# **The Nature of Pulsar Subpulse Drift**

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#### Abstract

A possible mechanism for the explanation of pulsar subpulse drift is suggested. In the region of the open magnetic field lines the existence of an electron-positron plasma penetrated by a primary particle beam is assumed. There is a possibility of excitation of large-scale drift waves propagating transversely to the magnetic field lines. These waves can affect the fulfilment of the radio-wave generation conditions. If the pulsar angular velocity is near to the frequency of the drift waves one should observe regular drift phenomena.

### 1. Introduction

The subpulse drift phenomenon or the periodic subpulse phase variation in the subsequent individual pulses was discovered by Drake and Craft (1968) at the beginning of the pulsar era. Numerous observations carried out since then have showed that the subpulse drift occurs only for some pulsars and is characterised by a definite drift rate (Huguenin *et al.* 1970; Backer 1970; Taylor and Huguenin 1971).

A consistent model explaining subpulse drift was presented by Ruderman and Sutherland (1975) (see Cheng and Ruderman 1980; Allen and Melrose 1982; Allen 1985 as well). This model is based on the idea of sparks appearing in definite places at the stellar surface within the polar gap. The sparks undergo  $E_0 \perp B_0$  drift which manifests itself as a drifting subpulse phenomenon. Sparks provoke discharges that serve as a source of the subpulse associated plasma columns. Therefore the information about the spark drift is preserved.

The supposition that within a gap a spiral wave is being excited was made by Arons (1981). The wave modulates the particle density and this explains the subpulse presence and their drift. Jones (1984) suggested a polar cap model consisting of a dense positron plasma and one-dimensional ion beam for which the excitation of such spiral waves is possible.

The aim of this paper is to explain the phenomenon of drifting subpulses in pulsar radio emission within the framework of the theory developed previously (Kazbegi *et al.* 1989*a*, 1989*b*, 1991). The pulsar magnetosphere is filled with an electron–positron plasma flowing along the open magnetic field lines. The distribution function of such a plasma has a tail stretched in one direction (see Fig. 1 which is from Arons 1981). For typical pulsars ( $P \approx 1$  s) at a stellar surface the bulk of plasma particles has an average Lorentz factor  $\gamma_p \approx 3$  and



**Fig. 1.** Distribution function of particle energy. The solid curve corresponds to electrons, while the dashed curve corresponds to particles with positive charge.

concentration  $n_p \approx 10^{16}$  cm<sup>-3</sup> (Kazbegi *et al.* 1989*a*; Goldreich and Julian 1969). The primary beam is characterised by a Goldreich–Julian density  $n_b \approx 10^{11}$  cm<sup>-3</sup> and Lorentz factor  $y_b \approx 10^6$ . In such a plasma there exist two types of wave—a purely transverse electromagnetic t-wave and an electrostatic–electromagnetic lt-wave (Volokitin *et al.* 1985; Lominadze *et al.* 1986). Both have high and low frequency branches. However, in the limit of large wavevectors **k** and frequencies  $\omega$  these branches combine into a 'vacuum' one ( $\omega = kc$ ). The spectra of the high-frequency waves is in the superluminous region (the phase velocity of waves exceeds the speed of light) and they cannot be excited (Fig. 2). The spectra of the low-frequency modes are as follows:

$$\omega^t = kc(1-\delta), \tag{1}$$

$$\omega_0^{lt} = k_{\varphi} c \left( 1 - \delta - \frac{k_{\perp}^2 c^2}{2k_{\varphi}^2} \frac{1}{8(\omega_p^2/k_{\varphi}^2)\gamma_p - 1} \right).$$
(2)

Here and below the cylindrical coordinate system *x*, *r*,  $\varphi$  has been chosen, with the *x*-axis directed transversely to the plane of the magnetic field curvature, *r* and  $\varphi$  are the radical and azimuthal coordinates;  $k_{\varphi}$  is the component of the wavevector along the magnetic field,  $k_{\perp} = (k_r^2 + k_x^2)^{1/2}$ ,  $\delta = \omega_p^2/4\omega_B^2 \gamma_p^3$ ,  $\omega_p^2 = 4\pi e^2 n/m$ ,  $\omega_B = eB/mc$ , *e* and *m* are the charge and mass of particles respectively, *c* is the speed of light; *B* is the intensity of the dipolar magnetic field obeying the law  $B = B_0(R_0/R)^3$ , where  $R_0 = 10^6$  cm is the pulsar radius and  $B_0 = 10^{12}$  G is the magnetic field intensity at the stellar surface [note that the particle concentration changes according to the same law  $n = n_0(R_0/R)^3$ ].

The distribution function given in Fig. 1 tends to be unstable relative to some plasma instabilities. It appears that only the following three types of instabilities develop in a pulsar electron-positron plasma:

- the cyclotron instability of the electromagnetic, purely transverse t-waves;
- the Cherenkov instability developing due to the transverse momentum of particles achieved after the quasi-linear diffusion of the cyclotron instability;



**Fig. 2.** Spectrum of t-waves: 1—the high-frequency mode; 2—the low-frequency mode. (b) Spectrum of lt-waves: 1—the high-frequency mode,  $\omega_1 = \omega_p / \gamma_p^{3/2}$ ; 2—the low-frequency mode. Dashed curves correspond to the region of strong damping.

- the Cherenkov instability developing due to the particle drift motion in the inhomogeneous magnetic field of the pulsar.

The latter two mechanisms excite *t*-waves along with the electrostaticelectromagnetic lt-waves (Kazbegi *et al.* 1989*a*).

The excited waves propagate along the magnetic field lines. The frequency of the generated waves for typical pulsars falls within the radio band. The perturbations are excited if one of the following resonance conditions is fulfilled:

$$\omega - k_{\varphi} v_{\varphi} - k_{x} u_{x} = -\frac{|\omega_{B}|}{\gamma_{res}}, \qquad (3)$$

$$\omega - k_{\varphi} v_{\varphi} - k_{x} u_{x} = 0.$$
<sup>(4)</sup>

When the resonance (3) is fulfilled, purely transverse t-waves are excited. Here  $u = v_{\varphi} p_{\varphi} c/\omega_B R_B$  is the particle drift velocity caused by the magnetic field inhomogeneity and directed along the positive direction of the *x*-axis for the beam particles,  $R_B$  is the curvature radius,  $\gamma_{res}$  is the Lorentz factor of the resonant particles, and  $p_{\varphi}$  is normalised  $\varphi$ -component (along the curved field lines) of the particle momentum. Given that  $v_{\varphi} = c(1-1/2\gamma_{res}^2 - u_x^2/2c^2)$  equations (3) and (4) can be rewritten in the form

$$\frac{1}{2}\left(\frac{k_x}{k_\varphi} - \frac{u_x}{c}\right)^2 + \frac{1}{2}\frac{k_r^2}{k_\varphi} + \frac{1}{2\gamma_{res}^2} - \delta = -\frac{|\omega_B|}{\gamma_{res}k_\varphi c},$$
(5)

$$\frac{1}{2} \left( \frac{k_x}{k_\varphi} - \frac{u_x}{c} \right)^2 + \frac{1}{2} \frac{k_r^2}{k_\varphi} + \frac{1}{2\gamma_{res}^2} - \delta = 0.$$
 (6)

The condition (5) can be fulfilled both for the particles in the tail of the distribution function with  $\gamma_{res} = \gamma_t \approx 10^5$  and for the beam particles with  $\gamma_{res} = \gamma_b \approx 10^6$ . The condition (6) can be fulfilled exclusively at the resonance of the waves with the beam particles. As for the case  $\gamma_{res} = \gamma_p$ , neither of the resonances (5) and (6) is possible because the term  $1/2\gamma_p^2$  is the largest. We believe that the t-wave generation at the anomalous Doppler-effect resonance explains 'core'-type emission in pulsar radiation, and the wave generation at the Cherenkov resonance explains the 'cone' one (Rankin 1983). Note that the wave generation occurs at the distances  $R \approx 10^9$  cm (for  $P \approx 1$  s) and the frequencies fall within the radio range. The observational data show that the subpulse drift is pronounced in conal single profiles (Rankin 1983, 1986). As our paper deals with the subpulse drift phenomenon we shall focus below on the condition (6).

The waves are strongly damped at the resonance

$$\omega - k_{\varphi} v_{\varphi} - k_{x} u_{x} - \frac{|\omega_{B\alpha}|}{\gamma_{res}} = 0, \qquad (7)$$

which is fulfilled only for the particles of the bulk plasma ( $\gamma_{res} = \gamma_p$ ) and if  $\omega \approx 2\gamma_p \omega_B$ . Because the frequency of the generated waves is less than  $\gamma_p \omega_B$  the condition (7) cannot be fulfilled in the generation region (Kazbegi *et al.* 1989*a*). However, the excited waves propagating to the light cylinder can reach the damping region (Fig. 2). Hence only the waves that do not propagate through the damping region can leave the magnetosphere. The latter is possible only for the waves generated in the vicinity of closed field lines at the magnetosphere boundary. The particle mass increases near the light cylinder and they deviate in the direction opposite to rotation together with the 'frozen in' field lines (Fig. 3). Therefore only the waves that find themselves in the

region between the open and closed field lines where the density is greatly reduced can leave the magnetosphere. In the regions with reduced density both t- and lt-waves turn into the vacuum one without transformation. It is clear from the expressions (1) and (2) that, at  $n \to 0$ ,  $\omega_p \to 0$ ,  $\omega^t$  and  $\omega^{lt} \to kc$ , retaining all their polarisation properties.



**Fig. 3.** Schematic of a pulsar magnetosphere with the emission cone. The curves denote magnetic field lines: 1—the open field line region; 2—the closed field line region.

# 2. Generation of Low-frequency Waves Propagating across the Magnetic Field

Until now we discussed only the generation and propagation of waves with frequencies falling within the radio band (Kazbegi *et al.* 1989*a*). The angles of the excited waves are rather narrow  $\theta = k_{\perp}/k_{\varphi} \ll 1$ . For the Cherenkov mechanisms the angle is restricted both from above and below. The intersection of the emission diagram gives a ring of definite thickness—a cone.

Let us investigate now the case of low-frequency waves propagating nearly transversely to the magnetic field:  $\theta = \pi/2$ . The dielectric permeability tensor has been calculated by Kazbegi *et al.* (1991). In the zero approximation of the series expansion over a small parameter  $(\omega/\omega_{Bc}\gamma) \ll 1$  and at  $k_r \rightarrow 0$ , the form of the dispersion relation is as follows:

$$\left(\epsilon_{xx} - \frac{k_{\varphi}^2 c^2}{\omega^2}\right) \left(\epsilon_{\varphi\varphi} - \frac{k_x^2 c^2}{\omega^2}\right) - \left(\frac{k_x k_{\varphi} c^2}{\omega^2} + \epsilon_{\varphi x}\right)^2 = 0.$$
(8)

Here we have

$$\begin{aligned} \epsilon_{xx} &= 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega} \int \left(\frac{u_{x\alpha}}{c}\right)^2 \frac{1}{(\omega - k_x u_{x\alpha} - k_{\varphi} v_{\varphi})} \frac{\partial f}{\partial y} dp_{\varphi} ,\\ \epsilon_{\varphi x} &\equiv \epsilon_{x\varphi} = -\sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega} \int \frac{u_{x\alpha}}{c^2} \frac{v_{\varphi}}{(\omega - k_x u_{x\alpha} - k_{\varphi} v_{\varphi})} \frac{\partial f}{\partial y} dp_{\varphi} ,\\ \epsilon_{\varphi \varphi} &= 1 + \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega} \int \frac{v_{\varphi}/c}{(\omega - k_x u_{x\alpha} - k_{\varphi} v_{\varphi})} \frac{\partial f}{\partial y} dp_{\varphi} .\end{aligned}$$

The sum is taken over the particle species: electrons and positrons of the bulk plasma and electrons of the beam.

Pataraya *et al.* (1986) suggested using the Coriolis force in the sub-pulse drift explanation. In this case in the denominator of the expression for  $\epsilon_{ij}$  an additional term should be taken into account: i.e. instead of  $\omega - k_x u_x - k_{\varphi} v_{\varphi}$  one should treat  $\omega - k_x u_x - k_{\varphi} v_{\varphi} - (v_{\varphi} y / \omega_B) (\mathbf{k}_{\perp} \times \mathbf{\Omega})_{\varphi}$ . Comparing  $k_x u_x$  with  $(v_{\varphi} y / \omega_B)$  ( $\mathbf{k}_{\perp} \times \mathbf{\Omega})_{\varphi}$  one can see that the additional term is significant only if the following inequality holds:

$$\frac{c}{R_B\Omega_x} \leq \left| 1 - \frac{k_r}{k_x} \frac{\Omega_r}{\Omega_x} \right|.$$

If terms of the order of  $(u_{x\alpha}/c)^2 \ll 1$  and  $k_{\varphi}/k_x \ll 1$  are neglected one obtains from (8)

$$\epsilon_{\varphi\varphi} = \frac{k_x^2 c^2}{\omega^2 - k_w^2 c^2}.$$
(9)

We assume that  $\omega = k_x u_x^b + k_{\varphi} v_{\varphi} + a$ . The integration in parts of  $\epsilon_{\varphi\varphi}$  and the summation over a  $\alpha$  yields

$$1 - \frac{3}{2\gamma_p^3} \frac{\omega_p^2}{\omega^2} - \frac{1}{2} \frac{\omega_p^2}{\omega^3} \frac{k_x u_x^p}{\gamma_p} - \frac{\omega_b^2}{\omega a} \frac{1}{\gamma_b^3} - \frac{\omega_b^2}{\omega a^2} \frac{k_x u_x^b}{\gamma_b} = \frac{k_\perp^2 c^2}{\omega^2}.$$
 (10)

The indices *p* and *b* denote the bulk plasma and the beam respectively. Solving equation (10) one finds that *a* is a complex number and the growth rate ( $\Gamma_L = \text{Im}\omega = \text{Im}a$ ) is the greatest at

$$k_{\perp}^2 \ll \frac{\omega_p^2}{\gamma_p^3 c^2} \approx 10^{-6}$$
, (11)

and thus

$$\Gamma_L \approx \sqrt{\frac{3}{2}} \left(\frac{n_b}{n_p}\right)^{\frac{1}{2}} \frac{\gamma_p^{3/2}}{\gamma_b^{1/2}} k_x u_x \,. \tag{12}$$

Hence the low-frequency wave (drift wave hereafter)

$$\omega_0 \equiv \operatorname{Re}\omega = k_x u_x + k_\varphi v_\varphi \tag{13}$$

is excited at the left slope of the distribution function corresponding to the beam. The wave draws energy from the longitudinal motion of the beam particles as in the case of an ordinary Cherenkov wave-particle interaction. However, the wave is excited only if  $k_x u_x^b \neq 0$ , i.e. in the presence of the drift motion.

# 3. Nonlinear Energy Pumping in the Long-wave Region

The growth rate of the drift waves is rather small. However, the waves propagate nearly transversely to the magnetic field encircling the region of the open field lines of the magnetosphere and stay in the resonance region for time  $\tau$ . The duration of this period depends on the  $\varphi$ -component of the

wave vector and its direction. Computing the Poynting vector  $\boldsymbol{S}$  one can show that  $S_x/S_{\varphi} = k_x/k_{\varphi}$ , i.e. along the field lines the waves transfer only a small fraction of the whole energy defined by the ratio  $k_{\varphi}/k_x \ll 1$ . Let us choose the direction of particle propagation, i.e. the motion from the star, as positive. The particles give part of their energy to the waves and are carried out from the interaction region. New particles enter this region, in their turn give part of their energy to the waves and so on. The wave leaves the interaction region considerably slower than the particles. Hence there is no time for the inverse action of the waves upon the particles. The energy accumulation in the waves occurs without quasi-linear saturation. The wave amplitude grows until the nonlinear processes redistribute the energy over the spectra. From all nonlinear processes the probability of the three-wave interaction and the induced scattering of waves on plasma particles is the highest. However, the second-order current describing the decay interaction is proportional to  $e^3$ (charge in uneven power). Hence the contribution of electrons and positrons is compensated if their distribution functions coincide. At the same time the induced scattering is proportional to the even charge power ( $e^4$ ). Therefore the induced scattering occurs at the resonance on the bulk plasma

$$\omega - \omega' - (k_{\varphi} - k'_{\varphi})v_{\varphi} \approx 0.$$

Substituting (13) into this resonance condition one obtains

$$\frac{k_x - k'_x}{k_\varphi - k'_\varphi} = \mp \frac{c}{u_x^b} \frac{1}{2\gamma_p^2} \sim \mp 10.$$
 (14)

The upper sign corresponds to the waves propagating from the pulsar to the light cylinder ( $k_{\varphi} > 0$ ). For such waves the scattering increases the value of  $k_{\varphi}$  ( $k'_{\varphi} > k_{\varphi}$ ), i.e. each scattering event accelerates the energy outflow from the interaction region, whereas at  $k_{\varphi} < 0$  each scattering event decreases the longitudinal component of the wave vector  $k'_{\infty} < k_{\infty}$ , and  $k_{x}$  decreases too  $(k'_x < k_x)$ . This process leads to wave energy pumping in the long-wave region of the spectrum. Low-frequency waves with  $k_x \gg k_{\varphi}$  propagate along the spiral to the neutron star with the step defined by  $k_x/k_{\varphi} > -c/u_x \approx 10 \sim 10^2$ . In the generation region  $k_x^0 \sim 10^{-3}$  (equation 11),  $k_{\varphi}^0 \sim 10^{-10}$  (from the magnetosphere dimensions) and  $k_x^0/k_{\varphi}^0 \approx 10^7$ , consequently the  $10^{-7}$  energy fraction flows along the field lines with the speed of light and each scattering event increases the fraction of this energy because  $k'_x/k'_{\varphi} < k^0_x/k^0_{\varphi}$ . Therefore the low-frequency waves are excited in the vicinity of the light cylinder ( $R \approx 5 \times 10^9$  cm for  $P \approx 1$  s) where  $c/u_x \approx 10$ , and they propagate to the pulsar until  $c/u_x \approx 3 \times 10^2$  $(R \approx 10^9 \text{ cm})$ . The waves undergo scattering until  $k_x/k_{\varphi} \gg c/u_x \approx 10^2$  (for the fulfilment of the resonance 13). At these distances the minimal value of  $k_x$  is equal to  $10^{-8}$  cm<sup>-1</sup>. The latter coincides with the estimate  $k_x^{min} \sim 1/d$ , where d is the transverse dimension of the magnetosphere at the distance  $R \approx 10^9$  cm. i.e. one wavelength (minimal wavelength) can fit the transverse dimensions of the magnetosphere. In this case the frequency  $\omega \approx k_x u_x$  turns out to be of the order of the pulsar angular velocity  $\Omega \approx 2\pi$  s<sup>-1</sup>. For the energy to accumulate just in this region of the wavelength a sufficiently high rate of nonlinear pumping into the region with  $k \approx k_x^{min}$  is necessary. In this case the energy will not accumulate in the regions of intermediate wavelength and will have enough time to replenish from the 'reservoir' (in **k**-space) with  $k_x \approx 10^{-8}$  from which the energy is transferred along the field lines. The outflow of the  $k_{\varphi}/k_x \approx 10^{-2}$  fraction of energy takes a time  $r' \approx R/c \approx 3 \times 10^{-2}$ ; consequently the 'reservoir' will be emptied in a time  $r = 10^2 r' \approx 3$  s.

To estimate the rate of nonlinear pumping caused by induced scattering let us expand the distribution function in a series over a small argument  $|E|^2 / (mc^2n\gamma) \ll 1$ ;  $f = f_0 + f_1 + f_2 + ...$  From the kinetic equation using well-known methods (Galeev and Sagdeev 1967; Lominadze *et al.* 1979) one obtains

$$f_{\alpha}^{(3)} = \frac{e^{3}E_{kw}^{\varphi}}{(\omega - k_{x}u_{x\alpha})} \frac{\partial}{\partial p_{\varphi}} \int dk' d\omega' \frac{I(k', \omega')}{\omega - \omega' - (k_{\varphi} - k'_{\varphi})v_{\varphi} - (k_{x} - k'_{x})u_{x\alpha}} \\ \times \frac{\partial}{\partial p_{\varphi}} \left[ \frac{1}{\omega - k_{x}u_{x\alpha}} - \frac{1}{\omega' - k'_{x}u_{x\alpha}} \right] \frac{\partial f_{0}}{\partial p_{\varphi}}.$$
 (15)

Here

$$\langle E^{\varphi}_{k'\omega'}E^{\varphi}_{k\omega}\rangle = I(k,\omega)\delta(k+k')\delta(\omega+\omega'),$$

where  $E_{k\omega}^{\varphi}$  is the Fourier representation of the wave electric field E(r,t); angle brackets designate random phase averaging. After the substitution of these expressions in the definition of the third order current

$$J^{(3)} = \sum_{\alpha} e_{\alpha} \int \boldsymbol{v} f_{\alpha}^{(3)} d\boldsymbol{p} = \sigma^{(3)}(\boldsymbol{k}) E(\boldsymbol{k})$$
(16)

and taking into account that  $n_b \ll n_p$  one obtains the permeability tensor

$$Re\sigma^{(3)}(k) = \frac{\pi e^4 n_p}{m^2} \omega_k \int (k_{\varphi} - k'_{\varphi}) I(k') dk' \int \frac{d\mathbf{p}}{\gamma^6 (\omega_k - k_x u_x^p)^4} \\ \times \delta((\omega_k - \omega'_k - (k_{\varphi} - k'_{\varphi}) v_{\varphi}) \frac{\partial f_0}{\partial \mathbf{p}}.$$
(17)

Note that induced scattering in a plasma (or otherwise nonlinear Landau damping) is the nonlinear interaction of plasma particles and wave beatings with the frequency  $\omega - \omega'$  and the wavevector  $\mathbf{k} - \mathbf{k}'$ . The electric vectors of these waves are directed along the pulsar magnetic field and the interaction of the beating is analogous to the interaction of the potential wave directed along the magnetic field  $B_0$  with the particles. In such a case one can use the following Maxwell equation:

$$\frac{1}{4\pi}\frac{\partial E}{\partial t} + J^{(3)} = 0 \tag{18}$$

which can be reduced to the expression

$$\frac{\partial W(\boldsymbol{k},\omega)}{\partial t} = -Re[\sigma^{(3)}(k)]I(\boldsymbol{k}), \qquad (19)$$

where  $W(\mathbf{k},t) = \omega_k (I_k/8\pi)(\partial Re\epsilon_{\varphi\varphi}/\partial \omega_k)$  is the spectral energy density of the oscillations,  $I(k,\omega) = I_k \delta(\omega - \omega_k)$ ;  $\omega_k$  is the solution of the linear dispersion relation.

Defining the nonlinear decrement as  $\Gamma_{NL} = \frac{1}{2} \partial \ln W(k) / \partial t$  one obtains the estimate

$$\Gamma_{NL} \approx Re\sigma^{(3)}(k) \frac{I_k}{W(k)} \approx \frac{\int I_k dk}{mc^2 n_p \gamma_p} \frac{\omega_p^2}{\omega_k \gamma_p^2}.$$
 (20)

It follows from (20) that  $\Gamma_{NL}$  equals  $\Gamma_L$  (linear growth rate) for a sufficiently low turbulence level  $I_k dk/mc^2 n_p \gamma_p \ll 1$ . The pumping rate of the excited waves in the long-wave region  $\sim 1/\Gamma_{NL}$  appears to be of the order of the time  $\tau$ . Hence a balance sets in between the energy inflow and outflow in the region of maximal wavelength  $k_x = k_x^{min} = 10^{-8} \text{ cm}^{-1}$ . For the initial  $k_x^0$  and intermediate  $k_x$  the pumping rate is substantially higher  $(k_{\varphi}^0/k_x^0 \sim 10^{-7})$ . In the long-wave part of spectrum the pumping rate slows down. The frequency of the accumulated drift waves  $\omega = k_x u_x \sim 1 \text{ s}^{-1}$  can be of the same order as the pulsar angular velocity  $\Omega$ .

#### 4. Mechanism for Subpulse Drift

Consequently, at the distances ~  $10^9$  cm and at small angles with respect to the pulsar magnetic field, pulsar radio-wave generation can occur. Also, at the same distances the existence of accumulated drift waves propagating almost transversely to the magnetic field ( $k_x \gg k_{\varphi}$ ) with a frequency of the order of pulsar angular velocity

$$\omega \approx \Omega \pm \Delta \tag{21}$$

is possible. Note that the low-frequency wave propagating across  $B_0$  ( $\mathbf{k} \approx k_x$ ) is a purely transverse one with the electric vector directed along  $B_0$ . The Maxwell equation gives us the amplitude of the magnetic field of the wave:  $B_r = E_{\varphi}kc/\omega$ ; and  $B_r \gg E_{\varphi}$ . Thus, mainly the *r* component of the magnetic field is perturbed at the wave propagation.

Let us show that an increase of  $B_r$  is accompanied by a change of the magnetic field curvature. In a Cartesian coordinate system in an xy plane of the magnetic field curvature we have

$$\frac{dy}{dx} = \frac{B_y}{B_x}.$$
(22)

The curvature  $\rho = 1/R_B$  is defined by

$$\rho = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{-\frac{3}{2}} \frac{d^2y}{dx^2}.$$
(23)

Using  $div \mathbf{B} = 0$  and rewriting the expressions in the cylindrical coordinates at  $k_r \rightarrow 0$ ,  $(\partial/\partial r) \rightarrow 0$  one obtains

$$\rho = \frac{B_{\varphi}}{B} \frac{1}{r} - \frac{B_{\varphi}^2}{B^2} \frac{1}{r} \frac{1}{B} \frac{\partial B_r}{\partial \varphi}.$$
(24)

The change of value  $B = (B_{\varphi}^2 + B_r^2)^{1/2} \approx B_{\varphi}(1 + B_r^2/2B_{\varphi}^2)$  at  $B_r^2/B_{\varphi}^2 \ll 1$  is negligibly small. At the same time at  $k_{\varphi}r \gg 1$  the change of the value  $\rho = (1/r)(1 - k_{\varphi}rB_r/B_{\varphi})$ 

is significant. Thus a change of  $B_r$  changes significantly only  $u_x$ . Hence the resonance condition (6) will be fulfilled only for definite phases of the drift wave. If the frequency  $\omega$  coincides with  $\Omega$  exactly ( $\Delta = 0$ ), the generation region rotates together with the magnetosphere and an observer should see a steady subpulse. If  $\omega = \Omega + \Delta$  and  $\Delta \ll \Omega$  the necessary drift wave phase overcomes the magnetosphere rotation and a subpulse drift in the pulsar rotation direction should be observed. In the case  $\omega = \Omega - \Delta$ , the wave phase falls behind the pulsar rotation and a subpulse drift in an opposite direction should be observed. After definite time  $P_3 \approx \Omega/\Delta$  the phase returns to the original position and the process is repeated.



**Fig. 4.** Idealised picture of the emission pattern from one source. The solid line represents the line of sight trajectory. The dotted areas are regions of maximum intensity. Depending on the line of sight trajectory one can obtain from one to four subpulses with  $P_2$  defined by  $\theta_1 - \theta_2$ ;  $2\theta_1$  and  $2\theta_2$ .

Let us shortly discuss the question of the subpulse description within our model. Evidently it should be connected with the resonance condition (6). For this condition the term  $1/\gamma_{res}^2$  is negligibly small and if, for simplicity one neglects also the term  $k_r^2/2k_{\omega}^2$ , then one obtains from the quadratic equation (6)

$$\theta_{1,2} = \frac{u_{\chi}}{c} \pm (2\delta)^{\frac{1}{2}}.$$
 (25)

Consequently the intensity maximum should be within the angles  $\theta_1$  and  $\theta_2$ . Fig. 4 shows that depending on the place where the line of sight cuts the emission surface (in this idealised case) one can observe from one to four subpulses. The distance between the subpulses  $P_2$  is defined by the difference  $\theta_1 - \theta_2 \approx 2(2\delta)^{1/2}$  and by the values  $2\theta_1$  and  $2\theta_2$ . Recalling that  $u_x/c \approx 10^{-2}$  and  $\delta \approx 10^{-2}-10^{-3}$  one has an estimate  $P_2 \sim 3^\circ-30^\circ$  confirmed by the observations (Manchester and Taylor 1977). We have discussed the resonance conditions only for t-waves. However, the excitation conditions for the lt-mode could be satisfied as well (with  $E^{lt}$  perpendicular to  $E^{t}$ ). This condition has the following form:

$$\theta = \pm \frac{u_x}{c} \frac{\omega_p}{\omega^{lt}} \sqrt{2} \gamma^{1/2} \,. \tag{26}$$

Of course the wave excitation should not be obligatory at the same distances. At different heights the frequencies of the low-frequency drift waves should differ too; e.g.

$$\omega \approx \Omega \pm \Delta_{1,2,3,\dots}$$
,

where  $\Delta_1 \neq \Delta_2 \neq \Delta_3$ . We believe that PSR 2319+60 is just such an example of this situation (Wright and Fowler 1981), where all three components have different values of  $P_3$ ; 8P for the A-mode (i.e.  $\Omega/\Delta_1 = 8$ ), 4P for the B-mode ( $\Omega/\Delta_2 = 4$ ). The central component does not drift at all. This means that at the heights where the central component is generated either  $\Delta_3 = 0$  (less probable) or  $\omega \gg \Omega$  and we observe some smeared picture (quite probable) or there is no drift wave at all (most probable).

## 5. Conclusions

We believe that we have found a possible mechanism that can serve as a qualitative explanation of the subpulse drift phenomenon. Note that this mechanism does not need any additional assumptions. We assume the presence of the plasma and the beam particles only.

The drift wave propagating almost transversely to the magnetic field can affect the fulfilment of the radio-wave generation conditions. If the pulsar angular velocity is near to the frequency of the drift wave ( $\Omega \approx \omega$ ) one should observe regular drift phenomena. Otherwise ( $\Omega \ll \text{or} \gg \omega$ ) random appearances of subpulses along the pulse window should be observed.

Another model for the drift phenomenon is the spark model (Cheng and Ruderman 1980; Ruderman and Sutherland 1975). In the framework of the spark model the subpulse drift phenomenon is explained by the perpendicular component of electric field in the region of the polar cap. The value of the drift velocity of sparks in this is approximately

$$V_d = \Omega(r\cos\phi - z\sin\phi),$$

where r and z are coordinates across the polar cap and along the magnetic axis, and  $\Phi$  is the angle between the rotation axis and the magnetic axis of the pulsar dipole field. Details of the distribution of electric potential in the polar cap were given by Smith *et al.* (1981). For small values of  $\Phi$  it is difficult to explain different values of  $P_3$  for the same pulsar. If  $\Phi \approx \frac{1}{2}\pi$ , in our opinion it seems unlikely that sparks can exist for a sufficiently long time without spatial broadening (Filipenko and Radhakrishnan 1982).

Within our mechanism it is possible to explain different directions and reversals of subpulse drifting, and different values of  $P_3$  even for the same pulsar. Surely more work is required to transform this result into a model of subpulse drift.

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### References

- Allen, M. C. (1985). Mon. Not. R. Astron. Soc. 215, 111.
- Allen, M. C., and Melrose, D. B. (1982). Proc. Astron. Soc. Aust. 4, 365.
- Arons, J. (1981). Proc. Varenna Summer School & Workshop on Plasma Astrophysics, ESA SP-161, 288.
- Backer, D. S. (1970). Nature 227, 692.
- Cheng, A. F., and Ruderman, M. A. (1980). Astrophys. J. 235, 576.
- Drake, F. D., and Craft, H. D. (1968). Nature 220, 231.
- Filipenko, A. V., and Radhakrishnan, V. (1982). Astrophys. J. 263, 828.
- Galeev, A. A., and Sagdeev, R. Z. (1967). 'Reviews on Plasma Physics', Vol. 7.
- Goldreich, P., and Julian, W. (1969). Astrophys. J. 157, 869.
- Huguenin, G. R., Taylor, J. H., and Troland, T. H. (1970). Astrophys. J. 162, 727.
- Jones, P. B. (1984). Mon. Not. R. Astron. Soc. 209, 569.
- Kazbegi, A. Z., Machabeli, G. Z., Melikidze, G. I., and Usov, V. V. (1989*a*). Proc. Varenna-Abastumani International School & Workshop on Plasma Astrophysics, ESA SP-285 1, 271.
- Kazbegi, A. Z., Machabeli, G. Z., and Melikidze, G. I. (1989b). Proc. Varenna-Abastumani International School & Workshop on Plasma Astrophysics, ESA SP-285 1, 277.
- Kazbegi, A. Z., Machabeli, G. Z., and Melikidze, G. I. (1991). Mon. Not. R. Astron. Soc. (submitted).
- Lominadze, J. G., Mikhailovskii, A. B., and Sagdeev, R. Z. (1979). ZhETF Sov. Phys. 76, 959.
- Lominadze, J. G., Machabeli, G. Z., Melikidze, G. I., and Pataraya, A. D. (1986). Sov. J. Plasma Phys. 12, 712.
- Manchester, R. N., and Taylor, J. H. (1977). 'Pulsars' (Freeman: San Francisco).
- Pataraya, A. D., Chargeishvili, B. B., and Zedginidze, M. Z. (1986). Proc. Varenna-Abastumani Summer School & Workshop, ESA, SP-251, 417.
- Rankin, J. M. (1983). Astrophys. J. 274, 333.
- Rankin, J. M. (1986). Astrophys. J. 301, 901.
- Ruderman, M. A., and Sutherland, P. G. (1975). Astrophys. J. 196, 51.
- Smith, D. F., Muth, L. A., and Arons, J. (1981). Proc. Varenna Summer School & Workshop on Plasma Astrophysics, ESA SP-161, 288.
- Taylor, J. H., and Huguenin, G. R. (1971). Astrophys. J. 167, 273.
- Volokitin, A. S., Krasnoselskikh, V. V., and Machabeli, G. Z. (1985). Sov. J. Plasma Phys. 11, 310.
- Wright, G. A., and Fowler, L. A. (1981). Astron. Astrophys 101, 356.

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