

where

$$d\Psi_{-1}^2 = d\theta^2 - \sinh^2 \theta d\tau^2, \quad d\Psi_0^2 = d\theta^2 - \theta^2 d\tau^2, \quad d\Psi_{+1}^2 = d\theta^2 - \sin^2 \theta d\tau^2. \quad (32)$$

Note that for these NGT class B solutions, as opposed to the NGT class A solutions above, the two NGT parameters α_1 and c_2 are no longer arbitrary. In fact, $-1 < c_2 < 1$ and α_1 must satisfy the inequality $r^4(1 - c_2^2) - \alpha_1^2 \geq 0$ in order to preserve signature.

5. NGT Plane-Symmetric Bianchi Type I Solution with EM Field

In order to obtain the desired solution we start from the NGT generalisation of the class AI (Kramer *et al.* 1980) GR solution, i.e. (29) with $k = +1$. As in Section 3, we introduce a new parameter ϵ such that

$$2m = m_1\epsilon^{-3}, \quad \alpha_1 = -c_1\epsilon^{-2}, \quad \alpha_2 = -q_e\epsilon^{-2}, \quad \alpha_3 = -q_m\epsilon^{-2}, \quad (33)$$

apply the coordinate transformation

$$r = \epsilon^{-1}T, \quad \tau = \epsilon x, \quad \theta = \epsilon \sqrt{y^2 + z^2}, \quad \phi = \tan^{-1} \left(\frac{z}{y} \right), \quad (34)$$

and take the limit $\epsilon \rightarrow 0$. The result of these operations is

$$ds^2 = -A(T) \left\{ 1 + \frac{c_1^2}{T^4(1+c_2^2)} \right\} dx^2 - T^2(dy^2 + dz^2) + A(T) dT^2, \quad (35a)$$

$$g_{[14]} = \frac{c_1}{T^2 \sqrt{1+c_2^2}}, \quad g_{[23]} = -c_2 T^2, \\ f_{14} = \frac{c_e}{T^2}, \quad f_{23} = -c_m, \quad (35b)$$

where

$$A(T) \equiv \frac{m_1}{T} - \frac{4\pi}{T^2} \left(c_e^2 + \frac{c_m^2}{1+c_2^2} \right) + \frac{\lambda T^2}{3}. \quad (36)$$

In order to interpret this as a plane-symmetric cosmological solution we require T to be the time coordinate. This implies that T can take only those values for which $A > 0$.

In terms of the comoving coordinate system (x, y, z, t) we can write our six-parameter plane-symmetric Bianchi type I solution as follows:

$$ds^2 = -A(T) \left\{ 1 + \frac{c_1^2}{T^4(1+c_2^2)} \right\} dx^2 - T^2(dy^2 + dz^2) + dt^2, \quad (37a)$$

$$g_{[14]} = \frac{c_1}{T^2 \sqrt{A(T)(1+c_2^2)}}, \quad g_{[23]} = -c_2 T^2, \\ f_{14} = \frac{c_e}{T^2 \sqrt{A(T)}}, \quad f_{23} = -c_m, \quad (37b)$$

with $T(t)$ given, implicitly, by

$$t = \int^T \sqrt{A(u)} du, \quad (38)$$

where A is defined in (36). Among the parameters λ is the cosmological constant and c_e and c_m are the electric and magnetic charges, respectively. There are two NGT parameters, c_1 and c_2 , which distinguish this solution from the corresponding Einstein–Maxwell solution. If we set, for example, $c_1 = c_2 = c_e = m_1 = \lambda = 0$, in (35a,b) and (36), then we obtain a general relativistic Bianchi type I cosmological model with pure magnetic field discussed by De (1975) (case D, solution 1).

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