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Nonlocality and the D-state Probability of the Deuteron

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Abstract

Attractive short-range nonlocality incorporated only into the D state of the deuteron decreases the D-state probability P_D. The variation of P_D versus nonlocality strength is a characteristic curve.

1. Introduction

Kermode *et al.* (1991*a*, 1991*b*) have shown recently that the inclusion of attractive short-range nonlocality in the nucleon–nucleon potential helps to give smaller values of the deuteron radius r_D. This removes the inconsistency between the r_D–a_t linear relation found for potential models by Klarsfeld *et al.* (1986) and the point (r_D, a_t) representing the experimental values of the deuteron radius $r_D = 1\cdot953 \pm 0\cdot003$ fm (Klarsfeld *et al.* 1986) and the triplet scattering length $a_t = 5\cdot419 \pm 0\cdot007$ fm (Klarsfeld *et al.* 1984). Also, the radial deuteron wavefunctions of this class of potential (Kermode *et al.* 1991*a*, 1991*b*) do not have simple shapes at smaller radii, in contrast to those of the standard potential models, e.g. Fig. 2 in Kermode *et al.* (1991*a*) and Figs 2 and 3 in Kermode *et al.* (1991*b*). These short-range structures in the radial deuteron wavefunctions were associated with the non-pointlike structure of the nucleon (Kermode *et al.* 1991*b*; Mustafa and Kermode 1991).

Attractive short-range nonlocality was incorporated into the S channel in the deuteron potential of Kermode *et al.* (1991*a*, 1991*b*) and in 'both' the S and D channels in the deuteron potential of Mustafa *et al.* (1992). In the latter case, no pronounced effect was found in deuteron properties as a result of introducing attractive nonlocality into the D state (in addition to the S state) because of the relatively small role of the D state in comparison with that of the S state. Hence, it is necessary to 'switch off' the S-state nonlocality in order to see the relatively small effect of the D-state attractive nonlocality on deuteron properties. In this paper, attractive short-range nonlocality is included *only* in the D-state radial equation to isolate the effect of D-state nonlocality. For this purpose, seven potential models having different strengths for the D-state nonlocality are considered.

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2. Potential Model

The potential is assumed to be the sum of a nonlocal separable part, which operates only in the D state and has the form $\lambda_w\, g(r)\, g(r')$, plus a local part consisting of central (C), spin–orbit (LS) and tensor (T) components of the Reid (1968) hard-core potential,

$$V_{\mathrm{C}} + V_{LS}\boldsymbol{L}\,.\,\boldsymbol{S} + V_{\mathrm{T}}\, S_{12}\,, \tag{1}$$

where

$$V_{\mathrm{C}} = V_{\mathrm{C}}^{\mathrm{OPEP}} + \sum_{m=2}^{n} A_{\mathrm{C}}^{(m)}\, r^{-1} \exp(-m\mu r)\,, \tag{2a}$$

$$V_{LS} = \sum_{m=2}^{n} A_{LS}^{(m)}\, r^{-1} \exp(-m\mu r)\,, \tag{2b}$$

$$V_{\mathrm{T}} = V_{\mathrm{T}}^{\mathrm{OPEP}} - B_{\mathrm{T}}\, N^2\{1 + 3/N\mu r + 3/(N\mu r)^2\} r^{-1} \exp(-N\mu r) + \sum_{m=2}^{n} A_{\mathrm{T}}^{(m)}\, r^{-1} \exp(-m\mu r)\,, \tag{2c}$$

and where $V_{\mathrm{C}}^{\mathrm{OPEP}}$ and $V_{\mathrm{T}}^{\mathrm{OPEP}}$ are the central and tensor parts of the one-pion exchange potential (OPEP), $B_{\mathrm{C}} = B_{\mathrm{T}} = -14{\cdot}94714$ MeV fm, $\mu = 0{\cdot}7\ \mathrm{fm}^{-1}$ and $N = n = 6$. The radial coupled Schrödinger equations in this case are

$$u'' = (P - k^2)u + Sw\,, \tag{3a}$$

$$w'' = (Q - k^2)w + Su + \lambda_w\, g(r) \int_{r_c}^{\infty} w(r')g(r')\, \mathrm{d}r'\,, \tag{3b}$$

where

$$P = V_{\mathrm{C}}\,, \qquad S = 2\surd 2 V_{\mathrm{T}}\,, \qquad Q = 6/r^2 + V_{\mathrm{C}} - 3V_{LS} - 2V_{\mathrm{T}}\,,$$

and where $r_{\mathrm{c}} = 0{\cdot}54833$ fm is the hard-core radius, $g(r) = \exp(-\alpha r)$ with $\alpha = 2{\cdot}1\ \mathrm{fm}^{-1}$, and k^2 is the energy in units of fm^{-2}. The coefficient of the nonlocality strength λ_w is fixed at a certain value and the free parameters $A_{\mathrm{C}}^{(m)}$, $A_{LS}^{(m)}$ and $A_{\mathrm{T}}^{(m)}$ are varied by the computer search to give the experimental value of the deuteron binding energy of $-2{\cdot}2246$ MeV and the scattering parameters (χ^2/datum $\sim 0{\cdot}005$) of the Reid (1968) hard-core potential (at lab energies of 1, 5, 10, 50, 100, 180 and 300 MeV). Values of the nonlocality strength parameter λ_w together with the coefficients $A_{\mathrm{C}}^{(m)}$, $A_{LS}^{(m)}$ and $A_{\mathrm{T}}^{(m)}$ are listed in Table 1. The radial dependencies of the potentials are compared with those of the Reid hard-core potential in Fig. 1.

3. Deuteron D-state Probability

The variation of the deuteron D-state probability

$$P_{\mathrm{D}} = \int_{r_c}^{\infty} w^2\, \mathrm{d}r \bigg/ \int_{r_c}^{\infty} (w^2 + u^2)\, \mathrm{d}r \tag{4}$$

Table 1. Nonlocality strength parameter λ_w (fm^{-3}) and coefficients $A_{\mathrm{C}}^{(m)}$, $A_{LS}^{(m)}$ and $A_{\mathrm{T}}^{(m)}$ (MeV fm)

λ_w	m	$A_{\mathrm{C}}^{(m)}$	$A_{LS}^{(m)}$	$A_{\mathrm{T}}^{(m)}$
−100	2	−2188643(+1)	2214136(+1)	3407503(+1)
	3	2659548(+1)	−1321324(+3)	5502787(+1)
	4	3296096(+3)	1703838(+4)	4432382(+3)
	5	−2026089(+4)	−1261635(+5)	7095131(+2)
	6	−3794288(+4)	1882653(+5)	−5896876(+3)
−250	2	7101766(+2)	2333506(+2)	−5093599(+2)
	3	−2421553(+4)	−1815615(+3)	1717855(+4)
	4	2388450(+5)	−1929453(+4)	−1644543(+5)
	5	−8448221(+5)	2398685(+4)	6173661(+5)
	6	8618588(+5)	−1380646(+4)	−6996353(+5)
−500	2	1037240(+3)	4985464(+2)	−1017186(+3)
	3	−3385887(+4)	−5253423(+3)	3093014(+4)
	4	3232449(+5)	−1788165(+4)	−2822859(+5)
	5	−1114258(+6)	−5347893(+4)	1005358(+6)
	6	1124022(+6)	1029453(+5)	−1092855(+6)
−750	2	6357362(+2)	2683646(+1)	−1056010(+3)
	3	−2221789(+4)	1218378(+4)	3295575(+4)
	4	2272786(+5)	−2045457(+5)	−3076883(+5)
	5	−8315989(+5)	4935116(+5)	1117750(+6)
	6	8520294(+5)	−4255065(+5)	−1225391(+6)
−1000	2	4689050(+1)	−7018797(+2)	−1021974(+3)
	3	−7129643(+3)	4214992(+4)	3311404(+4)
	4	1200168(+5)	−5451785(+5)	−3296493(+5)
	5	−5660877(+5)	1729881(+6)	1270238(+6)
	6	6500746(+5)	−1856039(+6)	−1461277(+6)
−1500	2	−9720935(+2)	−5378061(+3)	−4261068(+2)
	3	1947312(+4)	1626426(+5)	2369893(+4)
	4	−9658759(+4)	−1557791(+6)	−2730245(+5)
	5	8988391(+4)	4652455(+6)	1131157(+6)
	6	−2073809(+3)	−4597195(+6)	−1332879(+6)
−2000	2	−6985243(+2)	−3501867(+3)	−2036184(+3)
	3	1780327(+4)	1288668(+5)	5278521(+4)
	4	−1074853(+5)	−1375061(+6)	−4450345(+5)
	5	1873720(+5)	3573516(+6)	1477409(+6)
	6	−1581671(+5)	−2906974(+6)	−1498138(+6)

for these potentials with nonlocality strength parameter λ_w is shown in Fig. 2. In equation (4) u and w are the radial deuteron wavefunctions of the S and D states respectively. The probability P_{D} decreases as the magnitude of the nonlocality strength parameter $\mid \lambda_w \mid$ is increased. It is interesting that the points $(P_{\mathrm{D}}, \lambda_w)$ representing the potentials lie on a characteristic curve.

Interesting features are also found in the shapes of the radial deuteron wavefunctions u and w. A gradual increase of the short-range nonlocal attraction from one potential to the other one implied by a gradual increase in the value of $\mid \lambda_w \mid$ can also be seen from the ordering of the graphs representing the u wavefunctions outside and close to the hard-core radius in Figs 3a and 3b respectively. The slopes of the u wavefunctions monotonically increase with increasing $\mid \lambda_w \mid$. The nonlocal attraction can also be seen by switching off the nonlocality ($\lambda_w = 0$)

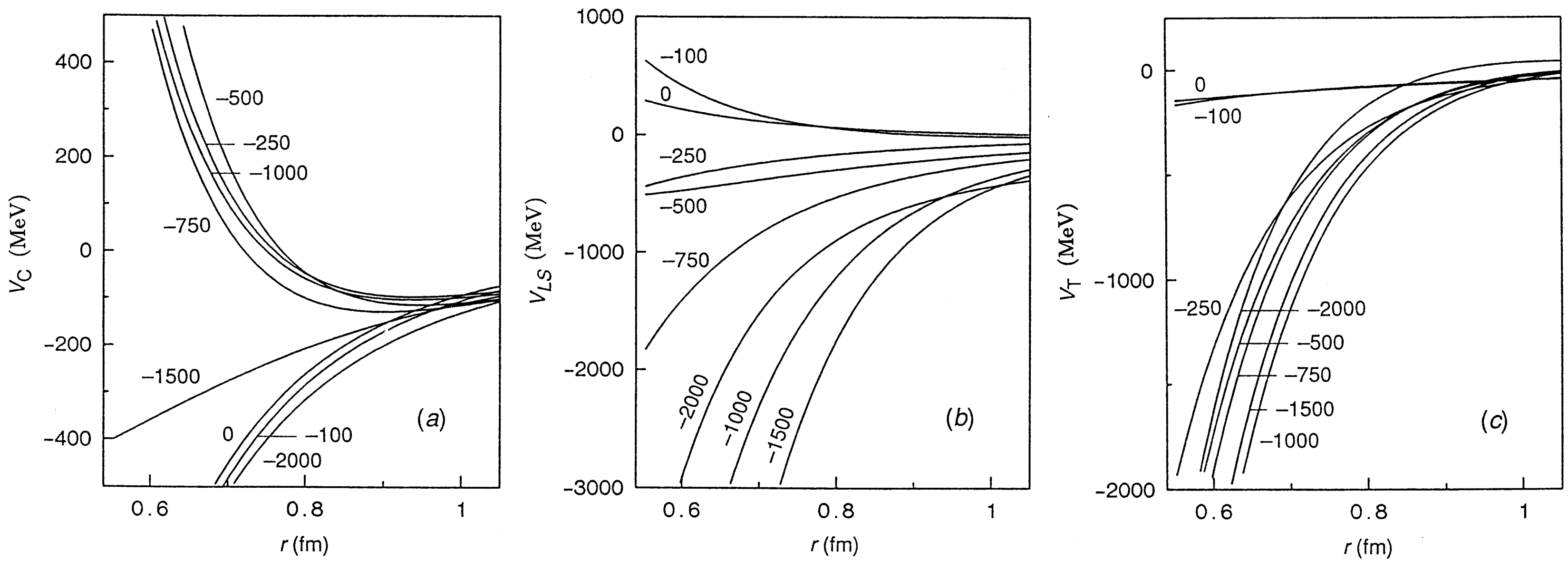

Fig. 1. Radial dependencies of the (*a*) central, (*b*) spin–orbit and (*c*) tensor components of the potentials of Table 1 compared with those of the Reid (1968) hard-core potential. Each graph is labelled with its value of λ_w (fm^{-3}).

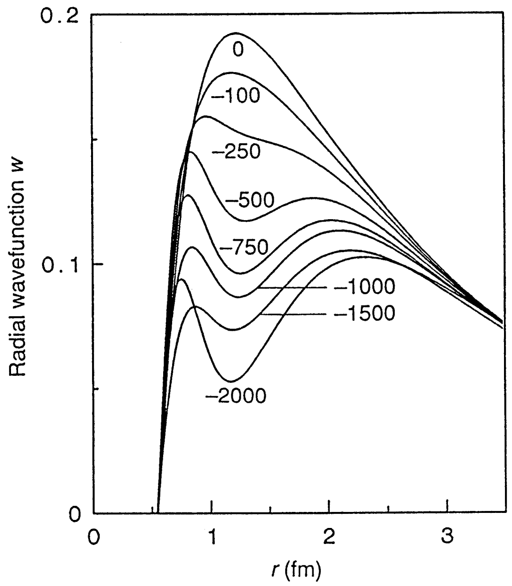

Fig. 2. Variation of the D-state probability P_D of the potentials with the nonlocality strength λ_w.

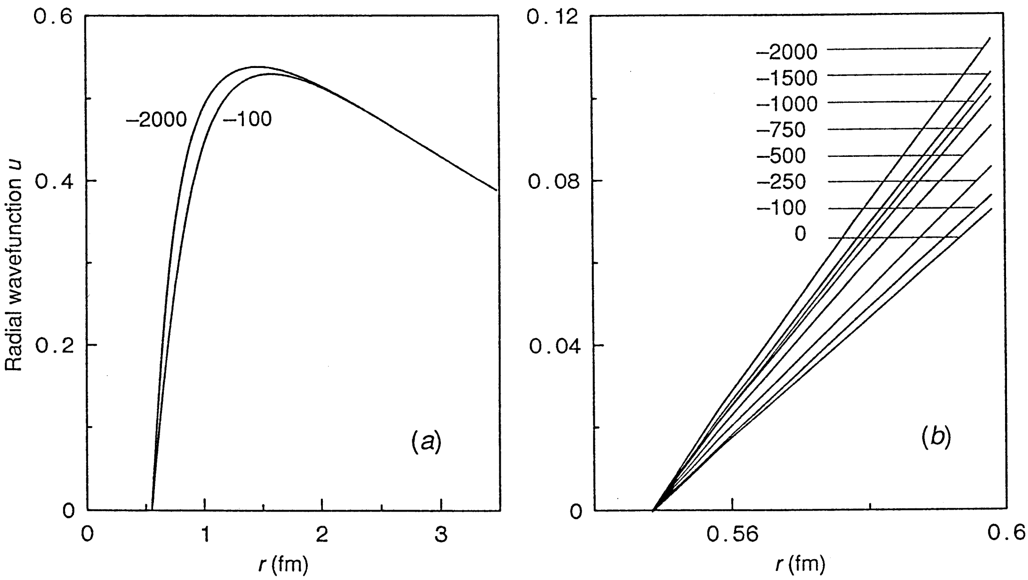

Fig. 3. Radial dependencies of the u wavefunctions of the potentials with (a) $\lambda_w = -100\ \text{fm}^{-3}$ and $-2000\ \text{fm}^{-3}$ and (b) the potentials of Table 1 and the Reid (1968) hard-core potential near the hard-core radius.

and calculating the binding energy of the local part alone. The local part is relatively repulsive, e.g. the local part of the potential with the smallest $|\lambda_w|$ has a small binding energy of $-1{\cdot}087$ MeV and the local parts with larger $|\lambda_w|$ are repulsive enough to exclude bound states.

The short-range structure found in the u wavefunctions of the potentials of Kermode *et al.* (1991*a*, 1991*b*) and in both the u and w waves of the potential

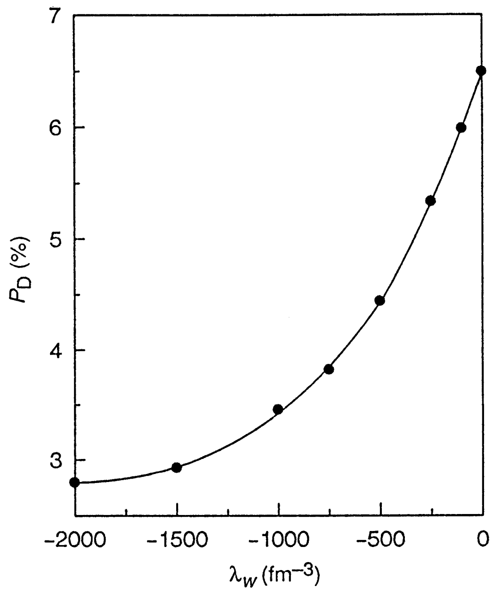

Fig. 4. Radial dependencies of the w wavefunctions of the potentials of Table 1 and of the Reid (1968) hard-core potential. Each graph is labelled with its value of λ_w (fm^{-3}).

of Mustafa *et al.* (1992) has also been found in the w wavefunctions of the present work, as shown in Fig. 4. It is clear from Fig. 4 that the attractive short-range nonlocality which acts only in the D state increases the complexity of these short-range structures. Also, within the radial range $r = 0\cdot88$ to $2\cdot5$ fm, the graphs in Fig. 4 representing the wavefunctions w of the D state are ordered monotonically by the nonlocality strength parameter λ_w. The Reid hard-core potential with no nonlocality ($\lambda_w = 0$) is at the top and the potential with $\lambda_w = -2000\ \text{fm}^{-3}$, having the largest nonlocal attraction, is at the bottom.

4. Conclusions

We have found that the D-state probability of the deuteron is sensitive in particular to the decrease caused by nonlocality in the w wavefunction at short radii where short-range structure occurs. The probability decreases smoothly with increasing strength of the attractive short-range nonlocality which acts only in the D state.

The potential of Kermode *et al.* (1991*a*) which incorporates attractive short-range nonlocality only in the S state has a relatively large value for the D-state probability of $P_D = 7\cdot64\%$. The inclusion of this nonlocality in the D state, in addition to the S state, in the potential of Mustafa *et al.* (1992) may be partially the reason for its giving the smaller value of $P_D = 6\cdot29\%$, a result that one would expect from the findings of the present work.

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