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Quantum Electrodynamic Processes in Laser Beams Focused by Lenses with $f \leq 1$

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Abstract

It is shown how the enhancement of Compton scattering and allied QED processes in the $f \leq 1$ lens focus of a high-power laser beam can be calculated, notwithstanding the inapplicability of the Volkov solution, by moving from the bound interaction picture to the first Born approximation in the total interaction picture.

1. Introduction

The Volkov (1935) solution for a Dirac electron in a plane wave has been used to investigate the possibility of enhancing the cross section for various quantum electrodynamic processes by illuminating an appropriate system with a focussed laser beam (Oleinik 1968). The representation of the electromagnetic field at the focus of a lens by a plane wave is however only adequate if the f number of the lens is $f \geq 4$. The field at the focus of an $f \approx 1$ lens for example cannot be adequately represented by a plane wave (Boivin and Wolf 1965). But it is precisely the laser field of an $f \approx 1$ lens, both because it is more intense and because, being non-null and non-wrenchless (Synge 1958), it can contribute to real e^+, e^- pair production (Bunkin and Tugov 1970), that is more likely to modify significantly the particular cross section.

It is therefore of interest to investigate the case of a low f number focused field impinging on processes such as Compton scattering which, by the substitution theorem, also includes two-photon pair creation and destruction.

We show how by moving from the bound interaction picture (BIP) to the first Born approximation in the total interaction picture (TIP) it is possible to overcome the unavailability of the Volkov solution in this case.

2. Procedure

Let $\mathcal{L}$ denote the total Lagrangian density for Compton scattering, a paradigm here for any QED process, in an external electromagmetic field. In the ordinary

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interaction picture or total interaction picture (TIP), $\mathcal{L}$ is written as

$$\mathcal{L} = \mathcal{L}^0 + \mathcal{L}', \tag{1}$$

where $\mathcal{L}^0 \equiv \mathcal{L}^0_{\mathrm{D}} + \mathcal{L}_{\mathrm{M}}$ is the total free field Lagrangian density and $\mathcal{L}' \equiv \mathcal{L}'_{\mathrm{F}} + \mathcal{L}'_{\mathrm{E}}$, the total interaction Lagrangian density, is regarded as the perturbation. Here $\mathcal{L}^0_{\mathrm{D}}$, $\mathcal{L}'_{\mathrm{M}}$, $\mathcal{L}'_{\mathrm{F}}$ and $\mathcal{L}'_{\mathrm{E}}$ denote the free Dirac field, the Maxwell field, the Dirac–Maxwell interaction, and the Dirac-external field (i.e. laser field) interaction Lagrangian densities respectively. In the TIP, the calculation of the scattering matrix $\langle S_{\mathrm{TIP}} \rangle$ is complicated by the $\mathcal{L}'_{\mathrm{E}}$ term in the perturbation $\mathcal{L}'$.

In the Furry picture or bound interaction picture (BIP) $\mathcal{L}$ is written as

$$\mathcal{L} = \overline{\mathcal{L}}^0 + \overline{\mathcal{L}}', \tag{2}$$

where $\overline{\mathcal{L}}^0 \equiv \mathcal{L}_{\mathrm{M}} + \mathcal{L}_{\mathrm{BD}}$, with $\mathcal{L}_{\mathrm{BD}} \equiv \mathcal{L}^0_{\mathrm{D}} + \mathcal{L}'_{\mathrm{E}}$ being the 'bound' (in the laser field) Dirac Lagrangian density, while $\overline{\mathcal{L}}' \equiv \mathcal{L}'_{\mathrm{F}}$, the Dirac–Maxwell interaction Lagrangian density, is regarded as the perturbation. Although the bound Dirac field equations of the BIP are more complicated than the free Dirac field equations of the TIP, the absence of the $\mathcal{L}'_{\mathrm{E}}$ term in $\overline{\mathcal{L}}'$ simplifies the calculation of $\langle S_{\mathrm{BIP}} \rangle$.

This is a satisfactory procedure if the laser field is a plane electromagnetic wave, for in that case the Dirac electron is represented by the Volkov solution. But the field at the focus of a lens is a plane wave only for lenses with f numbers ≥ 4. The more intense focused field of a lens with $f \approx 1$ is neither plane nor null nor wrenchless (Bunkin and Tugov 1970). Indeed, the electric and magnetic field strengths

$$\boldsymbol{E}(P, t) = \mathrm{Re}[\boldsymbol{e}(P)\,\mathrm{e}^{-\mathrm{i}\omega t}], \qquad \boldsymbol{H}(P, t) = \mathrm{Re}[\boldsymbol{h}(P)\mathrm{e}^{-\mathrm{i}\omega t}],$$

at a point P near the focus of an ideal lens are given by the equations (Boivin and Wolf 1965):

$$e_x(P) = -\mathrm{i}\,\alpha(I_0 + I_2\cos 2\psi), \quad e_y(P) = \mathrm{i}\alpha I_2 \sin 2\psi, \quad e_z(P) = -2\alpha I_1 \cos\psi,$$

$$h_x(P) = -\mathrm{i}\,\alpha I_2 \sin 2\psi, \quad h_y(P) = -\mathrm{i}\,\alpha(I_0 - I_2\cos 2\psi), \quad h_z(P) = -2\alpha I_1 \sin\psi. \tag{3}$$

Here $\alpha = kfE_0$, E_0 is the amplitude of the electric field strength in the incident plane wave, f is the focal length of the lens, $k = \omega/c$, and

$$I_\mu(k\rho, kz, \theta_0) = 4\int_0^{\theta_0} \cos^{1/2}\theta\,\sin^{\mu+1}\left(\frac{\theta}{2}\right)\cos^{3-\mu}\left(\frac{\theta}{2}\right)J_\mu(k\rho\sin\theta)\,\mathrm{e}^{\mathrm{i}kz\cos\theta}\,\mathrm{d}\theta,$$

where J_μ is a Bessel function ($\mu = 0, 1, 2$), $\theta_0 = \mathrm{arc\,tg}(d/2f)$, d is the lens diameter, and (ρ, z, ψ) are the coordinates of a point in the cylindrical coordinate system with centre at the focus and polar axis coinciding with the optical axis of the lens. It is clear that Compton scattering in such a field cannot be calculated using the bound interaction picture and the Volkov solution.

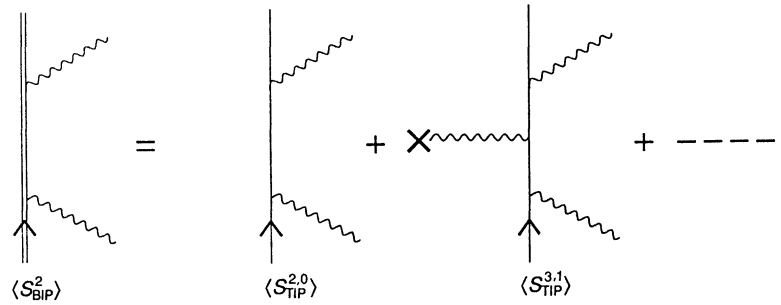

Fig. 1. Equivalence of transition amplitudes from equations (4)–(6).

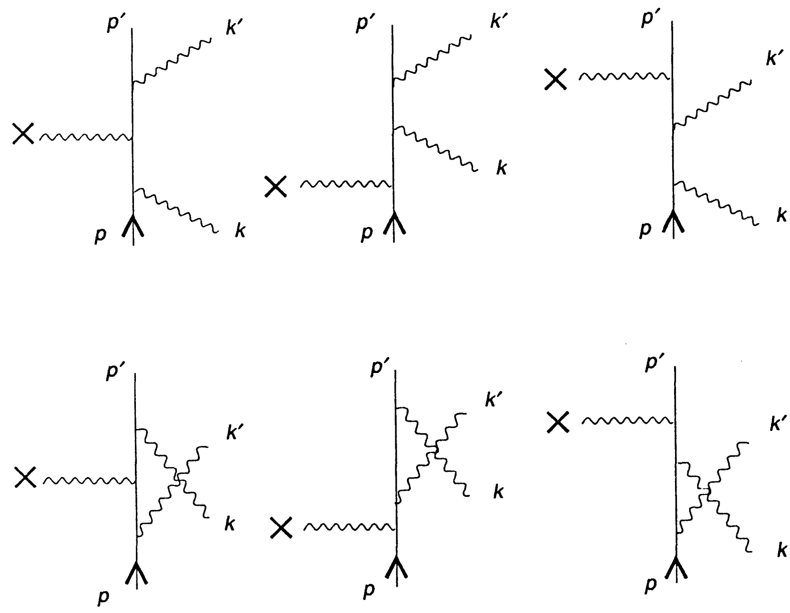

Fig. 2. Six topologically different diagrams associated with the transition amplitude $\langle p'k' | S^{3,1} | pk\rangle$.

The simplest practicable approach in this case is to consider *not*

$$ {}_{\mathrm{BIP}}\langle p'k' \mid S_{\mathrm{BIP}} \mid pk\rangle_{\mathrm{BIP}} = {}_{\mathrm{BIP}}\langle p'k' \mid \sum_n S^n_{\mathrm{BIP}} \mid pk\rangle_{\mathrm{BIP}} \approx {}_{\mathrm{BIP}}\langle p'k' \mid S^2_{\mathrm{BIP}} \mid pk\rangle_{\mathrm{BIP}}, \tag{4} $$

but *rather*, in the total interaction picture (TIP),

$$ {}_{\mathrm{TIP}}\langle p'k' \mid S_{\mathrm{TIP}} \mid pk\rangle_{\mathrm{TIP}} = {}_{\mathrm{TIP}}\langle p'k' \mid \sum_n S^n_{\mathrm{TIP}} \mid pk\rangle_{\mathrm{TIP}} \tag{5} $$

and to expand the latter in the Born expansion according to

$$
\begin{aligned}
{}_{\mathrm{TIP}}\langle p'k' \mid \sum_n S^n_{\mathrm{TIP}} \mid pk\rangle_{\mathrm{TIP}} &= {}_{\mathrm{TIP}}\langle p'k' \mid \sum_{n,m} S^{n,m}_{\mathrm{TIP}} \mid pk\rangle_{\mathrm{TIP}} \\
&\approx \sum_n {}_{\mathrm{TIP}}\langle p'k' \mid S^{n,n-2)}_{\mathrm{TIP}} \mid pk\rangle_{\mathrm{TIP}} \\
&= {}_{\mathrm{TIP}}\langle p'k' \mid S^{2,0}_{\mathrm{TIP}} \mid pk\rangle_{\mathrm{TIP}} + {}_{\mathrm{TIP}}\langle p'k' \mid S^{3,1}_{\mathrm{TIP}} \mid pk\rangle_{\mathrm{TIP}} + \dots \qquad (6)
\end{aligned}
$$

Hence, from the diagrammatic representations of the RHS of (4) and the RHS of (6) and therefore of (5), and from the unitary equivalence of (4) and (5) we have the equivalence of transition amplitudes shown in Fig. 1. With the TIP suffix understood, we therefore have to calculate $\langle p'k'|S^{3,1}|pk\rangle$. This transition amplitude has six topologically different diagrams associated with it, as shown in Fig. 2. Summing over all the diagrams gives

$$
\begin{aligned}
\langle p'k' \mid S^{3,1} \mid pk\rangle = &-\frac{\mathrm{i}\,e^3 m}{(2\pi)^{7/2}}\left(\frac{1}{4p'^0p^0k'^0k^0}\right)^{1/2} \\
&\times \bar{u}(\boldsymbol{p}')\Big(\not\epsilon(\boldsymbol{k}')\frac{1}{\not p'+\not k'-m}\not A^{(\mathrm{e})}(p+k-p'-k')\frac{1}{\not p+\not k-m}\not\epsilon(\boldsymbol{k}) \\
&+\not\epsilon(\boldsymbol{k})\frac{1}{\not p'+\not k-m}\not A^{(\mathrm{e})}(p+k-p'-k')\frac{1}{\not p+\not k'-m}\not\epsilon(\boldsymbol{k}') \\
&+\not\epsilon(\boldsymbol{k}')\frac{1}{\not p'+\not k'-m}\not\epsilon(\boldsymbol{k})\frac{1}{\not p'+\not k'-\not k-m}\not A^{(\mathrm{e})}(p+k-p'-k') \\
&+\not\epsilon(\boldsymbol{k})\frac{1}{\not p'+\not k-m}\not\epsilon(\boldsymbol{k}')\frac{1}{\not p'+\not k'-\not k-m}\not A^{(\mathrm{e})}(p+k-p'-k') \\
&+\not A^{(\mathrm{e})}(p+k-p'-k')\frac{1}{\not p+\not k-\not k'-m}\not\epsilon(\boldsymbol{k}')\frac{1}{\not p+\not k-m}\not\epsilon(\boldsymbol{k}) \\
&+\not A^{(\mathrm{e})}(p+k-p'-k')\frac{1}{\not p+\not k-\not k'-m}\not\epsilon(\boldsymbol{k})\frac{1}{\not p+\not k'-m}\not\epsilon(\boldsymbol{k}')\Big)u(\boldsymbol{p}), \qquad (7)
\end{aligned}
$$

where

$$
\not A^{(\mathrm{e})}(k'') \equiv \gamma^\nu\left(\frac{1}{2\pi}\right)^{3/2}\int A^{(\mathrm{e})}_\nu(x)\,\mathrm{e}^{-\mathrm{i}k''x}\,\mathrm{d}^4x,
$$

$$
\boldsymbol{E} = -\nabla A^{(\mathrm{e})}_0 - \boldsymbol{A}^{(\mathrm{e})}_{,0}, \qquad \boldsymbol{B} = \nabla\times\boldsymbol{A}^{(\mathrm{e})}
$$

is the focussed field (3).

3. Comments

A comparison of the first Born Compton cross section entailed by (7) and the free field $\langle p'k' | S^{2,0} | pk \rangle$ with the Volkov cross section would show precisely how the effect of a focused field differs from that of a plane wave. A calculation of this first Born cross section is currently under way (Perlman, Troup and Derlet, to be submitted).

References

Boivin, A., and Wolf, E. (1965). *Phys. Rev.* **138**, B1561.
Bunkin, F. V., and Tugov, I. I. (1970). *Sov. Phys. Dokl.* **14**, 678.
Oleinik, V. P. (1968). *Sov. Phys. JETP* **26**, 1132.
Synge, J. M. (1958). 'Relativity: The Special Theory', Ch. IX (North Holland: Amsterdam).
Volkov, D. M. (1935). *Z. Phys.* **94**, 250.

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