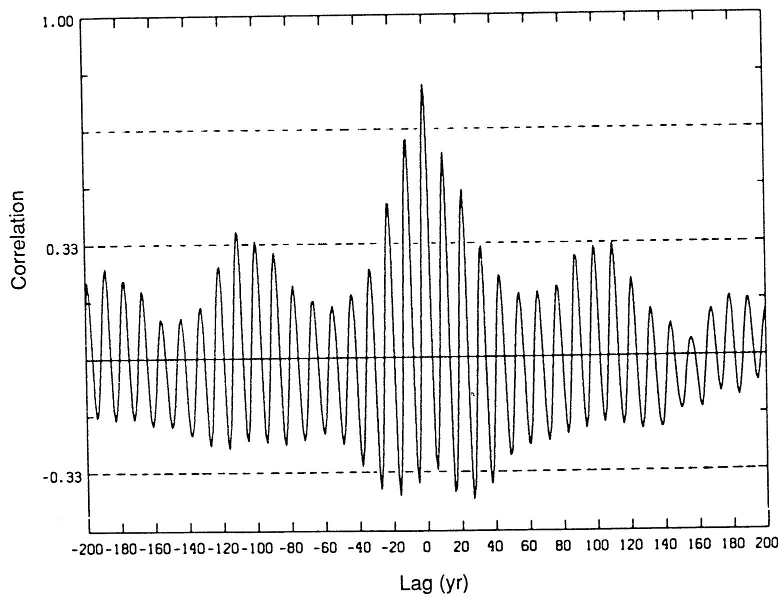

Fig. 3. Linear cross-correlation coefficients, at various lags, for the actual and reconstructed solar variation, based on nine cycles in 100 years, over cycles −4 to 21. The 11-year cyclicity and longer term modulation is similar to that demonstrated by the lagged auto-correlation coefficient for the actual data.

coefficients between the two sets, as illustrated in Fig. 3. There are two significant features apparent in these results which support the validity of the reconstructed cycle. Firstly, the maximum value of the cross-correlation coefficient occurring at lag zero provides confirmation that the phase between the two sets is also satisfactory. Secondly, the curve representing the lagged values replicates the significant features of the lagged auto-correlation function for the actual sunspot number data. On the basis established above it is now possible to undertake the detailed reconstruction of solar cycles based on W_M values provided by the radio carbon data. How representative these

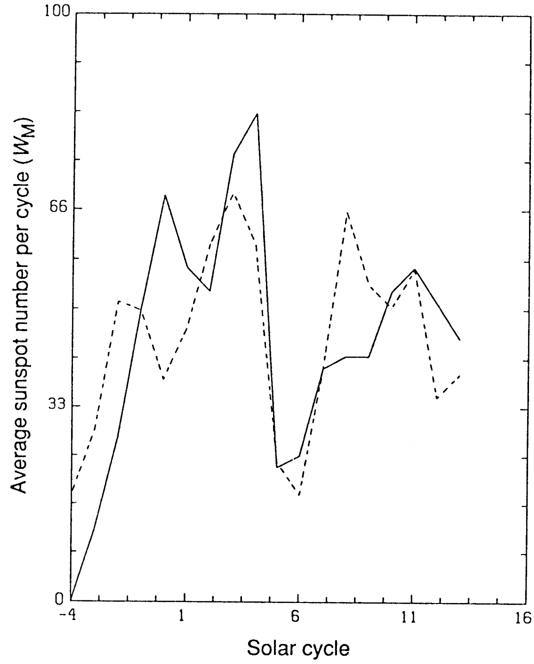

Fig. 4. Values of W_M for solar cycles −4 to 13: dashed line, actual; solid line, that derived from the ^{14}C model of Stuiver and Quay (1980).

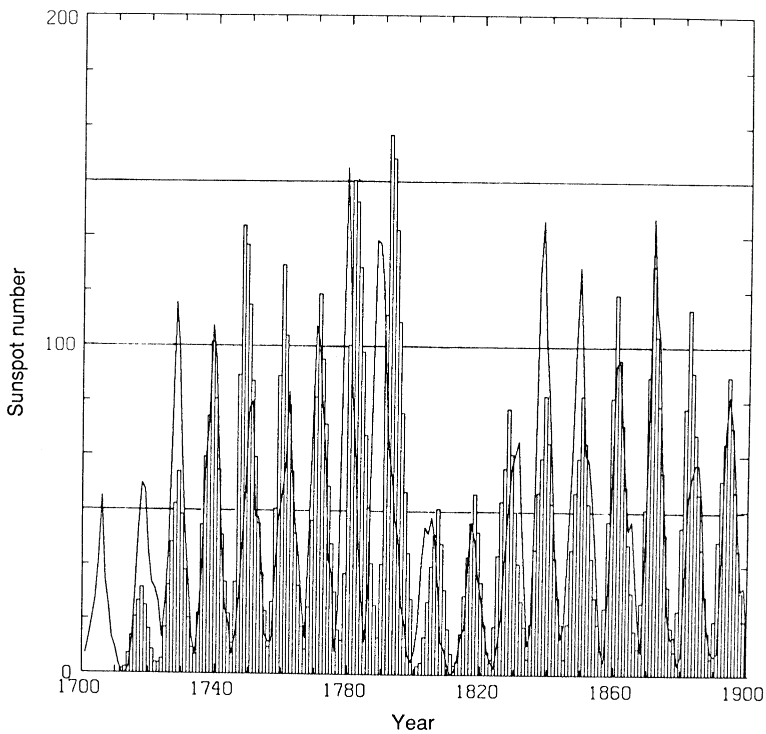

Fig. 5. Annual profiles of sunspot cycles −4 to 13 as determined from the reconstruction model with W_M values obtained from the carbon record and given in Fig 4. The replication of the downturn in solar activity following cycle 4 is of particular interest; W_M is zero for cycle −4. The overdrawn line represents the actual sunspot numbers over 18 cycles.

measurements are of actual sunspot activity can be ascertained from the following detailed reconstruction results.

A record of atmospheric ^{14}C variability over recent centuries can be established from ^{14}C measurements of dated tree ring samples. These are sometimes tabulated as decade averages, obtained from the analysis of 10-year tree ring sections, or by determining an average or measurements of 10 single year rings. Atmospheric ^{14}C values, representing the relative deviations of the measured ^{14}C values after correction for long term trends, based on wood samples of Douglas fir trees from the Pacific North West of America, have been compiled over a 860-year period, 1000 to 1860, by Stuiver and Quay (1980). They have identified three prominent maximum epochs in the ^{14}C record which translate to prolonged periods of minimum sunspot activity.

Relative changes in the ^{14}C production rates, which in turn are significantly linked with solar activity levels, have been calculated from a reservoir model by Stuiver and Quay (1980). They have established a correlation of $r = 0 \cdot 89$ between the average sunspot number per cycle, W_M, and the change in ^{14}C production rate. Consequently, approximate values for W_M have been computed over an 860-year interval utilising an analytical relationship derived from the sunspot record over 1720 to 1860, which is acknowledged to be reliable. The W_M magnitudes for solar cycles −4 to 13 (AD1700 to 1900), using the criterion of nine cycles per century, have been interpolated from the decadal values given by Stuiver and Quay (1980) and are plotted, along with the corresponding actual sunspot average per cycle, in Fig. 4. As W_M varies only on a long time scale, linear interpolation gives an adequate approximation.

Even though average sunspot numbers per cycle have been calculated back to AD1000 by Stuiver and Quay (1980), and more recently they have established ^{14}C production rate changes post AD300 (Stuiver and Quay 1981), our attention is confined to the period following the Maunder minimum to allow comparison with the record of observed sunspot numbers. Adopting the previous mode of cycle classification based on the magnitude of W_M from the ^{14}C data, and employing the same computational procedure, determines the detailed profile of cycles −4 to 13 as depicted in Fig. 5. The breakdown of the 18 cycles is 5–L, 5–M, 5–H and 3–V, compared with 7–L, 3–M, 4–H and 4–V actual.

4. Results

The reconstruction algorithm has duplicated the detail of solar variability over 26 cycles to a creditable extent in the case when the observed sunspot data values are utilised. Some variation in both phase and amplitude of R_M is evident when a comparison is made on a cycle by cycle basis between the reconstructed carbon ($^{14}C_R$) and actual series. However, with the exception of cycles −4 and −3, this series reproduces the broad characteristic features associated with the other cycles over two centuries. In particular, the $^{14}C_R$ series mimics the most notable characteristic of the contemporary record—the plunge from the very high values of R_M for cycles 3 and 4 to the very low values of R_M recorded for cycles 5 and 6, and then the upward trend of R_M after cycle 6. It is of interest to note that this glitch in the sunspot and ^{14}C record is also evident in some tree ring width chronologies obtained

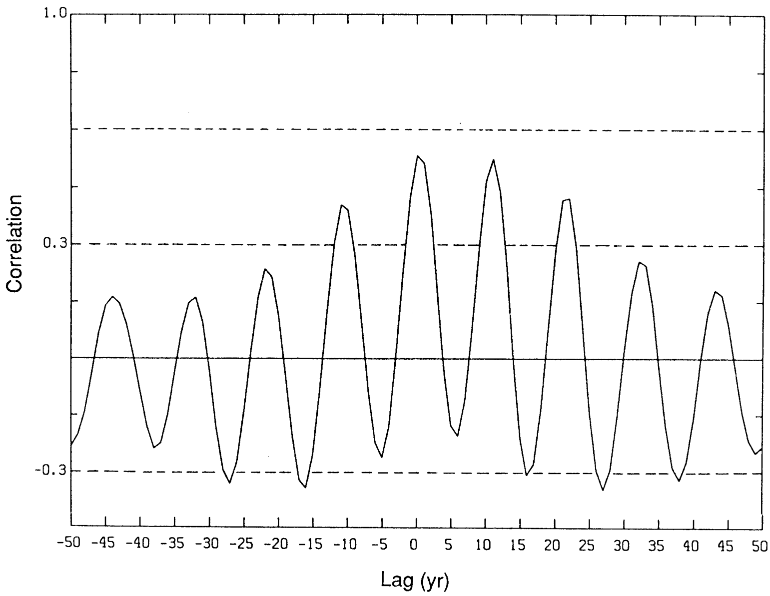

Fig. 6. Cross correlation between the actual sunspot sequences for cycles −4 to 13 and the ^{14}C reconstruction series. Here maximum correlation exists at both lag 0 and lag 11 and is an indicator of phase—the correlation coefficient peaks at lags of 11 years reflect the model cyclicity and are to be expected.

from high altitude sites (Murphy and Whetton 1989). The discrepancies in R_M values computed for $^{14}C_R$ cycles −4 and −3 reflects on the low values of W_M obtained from the ^{14}C model calculations, rather than on the viability of the reconstruction technique. In view of the acceptable correspondence already attained between the other cycles, the value $W_M = 0$ for cycle −4 must be attributed to other factors. Is it possible that a substantial phase delay exists between the ^{14}C and observed records? Their parallel behaviour over cycles 3 to 7, as previously detailed, would tend to discount any large phase adjustment which would have to be of at least one or two cycles to compensate for this anomaly. One outcome of such an adjustment, which does not appear tenable, would be a very large discrepancy between the low values of R_M obtained from $^{14}C_R$ cycle 5, and the very high actual values of the corresponding cycle 4 or 3 in the observed record. Following numerous hydrogen bomb explosions in the early sixties a dramatic increase in the ^{14}C level was found in some tree rings obtained from locations remote from the testing site (Schweingruber 1988). While it is to be acknowledged that these detonations occurred in the lower atmosphere, the ^{14}C peak was found in rings formed very soon after the bomb peak period, indicating a short ^{14}C take up time. An examination of the cross correlation existing between the two sequences, which is given in Fig. 6, shows that, apart from the expected manifestation of the 11-year cyclicity, it is highest at lags 0 and +11. In this context only correlation coefficients at positive lags are relevant. While these results of near equal magnitude provide no definitive resolution of this issue of phase, they do indicate that on a year-interval basis the *model* $^{14}C_R$ structure is near optimally collimated.

Sunspot numbers for cycles −4 and −3 (Fig. 2) which are based on Wolf's calculations (Chernosky and Hagan 1958) exhibit a rapid, possibly overestimated, recovery from the Maunder minimum. This approximately 70-year period of near zero solar activity has been estimated as extending over 1645–1715 (Eddy 1976), 1654–1714 on the ^{14}C record (Stuiver and Quay 1980) and 1645–99 (Schove 1983). Eddy (1977) considered the reliability of this historical data over 1700–48 to be poor, and following a re-evaluation of past records his estimates of sunspot numbers, in particular over the period 1700–15, were substantially lower. If these figures are adopted solar cycle −4 has $W_M = 7$ (down from $W_M = 18$) which is somewhat closer to the ^{14}C value of zero. If the reductions in Eddy's sunspot numbers for the years 1713–15, the last years of his tabulation, should also apply to the rest of cycle −3 then the gradual rise, which he deemed reasonable, would now be emulated. Undoubtedly, there is considerable uncertainty about the quantification of these two cycles with Eddy (1976) even rating his own estimates as probably poor. Overall, the yearly sunspot numbers given by this reconstruction procedure must also rate as only reasonable estimates, even though they are somewhat in variance with those based on fragmented historical records. Presumably there are other possible explanations for the discrepancy between the two records for cycles −4 and −3 which could include a nonlinear association between sunspot numbers and ^{14}C rates of production. Also, an increase in the Stuiver and Quay (1980) model value adopted for ^{14}C production rate increases from the 'tentative' value of 10% of the average, which was used for determining periods where W_M was assumed to be zero, would affect cycle −4 values.

Aurorae results have been used as proxy data to infer epochs and the intensity magnitude of solar maximum activity over past centuries prior to the systematic recording of sunspot numbers. Schove (1955) has constructed a non-continuous record of probable years of sunspot extremes from 649BC to AD1500, and from 1500–1748 a year-by-year estimate of average sunspot numbers. While the ^{14}C measurements indicate the periods of extreme activity and have given a representative envelope of the solar cycle maximum since 1700, they do not determine the years of maximum activity if they have been derived from typically 10-year ring sections or based on 10-year averages of single ring sections. However, if the year-by-year cycle is reconstructed from this ^{14}C data giving W_M as already described, then correlation with actual tree ring chronologies, showing a significant 11-year spectral signal, should locate with reasonable accuracy the years of maximum activity and provide an independent check on the auroral estimates, which undoubtedly have errors associated with them. Clearly there are some implicit assumptions made in respect of the tree ring solar cycle interaction. In particular, the '11-year' cycle has been in operation over the previous centuries, that an average cycle length based on contemporary averages is representative of these earlier cycles and that the correlation lag between solar cycle and tree ring growth, determined over 1700 to the present, also applies to previous intervals of time. While not attempting, at this stage, to reconstruct the detail of sunspot activity prior to 1700 from the ^{14}C variation, the relationship between the actual and reconstructed records post-1700 with tree ring growth rates will now be explored. Normally one may expect any interdependence to be in vogue prior to 1700 and accordingly could be utilised to ascertain the validity of reconstruction from radio carbon data.

A significant relationship has been shown to exist, by Mori (1981), between a 516-year long record of tree ring width measurements and the solar cycle. While some reservations could be expressed about the fact that only a single disc sample was used to establish the chronology, multiple measurements along different radii were made and on comparison shown to exhibit similar periodicities (Outi 1961). Spectral analysis of the ring index time series established a peak at 11·1 years at greater than the 99% confidence level while, in addition, cross-spectral analysis indicated a phase delay between growth and the solar cycle of about 2·8 years.

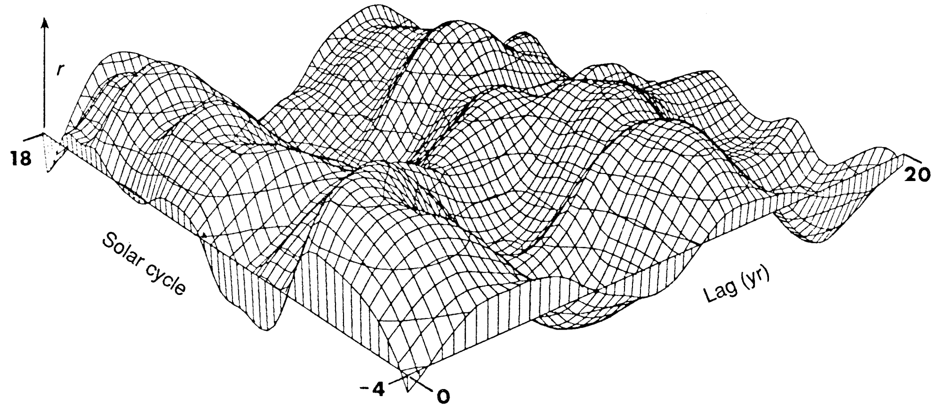

Fig. 7. Lagged cross correlation between actual four cycle blocks of annual sunspot numbers, incremented by one cycle, commencing cycles –4 to –1 through to cycles 15 to 18, and the annual tree ring width indices established from a Formosan Cypress from Taipingshan. The maximum values occur at lags of about 2–3 and the 11-year cyclicity is also evident.

Linear correlation coefficients have been determined over the 255 years of data common to both the sunspot and tree ring chronologies on the basis of a progressive subinterval equal in length to four solar cycles, commencing from cycle –4. This variation of r with time and positive lag number is illustrated in Fig. 7. The maximum value for each computation occurs mainly at a lag of 2 years with an average given by 2·45 years, which is in reasonable agreement with Mori's (1981) 2·8 years for the total series. Moreover, the profile of the corresponding ridge gives a broad brush outline of the envelope of solar cycle maximum over the two and a half centuries. The second ridge of maximum values, at approximately lag 13, does not seem to have any relevance in the context of tree-growth response, but they do indicate in a positive way that a strong 11-year cyclic relationship exists over the intervals considered. There is a substantial variation in the maximum values of r, ranging from 0·182 to 0·594, with the 5% significance level set at approximately 0·30. Specifically, the correlation values derived from intervals containing solar cycles 5 and 6, over 1798 to 1823, reflected this sudden downsturn in solar activity, indicating that the tree ring rate of growth has a high correlation with periods of intense solar activity but drops off in quiescent periods.

Relationships, in the longer term, between this tree ring data and solar cycle variations can be established by determining the magnitude of their cross correlation over the total length of common record. Although correlations at zero or negative lags are the only ones that are relevant in these circumstances, taking into account the manner in which the lags are offset and the fact that any solar influence on tree growth will be in the current or later growth period, the results for positive lags have also been included to demonstrate the strong 11-year coupling between the two time series.

Examination of Fig. 8, which displays the calculations between the Taipingshan and solar data, shows that the maximum correlation occurs at a time lag of −3 years, meaning maximum ring width follows about three years after solar maximum. The relevance of this correlation can be established by comparing the value of r with the 1% significance level, which are given by 0·303 and 0·161 respectively. If the persistence in the ring chronology is taken into consideration, this 1% level would, of course, be higher. For the reconstructed series we have $r = 0 \cdot 279$ at lag −3 and for the carbon 14 reconstruction $r = 0 \cdot 306$ at a lag of −2 and 0·350 at a lag of −13 (Fig. 9), but in this case the 1% significance level is 0·183. Strictly, these values can only be interpreted as a measure of replication. These figures for the $^{14}C_R$ model would indicate a minimum possible ^{14}C absorption time of one year or some longer time constrained to a multiple of 11 plus 1. Nevertheless, recalling aspects in this context considered earlier, a maximum lag of no more than 12 years would still maintain consistency between the observational and measured carbon records.

5. Conclusions

Without excessive parametrisation it would be very difficult indeed to reproduce the salient features of the known solar cycle over the last three centuries. However, using a single determinable parameter such as R_M or W_M to model a single cycle achieves, in a relatively simple manner, a reasonable approximation to the actual cycle, as demonstrated earlier. Although the rise and fall components of the cycles appear to be of an exponential nature, they can be adequately described by low order approximating polynomials. As some solar cycles with near equal R_M or W_M have different rise and fall distributions of annual average sunspot numbers, clearly the best that can be accomplished is a model prototype for the category. This represents the principal objective of the study—namely, to provide a basis for future detailed reconstruction of earlier periods of strong solar activity using carbon 14 results, not to replicate the contemporary cycles.

As the annual sunspot number is only representative of the average level of solar activity throughout the year, there is little to be achieved in aiming to model such data precisely. These values have, afterall, been subject to some objective assessment in the process of identification and counting of actual groups and spots, followed by averaging when both applying the Wolf formula and in the calculation of the final annual number.

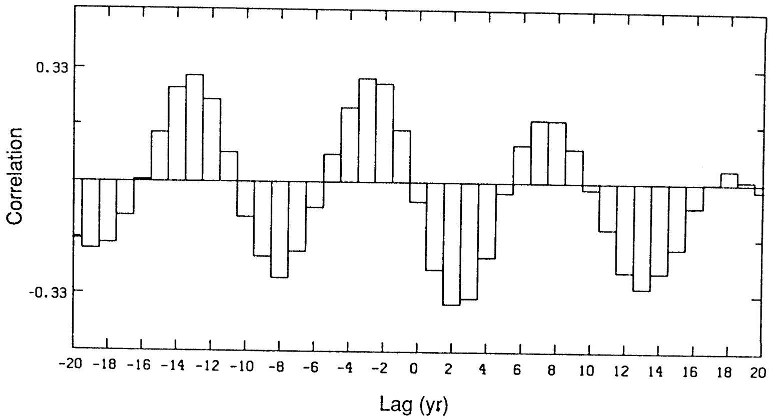

Fig. 8. Cross correlation between Taipingshan ring width index data and solar cycle data over 259 years indicates that there is a lag of −3 years between maximum ring width and solar maximum. Only correlations at negative lags relate to the influence of solar variability on growth patterns.

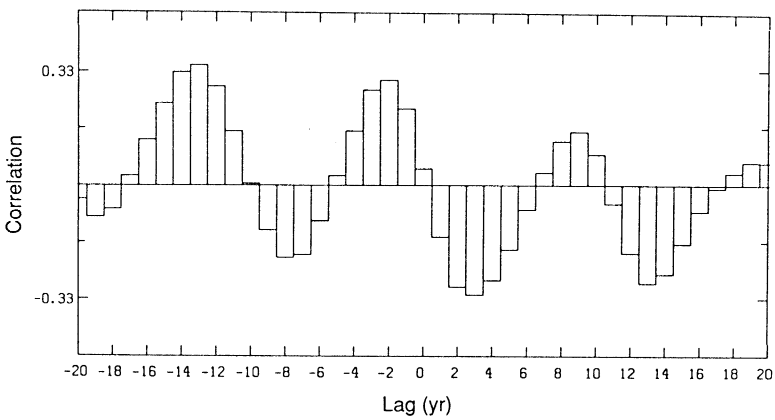

Fig. 9. Cross correlation between Taipingshan ring width index data and the ^{14}C reconstructed cycle data over 201 years indicates that the reconstructed series maintains a high and equivalent correlation to the actual data. It could be expected that reconstructions of earlier periods of high solar activity should show an overall similar correlation with tree ring data.

There seems little prospect of constructing, even in the long term, solar models which emulate the complex physical interactions which are manifested by the solar activity cycle. Accordingly, the estimation of the previous record and any forecasting of future activity must follow a more indirect path. If the major periodicities associated with the cycle can be identified from other sources—the current observation record being far too short to provide anything other than the short term variations—and the phase difference established, then such an approach would seem to offer better prospects. While the carbon 14 record appears to present the only feasible avenue to allow reconstruction of recent solar variability, between the periods of near zero activity, spectral analysis has also established the germane periodicities (Sonett and Finney 1989) imbedded in the cycle over recent millennia.

Tree ring data that exhibit a significant 11-year periodicity can be employed in the reconstruction of solar activity prior to 1700. Paralleling the techniques already introduced to reconstruct the post-1700 variations allows estimated annual sunspot numbers to be determined pre-1700, again based on the ^{14}C data from the Stuiver and Quay (1980) model. Correlations with a tree ring index series over the same interval can then be used, taking into account the established lags between the two series post-1700, to optimally adjust the reconstructed series to determine with reasonable accuracy epochs of maximum and minimum activity. In turn, these years can be compared with other estimates based on auroral activity (Schove 1955). However, extrapolation back in time must be based on principal current cycle periodicities; short periods of reconstruction should be adequately approximated by the imposition of nine cycles in 100 years. As a final test, short interval moving correlations should also reflect the magnitude of anomalous solar activity, in line with the ^{14}C variations.

Apart from the time of emergence from the Maunder minimum, progressive correlations between observed sunspot and tree ring data shown a dramatic decrease over the three decades covering solar cycles 5 and 6. Further, the carbon record over this period indicates a rapid decrease in solar activity which is also reflected in the ring widths when an 11-year band pass filter is applied. This recorded period is only a transient downturn in activity, unlike the Maunder minimum period where activity all but ceased. It is therefore rather surprising to note that no evidence of sustained diminution in the 11-year band pass filtered ring widths exists over the duration of the Maunder minimum, which appears somewhat contradictory.

Finally, additional tree ring chronologies should be processed to find the location of species which exhibit significant 11-year signals and from which new chronologies can be assembled, thereby assisting in the process of solar cycle reconstruction.

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