Aust. J. Phys., 1994, **47**, 59–76

Effect of Streaming Electrons on the Propagation of Electromagnetic Waves in a Magnetised Relativistic Plasma

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Abstract

Expressions for the first- and third-order dispersion relations have been derived for circularly polarised waves in a magnetised relativistic plasma containing electron streams. From the first-order dispersion relations it is seen that the waves are split into three parts, one of which is reflected causing the formation of standing waves in the streaming plasma. From the third-order dispersion relations, the expression for the shifts of wave number has been obtained. It is observed that the streaming of electrons has a significant contribution to the wave-number shift of the electromagnetic waves.

1. Introduction

In recent years the propagation of electromagnetic (EM) waves in relativistic plasmas has been studied by various authors (Kaw and Dawson 1970; Steiger and Woods 1972; Max and Perkins 1972; Max 1973; Akhiezer *et al.* 1975; Kennel and Pellat 1976; Stenflo *et al.* 1983; Shukla *et al.* 1986; Shivamoggi 1989; Paul 1990; Chakraborty 1992). It has been found that relativistic effects have a significant contribution to linear and nonlinear phenomena occurring in both laboratory and space plasmas (Kaplan and Tsytovich 1973; Tsytovich and Stenflo 1983). Self-action effects, e.g. wave-number shift or frequency shift, precessional rotation, filamentation, are the most important nonlinear phenomena arising from the nonlinear interaction of waves and particles in plasmas (Sodha *et al.* 1976; Max *et al.* 1974; Arons and Max 1974). Sluijter and Montgomery (1965) have shown that the relativistic mass correction effect is important for electron motion in a simple plasma for the investigation of the frequency shift of a plane-polarised transverse wave, and they improved the results of other authors (Sturrock 1957; Jackson 1960; Montgomery and Tidman 1964; Dawson 1969). Subsequently, Tidman and Stainer (1965) obtained the intensity dependence of the shift in frequency and wave number for waves in a finite temperature plasma by considering both EM waves and electron–plasma oscillations. Later, Boyd (1967), Das (1968, 1971), Goldstein and Salu (1973), Schindler and Janick (1973), Chandra (1974, 1979) and many other authors have investigated theoretically the frequency and wave-number shifts for EM waves propagating in a relativistic plasma under

0004-9506/94/010059$05.00

various physical situations. Chakraborty (1977) and coworkers (Chakraborty and Paul 1983; Bhattacharyya and Chakraborty 1979, 1982; Khan and Chakraborty 1979; Bhattacharyya 1983; Chakraborty *et al.* 1984) studied theoretically other types of self-action effects, i.e. precessional rotation, inverse Faraday effects, etc., in unmagnetised as well as magnetised plasma.

The presence of streaming of particles in a relativistic plasma has a special role in different aspects of wave phenomena. Das and Paul (1985) and subsequently other authors (Das *et al.* 1988; Roy Chowdhury *et al.* 1989, 1990; Nejoh 1987*a*, 1987*b*, 1988; Singh and Dahia 1990; Ghosh and Roy 1991; Salauddin 1990; Chakraborty *et al.* 1992) have shown that the streaming of ions in a relativistic plasma has an important contribnution to the formation of ion-acoustic solitons and shocks. The effect of streaming electrons on the propagation of electromagnetic or electrostatic waves is also important. Gold (1965) has shown that in the presence of streams the EM instability occurs. Sturrock (1958), Briggs (1964), Buneman (1959), Tanenbaum (1967), Bandyopadhyay and Paul (1973) and many others have discussed the stream instability in various kinds of plasma. Clemmow and Dougherty (1969) and others have discussed the stream instability considering relativistic effects in the plasma. Recently, Khalil (1988) considered the two-stream instability in an electron–ion anisotropic isothermal plasma. But there is much work yet to be done on nonlinear wave propagation in relativistic plasmas containing electron streams. It is to be noted that none of the above authors have considered streaming of electrons in the investigation of nonlinear phenomena such as wave-number shift or frequency shift, precessional rotation, inverse Faraday effect, etc.

In the present paper our motivation is to investigate the role of electron streaming in the propagation of EM waves in a relativistic plasma, particularly its effect on the stability of the wave, and the nonlinear shift of wave parameters. It has been found that, similar to the streaming of ions, the streaming of electrons has an important contribution to both linear and nonlinear propagation of waves in a relativistic plasma.

2. Basic Equations

We assume a cold, homogeneous and magnetic plasma. The static magnetic field is along the direction of wave propagation. The effect of collisions between the electrons is not considered. The electromagnetic power is very high and electrons attain relativistic velocities, but the ion velocity remains much below the electron velocity. The incident wave is circularly polarised, purely transverse and sinusoidal. Moreover, we assume that there is a streaming motion of electrons in the direction of wave propagation.

Therefore, we write the plasma equations as

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}_{\mathrm{e}} \cdot \nabla\right)\boldsymbol{p}_{\mathrm{e}} = -e\boldsymbol{E} - \frac{e}{c}\left(\boldsymbol{v}_{\mathrm{e}} \times \boldsymbol{H}\right), \tag{1}$$

$$\frac{\partial n_{\mathrm{e}}}{\partial t} + \nabla \cdot (n_{\mathrm{e}}\, v_{\mathrm{e}}) = 0\,, \tag{2}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t}, \tag{3}$$

$$\nabla \times \boldsymbol{H} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} - \frac{4\pi e n_e \boldsymbol{v}_e}{c}, \tag{4}$$

$$\nabla . \boldsymbol{H} = 0, \tag{5}$$

$$\nabla . \boldsymbol{E} = -4\pi e(n_i - n_e), \tag{6}$$

where $\boldsymbol{p}_e = m_0 \boldsymbol{v}_e/(1 - v_0^2/c^2)^{\frac{1}{2}}$, m_0 is the rest mass of the electron, v_e is the velocity, n_i and n_e are the number density of ions and electrons, $-e$ is the charge of an electron and the other parameters have their usual meanings.

3. First-order Dispersion Relations

We now assume that the field parameters are perturbed as

$$\begin{aligned} E &= 0 + \epsilon E^{(1)} + \epsilon^2 E^{(2)} + \epsilon^3 E^{(3)} + \dots, \\ H &= H_0 + \epsilon H^{(1)} + \epsilon^2 H^{(2)} + \epsilon^3 H^{(3)} + \dots, \\ v &= v_0 + \epsilon v^{(1)} + \epsilon^2 v^{(2)} + \epsilon^3 v^{(3)} + \dots, \\ n &= n_0 + \epsilon n^{(1)} + \epsilon^2 n^{(2)} + \epsilon^3 n^{(3)} + \dots, \end{aligned} \tag{7}$$

where ϵ is the expansion parameter and the field variables with superscripts $1, 2, 3$, etc. correspond to first-order, second-order and third-order approximations respectively. Further, H_0, v_0, n_0 are the static magnetic field, electron stream velocity and equilibrium electron density respectively.

Let us consider the electromagnetic wave which is circularly polarised in the form

$$E_{\pm}^{(1)} = a(\mathrm{e}^{\mathrm{i}\theta_\pm} + \mathrm{e}^{-\mathrm{i}\theta_\pm}), \tag{8}$$

where $E_{\pm}^{(1)} = E_x^{(1)} \pm \mathrm{i}\, E_y^{(1)}$, a is the amplitude of the wave, $\theta_\pm = (k_\pm z - \omega t)$, k and ω are the wave number and frequency of the wave, and the subscript $+$ and $-$ signs represent the left circularly polarised (LCP) and right circularly polarised (RCP) components of the wave respectively.

Therefore, using (7) and (8) in (1) to (6), the first-order field parameters are obtained as

$$v_{\pm}^{(1)} = -\frac{\mathrm{i}\, ea}{m_0\, \omega}\left(\frac{(\omega \mp k_\pm v_{0z})\mathrm{e}^{\mathrm{i}\theta_\pm}}{\gamma_0(\omega - k_\pm v_{0z}) \pm \Omega_e} - \frac{(\omega \mp k_\mp v_{0z})\mathrm{e}^{-\mathrm{i}\theta_\mp}}{\gamma_0(\omega - k_\mp v_{0z}) \mp \Omega_e}\right), \tag{9}$$

$$H_{\pm}^{(1)} = \pm\frac{\mathrm{i}\, ca}{\omega}(k_\pm \mathrm{e}^{\mathrm{i}\theta_\pm} + k_\mp \mathrm{e}^{-\mathrm{i}\theta_\mp}), \tag{10}$$

$$v_z^{(1)} = 0, \quad N^{(1)} = 0, \quad H_z^{(1)} = 0, \tag{11}$$

where

$$\Omega_{\mathrm{e}} = eH_0/m_0\,c\,, \qquad H_0 \equiv (0,0,H_0)\,,$$

$$\gamma_0 = 1 + v_{\mathrm{e}z}^{(0)^2}/2c^2\,.$$

Eliminating the field component from (9)–(11), the first-order dispersion relation is obtained as

$$n_{\pm}^2 = \frac{k_{\pm}^2\,c^2}{\omega^2} = 1 - \frac{X_{\mathrm{e}}(1 \mp v_{0z}\,k_{\pm}/\omega)}{\gamma_0(1 - v_{0z}\,k_{\pm}/\omega) \pm \gamma_{\mathrm{e}}}\,, \tag{12}$$

where

$$X_{\mathrm{e}} = \omega_{\mathrm{pe}}^2/\omega^2\,, \quad \omega_{\mathrm{pe}}^2 = \frac{4\pi N^{(0)}e^2}{m_0}\,, \quad \gamma_{\mathrm{e}} = \Omega_{\mathrm{e}}/\omega\,.$$

It is seen that the relativistic effect is introduced in the dispersion relation through the factor γ_0. In the absence of the streaming motion of electrons (i.e. $v_{0z} = 0$), then $\gamma_0 = 1$, which indicates that relativistic effects do not affect the dispersion relation, which in this case is reduced to a well-known form. However, from the dispersion relation (12), the nature of the propagating wave can be understood under different physical situations for different values of the streaming motion.

Let us write the relation (12) in the form

$$a_1\,k_{\pm}^3 + a_2\,k_{\pm}^2 + a_3\,k_{\pm} + a_4 = 0\,, \tag{13}$$

where

$$a_1 = \gamma_0\,c^2\omega^2 v_{\mathrm{e}z}^{(0)}\,,$$

$$a_2 = -\gamma_0\,c^2\omega^3 \mp c^2\omega^2\Omega_{\mathrm{e}}\,,$$

$$a_3 = -\gamma_0\,\omega^4 v_{\mathrm{e}z}^{(0)} \mp \omega_{\mathrm{pe}}^2\,v_{\mathrm{e}z}^{(0)}\,\omega^2\,,$$

$$a_4 = \gamma_0\,\omega^5 - \omega_{\mathrm{pe}}^2\,\omega^3 \pm \omega^4\Omega_{\mathrm{e}}\,.$$

It is seen that the dispersion relation becomes a cubic equation for k in the presence of stream electrons. But in the absence of a stream velocity, the relation is reduced to a familiar quadratic equation. To find $k_{\pm}$ from (13), we assume $k_{\pm} = X_{\pm} + \xi_{\pm}$. Using this value of k in (13) and setting the coefficient of X^2 to zero, we obtain

$$\xi_{\pm} = \frac{\gamma_0\,\omega \pm \Omega_{\mathrm{e}}}{3\gamma_0\,v_{\mathrm{e}z}^{(0)}} = \frac{\omega \pm \Omega_{\mathrm{e}}^{\mathrm{r}}}{3v_{\mathrm{e}z}^{(0)}}\,;$$

$$\Omega_{\mathrm{e}}^{\mathrm{r}} = \frac{\Omega_{\mathrm{e}}}{1 + v_{\mathrm{e}z}^{(0)^2}/2c^2}\,,$$

and finally we get an equation

$$X_\pm^3 + \beta_\pm X_\pm + \mu_\pm = 0\,, \tag{14}$$

where

$$\begin{aligned}\beta_\pm = & -\frac{\Omega_e^2}{3 v_{ez}^{(0)^2}\gamma_0^2} - \frac{\omega^2}{3v_{0z}^2} - \frac{\omega^2}{c^2} \\ & \mp \frac{2\omega\Omega_e}{3\gamma_0 v_{0z}^2} \pm \frac{\omega_{pe}^2}{c^2\gamma_0}\,,\end{aligned} \tag{15}$$

$$\begin{aligned}\mu_\pm = & \mp \frac{\Omega_e^3}{27 v_{0z}^3\gamma_0^3} - \frac{2\omega^3}{27 v_{0z}^3} \\ & - \frac{2\Omega_e^2\omega}{9v_{0z}^3\gamma_0^2} - \frac{2\omega_{pe}^2\omega}{3c^2 v_{0z}\gamma_0} \\ & \mp \frac{2\omega^2\Omega_e}{9v_{0z}^3\gamma_0} + \frac{\omega_{pe}^2\Omega_e}{3c^2 v_{0z}\gamma_0^2} \pm \frac{2\omega^2\Omega_e}{3c^2\gamma_0 v_{0z}} + \frac{2\omega^3}{3c^2 v_{0z}}\end{aligned} \tag{16}$$

If $\beta_\pm$ and $\mu_\pm$ are real, then $X_\pm$ will be real. Three values of $X_\pm$ are given by

$$\begin{aligned} k_\pm^{01} &= 2(-\tfrac{1}{3}\beta_\pm)^{\frac{1}{2}}\cos\theta_\pm + \xi_\pm\,, \\ k_\pm^{02} &= 2(-\tfrac{1}{3}\beta_\pm)^{\frac{1}{2}}\cos(\tfrac{2}{3}\pi + \theta_\pm) + \xi_\pm\,, \\ k_\pm^{03} &= 2(-\tfrac{1}{3}\beta_\pm)^{\frac{1}{2}}\cos(\tfrac{2}{3}\pi - \theta_\pm) + \xi_\pm\,, \end{aligned} \tag{17}$$

where

$$\theta = \tfrac{1}{3}\cos^{-1}[-\mu_\pm/2(-\beta_\pm^3/27)^{\frac{1}{2}}]\,.$$

It is seen that the modes of the LCP wave, i.e. $k_+^{01}, k_+^{02}, k_+^{03}$, are real when

$$\frac{\omega^2}{c^2} + \frac{\omega^2}{3v_{0z}^2} + \frac{\Omega_e^2}{3v_{ez}^{02}\gamma_0^2} + \frac{2\omega\Omega_e}{3\gamma_0 v_{0z}^2} > \frac{\omega_{pe}^2}{c^2\gamma_0}\,. \tag{18a}$$

But when

$$\frac{\omega^2}{c^2}\left(1 + \frac{c^2}{3v_{0z}^2}\right) + \frac{\Omega_e}{\gamma_0 v_{0z}^2}\left(\frac{\Omega_e}{3\gamma_0} + \tfrac{2}{3}\omega\right) < \frac{\omega_{pe}^2}{c^2\gamma_0}\,, \tag{18b}$$

all three modes will be unstable as the values of the three modes of the LCP waves become complex. However, for the RCP waves, the modes k_-^{01}, k_-^{02} and k_-^{03} are always real only when

$$\frac{\omega^2}{c^2}\left(1 + \frac{c^2}{3v_{0z}^2}\right) + \frac{\Omega_e}{\gamma_0 v_{0z}^2}\left(\frac{\Omega_e}{3\gamma_0} - \tfrac{2}{3}\omega\right) > -\omega_{pe}^2/c^2\gamma_0\,. \tag{19}$$

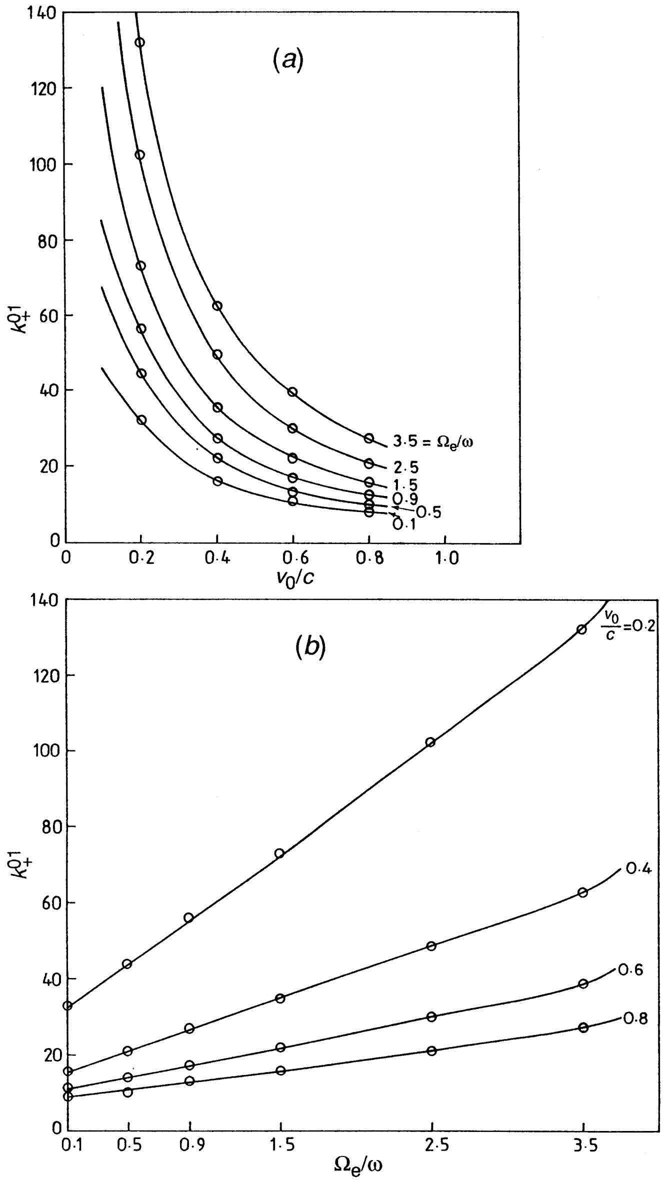
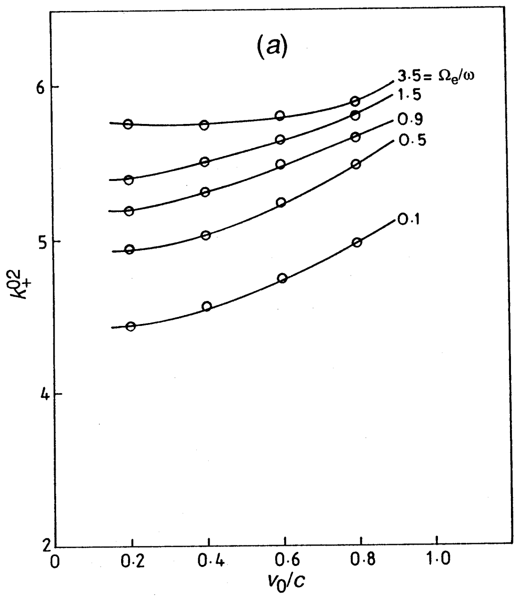


Fig. 1. Wave number of the first mode k_+^{01} of the LCP wave versus (*a*) the streaming velocity v_0/c and (*b*) the gyrofrequency Ω_e/ω.

To illustrate the nature of the wave number for all modes of LCP and RCP waves under various physical conditions, Figs 1–8 are presented below for different values of the stream velocity, gyrofrequency, etc. In Fig. 1*a* it is seen that the wave number of the first mode of the LCP wave, k_+^{01}, decreases as the stream velocity increases. But from Fig. 2*a* we see that the second mode k_+^{02} of the LCP wave increases with an increase in stream velocity. However, Fig. 2*b*



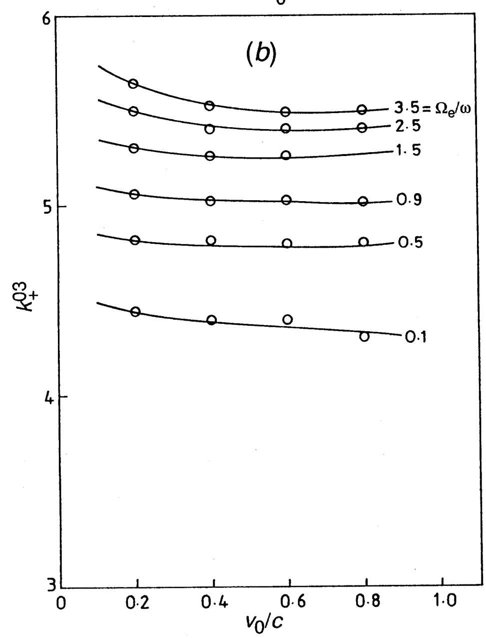


Fig. 2. Wave number of (a) the second mode k_+^{02} and (b) the third mode k_+^{03} versus the streaming velocity v_0/c.

shows that the third mode k_+^{03} of the LCP wave is always negative for any value of the stream velocity, which indicates that this mode will be reflected due to the streaming motion of electrons in the plasma. Reflection of the third mode of the propagating waves implies that standing waves may be formed due to superposition of the propagating and reflecting waves.

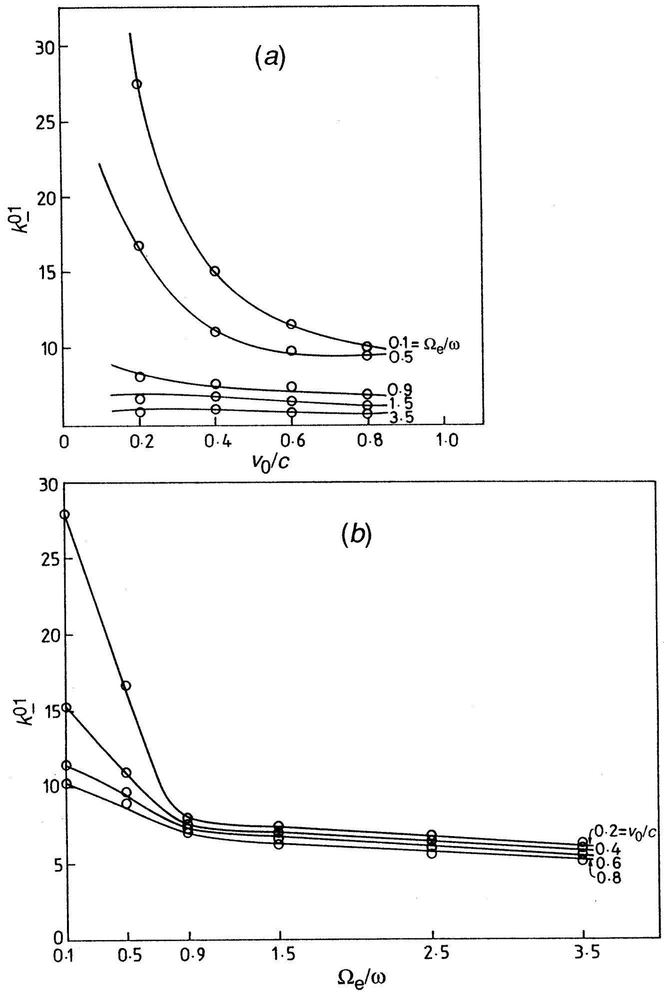


Fig. 3. Wave number of the first mode k_-^{01} of the RCP wave versus (a) the streaming velocity v_0/c and (b) the gyrofrequency Ω_e/ω.

4. Third-order Dispersion Relation and Wave-number Shift

Using equations (7) in (1)–(6), the second-order equations for the longitudinal field variables are obtained as

$$\left(1+\frac{3v_{0z}^2}{2c^2}\right)\frac{\partial v_{2z}}{\partial t}+\frac{v_{0z}}{2c^2}\frac{\partial}{\partial t}(v_+^{(1)}v_-^{(2)})+v_{0z}\left(1+\frac{3v_{0z}^2}{2c^2}\right)\frac{\partial v_{2z}}{\partial z}$$

$$+\frac{v_{0z}^2}{2c^2}\frac{\partial}{\partial z}(v_+^{(1)}v_-^{(2)})=-\frac{e}{m_0}E_z^{(2)}+\frac{\mathrm{i}\,e}{2m_0\,c}$$

$$\times\,(v_-^{(1)}H_+^{(1)}-v_+^{(1)}H_-^{(1)})\,, \tag{20}$$

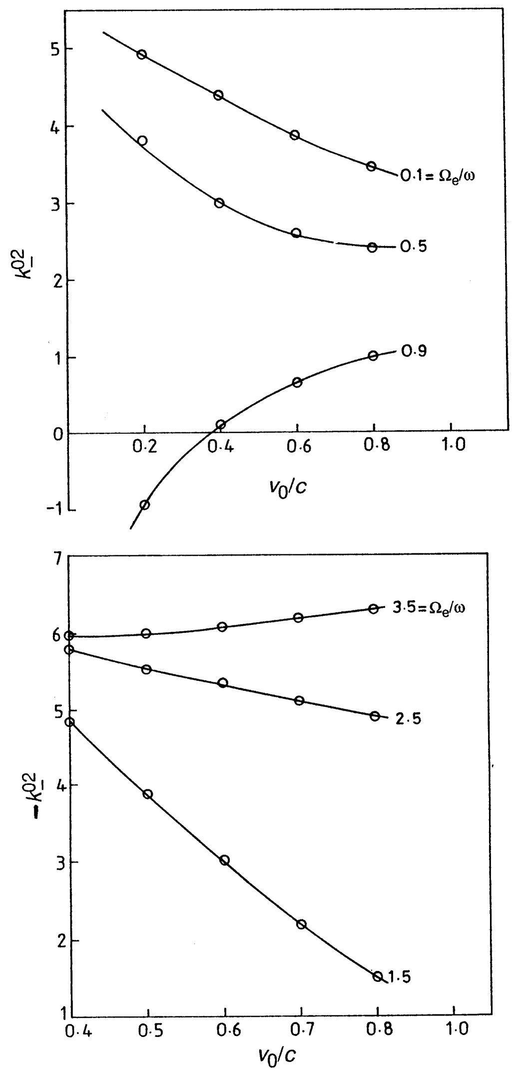


Fig. 4. Wave number of the second mode k_-^{02} of the RCP wave versus the streaming velocity v_0/c for the values of Ω_e/ω indicated.

$$\frac{\partial E_z^{(2)}}{\partial t} = 4\pi e v_{0z}\, n_e^{(2)} + 4\pi e n_0\, v_{2z}\,, \tag{21}$$

$$\frac{\partial E_z^{(2)}}{\partial z} = -\,4\pi e n_e^{(2)}\,. \tag{22}$$

Using the values of the first-order quantities, e.g. $v_\pm^{(1)}$ and $H_\pm^{(1)}$ in the above second-order equations, we get

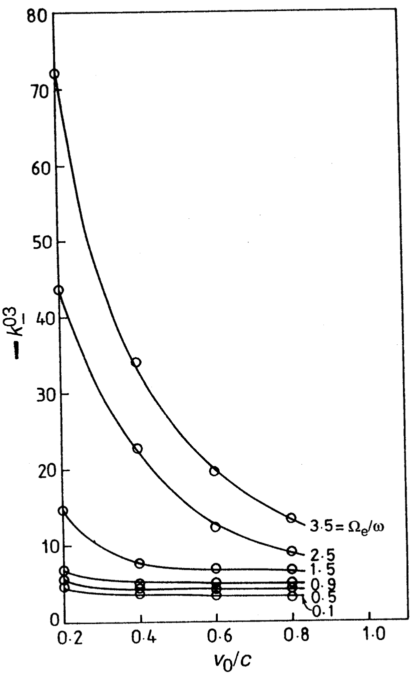


Fig. 5. Wave number of the third mode k_-^{03} of the RCP wave versus the streaming velocity v_e/c.

$$E_z^{(2)} = \mathrm{i}\, m_0\, \alpha^2 \Bigg(\frac{P_1\, \omega^2 v_{0z}^2 (k_+ + k_-) - 2\omega^3 v_{0z}}{2Q_1} + \frac{(\omega + k_-\, v_{0z})k_+}{\gamma_0(\omega - k_-\, v_{0z}) - \Omega_\mathrm{e}} - \frac{(\omega - k_+\, v_{0z})k_-}{\gamma_0(\omega - k_+\, v_{0z}) + \Omega_\mathrm{e}} \Bigg) \mathrm{e}^{\mathrm{i}(\theta_+ + \eta_-)} + \text{c.c.}\,, \tag{23}$$

$$n_\mathrm{e}^{(2)} = \frac{m_0\, \alpha^2 (k_+ + k_-)}{4\pi e^2 R} \Bigg(\frac{P_1\, \omega^2 v_{0z}^2 (k_+ + k_-) - 2\omega^3 v_{0z}}{2Q_1} + \frac{(\omega + k_-\, v_{0z})k_+}{\gamma_0(\omega - k_-\, v_{0z}) - \Omega_\mathrm{e}} - \frac{(\omega - k_+\, v_{0z})k_-}{\gamma_0(\omega - k_+\, v_{0z}) + \Omega_\mathrm{e}} \Bigg) \times \mathrm{e}^{\mathrm{i}(\theta_+ + \theta_-)} + \text{c.c.}\,, \tag{24}$$

$$v_z^{(2)} = \left(\frac{\mathrm{i}\, m_0\, \alpha^2 (2\mathrm{i}\, \omega)}{4\pi n_0\, e^2 R} - \frac{m_0\, \alpha^2 (k_+ + k_-)}{4\pi n_0\, e^2 R} \right) \times \Bigg(\frac{P_1}{2Q_1}\, \omega^2 v_{0z}^2 (k_+ + k_-) - 2\omega^3 v_{0z} + \frac{(\omega + k_-\, v_{0z})k_+}{\gamma_0(\omega - k_-\, v_{0z}) - \Omega_\mathrm{e}} - \frac{(\omega - k_+\, v_{0z})k_-}{\gamma_0(\omega - k_+\, v_{0z}) + \Omega_\mathrm{e}} \Bigg) \times \mathrm{e}^{\mathrm{i}(\theta_+ + \theta_-)} + \text{c.c.} \tag{25}$$

Third-order equations for the transverse field variables obtained from (1) to (7) are

$$\left[\left(1+\frac{v_{0z}^2}{2c^2}\right)\frac{\partial}{\partial t}+v_{0z}\left(1+\frac{v_{0z}^2}{2c^2}\right)\frac{\partial}{\partial z}\mp\mathrm{i}\,\Omega_{\mathrm{e}}^{(0)}\right]v_{\pm}^{(3)}$$

$$\begin{aligned}&=-\frac{e}{m_0}E_{\pm}^{(3)}\pm\mathrm{i}\,v_{0z}\,\Omega_{\pm}^{(3)}\pm v_z^{(2)}\,\Omega_{\pm}^{(1)}-\frac{\partial}{\partial t}(v_{\pm}^{(1)}\,v_{\mp}^{(1)})\frac{v_{\pm}^{(1)}}{2c^2}\\&+\frac{v_{0z}}{c^2}\frac{\partial}{\partial t}(v_z^{(2)}\,v_{\pm}^{(1)})-\frac{v_{0z}}{2c^2}\frac{\partial}{\partial z}(v_{\pm}^{(1)}\,v_{\mp}^{(1)})v_{\pm}^{(1)}\\&+\frac{v_{0z}^2}{c^2}\frac{\partial}{\partial z}(v_z^{(2)}\,v_{\pm}^{(1)})-v_z^{(2)}\frac{\partial}{\partial t}v_{\pm}^{(1)}-\frac{v_{0z}^2}{2c}v_z^{(2)}\frac{\partial}{\partial z}v_{\pm}^{(1)}\,,\end{aligned}\qquad(26)$$

$$\left(\frac{\partial^2}{\partial t^2}-c^2\frac{\partial^2}{\partial z^2}\right)E_{\pm}^{(3)}=4\pi e\left(n_{\mathrm{e}}^{(2)}\frac{\partial}{\partial t}v_{\pm}^{(1)}+v_{\pm}^{(1)}\frac{\partial n_{\mathrm{e}}^{(2)}}{\partial t}+n_{\mathrm{e}}^{(0)}\frac{\partial}{\partial t}v_{\pm}^{(3)}\right).\qquad(27)$$

Now, using the values of $v_{\pm}^{(1)}$, $\Omega_{\pm}^{(1)}$, $v_z^{(2)}$, $n_{\mathrm{e}}^{(2)}$ and $E_z^{(2)}$, we obtain the dispersion relation for the first harmonic part of the third-order field variables as

$$\begin{aligned}\frac{k_{\pm}^2\,c^2}{\omega^2}&=1-\frac{X_{\mathrm{e}}(1\mp v_{0z}\,k_{\pm}/\omega)}{\gamma_0(1-v_{0z}k_{\pm}/\omega)+\Omega_{\mathrm{e}}/\omega}+\frac{\omega P_1\,\alpha^2}{2RQ_1}\\&\times\frac{(1\mp v_{0z}\,k_{\mp})v_{0z}\,B(k_{+}+k_{-})}{\gamma_0\,\omega-\gamma_0\,k_{\mp}\,v_{0z}\mp\Omega_{\mathrm{e}}}\\&-\frac{\alpha^2A}{2R}\,\frac{(k_{+}+k_{-})(1\mp v_{0z}\,K_{\mp}/\omega)c^2}{\gamma_0\,\omega-\gamma_0\,k_{\mp}\,v_{0z}\mp\Omega_{\mathrm{e}}}\\&-\frac{P_1\,\alpha^2}{2RQ_1}\,\frac{k_{\mp}\,v_{0z}B^2}{Y}-\frac{\alpha^2A}{2R}\,\frac{c^2k_{\mp}\,B}{\omega Y}\\&+\frac{X_{\mathrm{e}}\,\alpha^2\omega^3(-\omega+v_{0z}\,k_{\pm})(X'+Y')}{2Y}-\frac{\alpha^2\omega^2P_1}{2Q_1\,R}\\&\times\frac{v_{0z}^2(1\mp v_{0z}\,k_{\mp}/\omega)(1-v_{0z}\,k_{\pm}/\omega)B^2}{c^2(\gamma_0\,\omega-\gamma_0\,v_{0z}\,k_{\mp}\mp\Omega_{\mathrm{e}})Y}\\&+\frac{\alpha^2\omega A}{2R}\,\frac{v_{0z}(1\mp v_{0z}\,k_{\mp}/\omega)(1-v_{0z}\,k_{\pm}/\omega)B}{(\gamma_0\,\omega-\gamma_0\,v_{0z}\,k_{\mp}\mp\Omega_{\mathrm{e}})Y}\\&-\frac{\alpha^2P_1\,\omega}{2RQ_1}\,\frac{\gamma_0\,k_{\mp}\,v_{0z}(1\mp v_{0z}\,k_{\mp}/\omega)B^2}{(\omega\gamma_0-\gamma_0\,v_{0z}\,k_{\mp}\mp\Omega_{\mathrm{e}})Y}\\&+\frac{\alpha^2A}{2R}\,\frac{\gamma_0\,k_{\mp}\,c^2(1\mp v_{0z}\,k_{\mp}/\omega)B}{(\omega\gamma_0-\gamma_0\,v_{0z}\,k_{\mp}\mp\Omega_{\mathrm{e}})Y}\,,\end{aligned}\qquad(28)$$

where

$$A = \frac{(1 - v_{0z}\, k_+/\omega)k_-}{\gamma_0(\omega - k_+\, v_{0z}) + \Omega_\mathrm{e}} - \frac{(1 + v_{0z}\, k_-/\omega)k_+}{\gamma_0(\omega - k_-\, v_{0z}) - \Omega_\mathrm{e}}\,,$$

$$B = v_{0z}(k_+ + k_-) - 2\omega\,.$$

To find the nonlinear wave-number shift we follow the usual procedure and write $k_+ = k_+^{(0)} + \delta k_+$ and $k_- = k_-^{(0)} + \delta k_-$ in the dispersion relation (28). After some simple calculation we get the wave-number shift of the LCP and RCP waves as

$$\begin{aligned}
\delta k_\pm = \frac{\omega^2}{2k_\pm^{(0)}\, c^2}\Bigg(&- \frac{\omega(k_+ + k_-)(1 \mp v_{0z}\, k_\mp/\omega)\alpha^2 v_{0z}\, B}{2[\gamma(\omega - k_\mp\, v_{0z}) \mp \Omega_\mathrm{e}]R}\,\frac{P_1}{Q_1} \\
&+ \frac{(k_+ + k_-)(1 \mp v_{0z}\, k_\mp/\omega)\alpha^2 c^2 A}{2[\gamma(\omega - k_\mp\, v_{0z}) \mp \Omega_\mathrm{e}]R} + \frac{k_\mp^{(0)}\, v_{0z}\, \alpha^2 B^2 P_1}{2YQ_1\, R} \\
&- \frac{c^2 k_\mp^{(0)}\, \alpha^2 BA}{2\omega YR} - \frac{X_\mathrm{e}\, \alpha^2\omega^3(\omega - k_\pm\, v_{0z})}{2Y}\,(X' + Y') \\
&- \frac{v_{0z}^2}{c^2}\,\frac{\alpha^2\omega^2}{2}\,\frac{(1 \mp v_{0z}\, k_\mp/\omega)(1 - v_{0z}\, k_\pm/\omega)B^2}{(\omega\gamma \mp \Omega_\mathrm{e} - \gamma v_{0z}\, k_\mp)YR}\,\frac{P_1}{Q_1} \\
&+ \frac{\alpha^2\omega v_{0z}(1 - v_{0z}\, k_\mp/\omega)(1 - v_{0z}\, k_\pm/\omega)BA}{2Y(\omega\gamma \mp \Omega_\mathrm{e} - \gamma v_{0z}\, k_\mp)R} \\
&- \frac{\alpha^2\gamma k_\mp\, B^2(1 \mp v_{0z}\, k_\mp/\omega)v_{0z}\omega P_1}{2Y(\omega\gamma \mp \Omega_\mathrm{e} - \gamma v_{0z}\, k_\mp)Q_1\, R} \\
&+ \frac{\alpha^2\gamma k_\mp\, c^2 B(1 \mp v_{0z}\, k_\mp/\omega)A}{2Y(\omega\gamma \mp \Omega_\mathrm{e} - \gamma v_{0z}\, k_\mp)R}\Bigg)\,,
\end{aligned} \tag{29}$$

where

$$P_1 = 1 - \frac{v_{0z}\, k_+}{\omega} + \frac{v_{0z}\, k_-}{\omega} - \frac{v_{0z}^2\, k_+\, k_-}{\omega^2}\,,$$

$$Y = \omega\gamma - k_\pm\, v_{0z}\gamma \pm \Omega_\mathrm{e}\,,$$

$$X_\mathrm{e} = 0{\cdot}5\,; \quad \alpha^2 = 4{\cdot}7 \times 10^{-3}\,; \quad \omega = 1{\cdot}789 \times 10^{11}\,,$$

$$\begin{aligned}
Q_1 = {}& \omega^2\gamma^2 - \omega\gamma^2 v_{0z}(k_+ + k_-) - \Omega_\mathrm{e}^2 \\
&+ v_{0z}\, \Omega_\mathrm{e}(k_+ - k_-)\gamma + \gamma^2 v_{0z}^2\, k_+\, k_-\,,
\end{aligned}$$

$$X' = \frac{(1 \mp v_{0z}\, k_\pm/\omega)^2(1 \pm v_{0z}\, k_\pm/\omega)}{(\omega\gamma - \gamma v_{0z}\, k_\pm \pm \Omega_\mathrm{e})^3}\,,$$

$$R = 1 - \frac{4\gamma_1}{X_\mathrm{e}} - \frac{\gamma_1\, v_{0z}^2(k_+ + k_-)^2}{X_\mathrm{e}\,\omega^2} + \frac{4\gamma_1\, v_{0z}(k_+ + k_-)}{X_\mathrm{e}\,\omega}\,,$$

$$Y' = \frac{2(1 \mp v_{0z}\, k_\pm/\omega)(1 \mp v_{0z}\, k_\mp/\omega)(1 \pm v_{0z}\, k_\mp/\omega)}{(\omega\gamma - \gamma v_{0z}\, k_\pm \pm \Omega_\mathrm{e})(\omega\gamma \mp \Omega_\mathrm{e} - \gamma v_{0z}\, k_\mp)^2}\,.$$

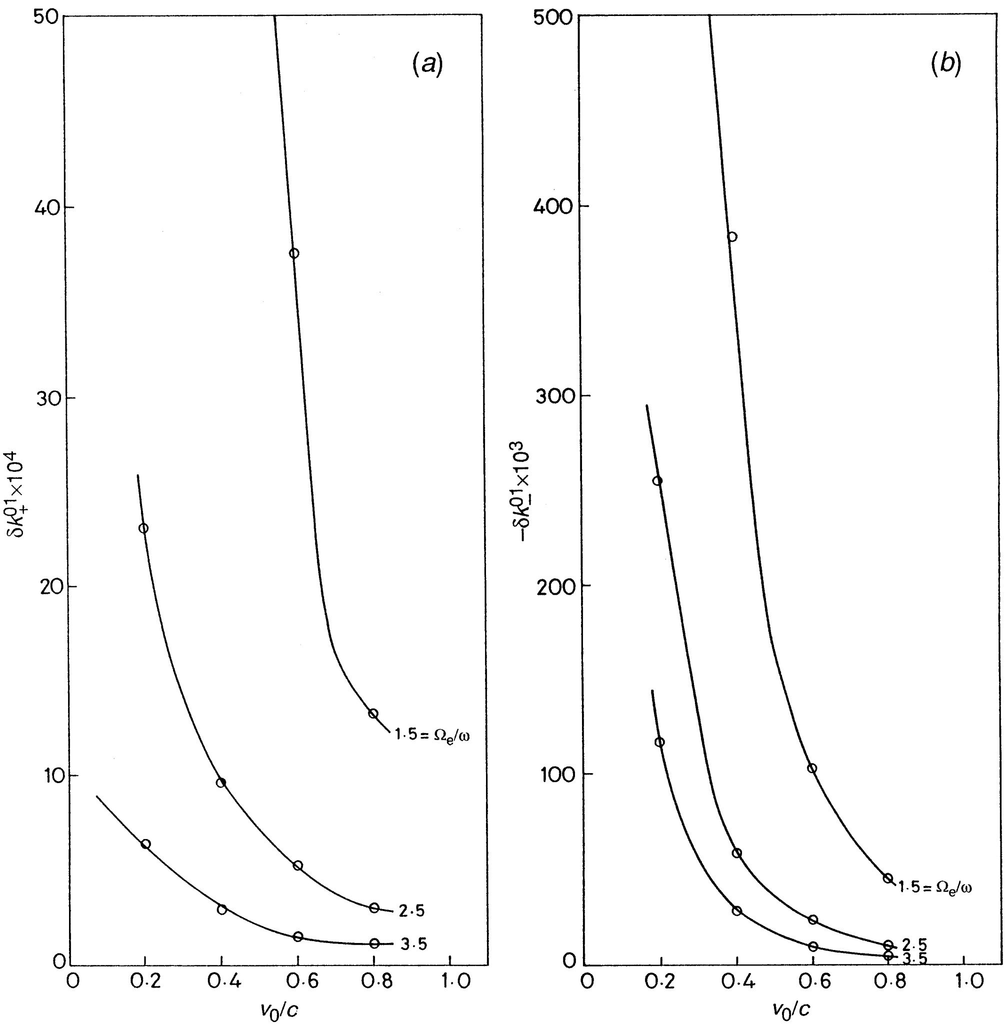


Fig. 6. Wave-number shift (a) δk_{+}^{01} and (b) δk_{-}^{01} of the first mode versus v_0/c when $\Omega_e/\omega > 1$.

From (29) we observe that streaming of electrons has a significant role in the wave-number shift of EM waves. Figs 6–8 illustrate the dependence of wave-number shift on the streaming velocity under various physical situations, e.g. gyrofrequency, wave frequency, etc. It is seen that the wave-number shift of the first mode of the EM wave decreases as the streaming velocity increases. When the gyrofrequency is near the wave frequency the variation of the shift is very rapid compared with when the gyrofrequency is greater than the wave frequency, i.e. $\Omega_e > \omega$. But for the reflecting mode we see that the nature of the variation of the wave-number shift is opposite to that of the propagating mode. For low values of Ω_e/ω the variation of the shift is very small and becomes almost zero for large values of the streaming velocity.

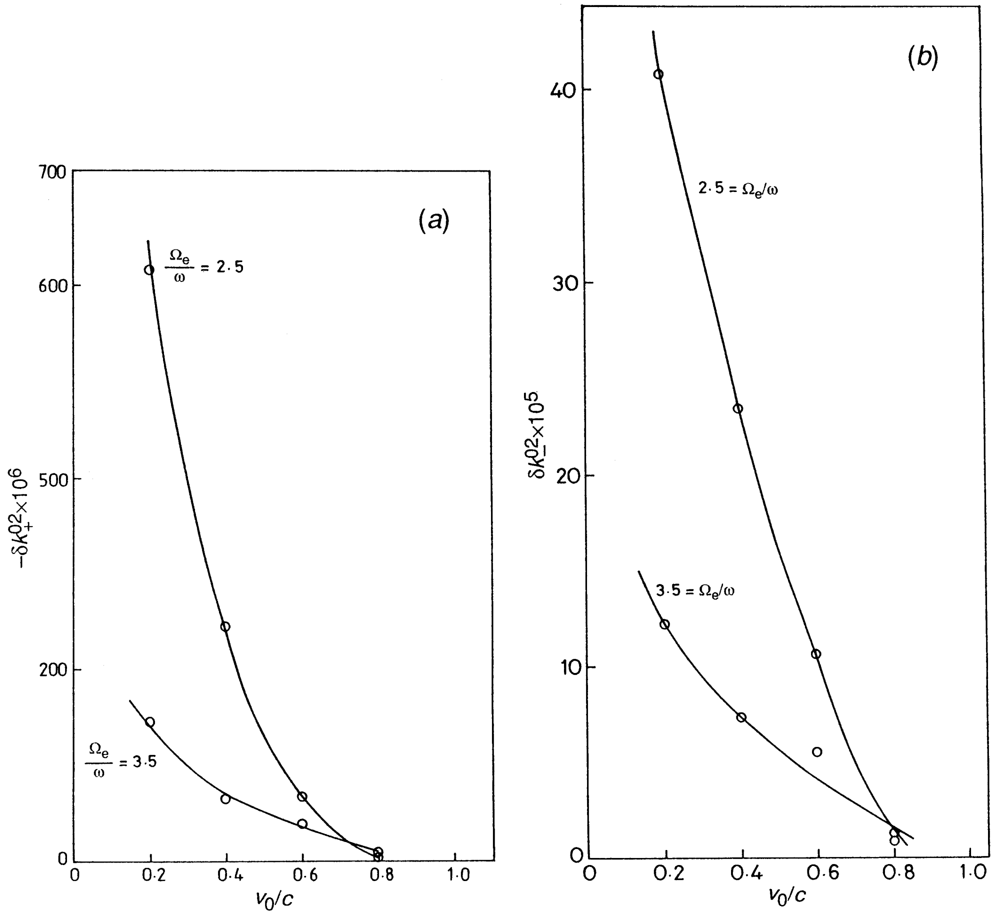


Fig. 7. Wave-number shift (a) δk_+^{02} and (b) δk_-^{02} of the second mode versus v_0/c when $\Omega_e/\omega > 1$.

5. Summary and Concluding Remarks

Our present work shows that streaming electrons have a significant effect on both linear and nonlinear propagation of EM waves in a relativistic plasma. With the presence of streaming electrons in the plasma there is every possibility of producing standing waves. It is known that the role of standing waves is important in various plasma phenomena (Montgomery and Tidman 1964).

Marbuger and Tooper (1975) showed that propagation of intense radiation in an overdense plasma can give rise to total reflection where a normally incident wave resembles more a standing wave than a travelling wave. For intensities of the order 10^{16} W cm^{-2} or more, the plasma species attain velocities of relativistic order. The relativistic effects are more important for standing waves than travelling waves (Bourdier and Fortin 1978, 1979).

Khan and Chakraborty (1979) investigated the precessional rotation of the standing wave in a magnetised relativistic plasma. Later, Chakraborty *et al.* (1990) evaluated theoretically the non-oscillating magnetic field for the Alfven

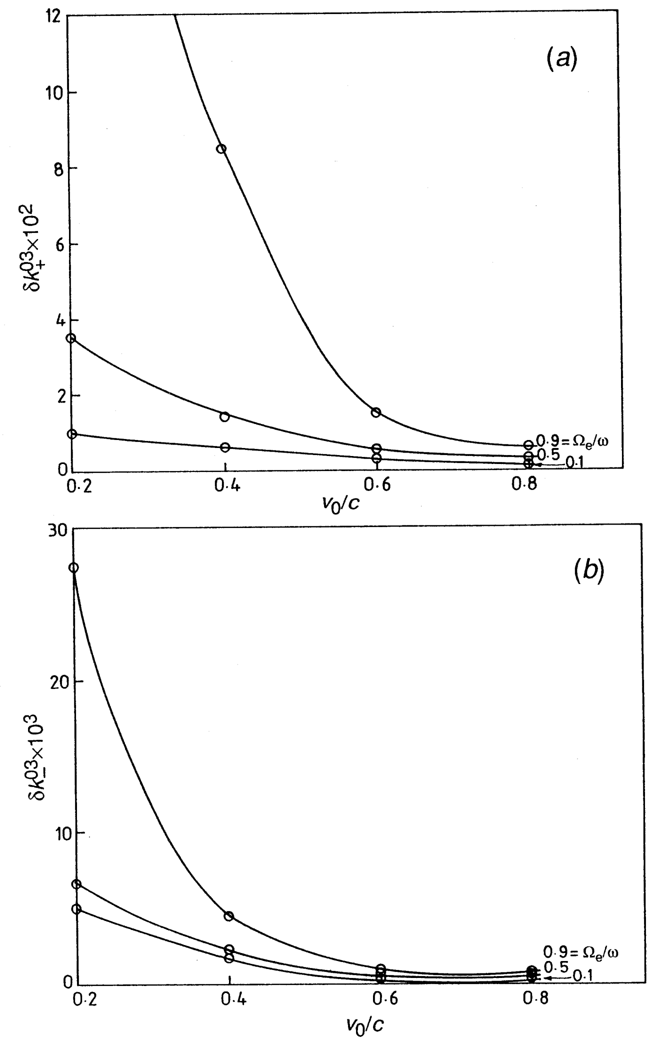


Fig. 8. Wave-number shift (a) δk_+^{03} and (b) δk_-^{03} of the third mode versus v_0/c.

standing wave but they did not give any explanation for the existence or formation of the standing waves in the plasma. In this paper, we have shown that standing waves may be formed even in relativistic plasmas under suitable physical conditions, particularly in the presence of streaming electrons and strong static magnetic fields. So to investigate self-action effects, such as precessional rotation or the inverse Faraday effect in a relativistic plasma, consideration of the presence of streaming electrons will yield more interesting results than those found by earlier authors.

From experimental observations it is a well-known fact that during solar bursts or the explosion of stars huge amounts of matter in the form of ionised gases are ejected from these astrophysical objects at very high velocities (Zeleznyakov 1964; Morton 1967a, 1967b, 1969; Kaplan and Tsytovich 1973; Snow 1979;

Harrison 1986; Kahler 1987). Moreover, various types of EM waves with low and high frequencies are emitted. The interaction of streaming electrons with the EM waves would thus be very important here and would give new ideas about the experimentally observed results during these phenomena. Consideration of non-uniform density distributions and the magnetic field of the stellar atmosphere (e.g. the solar corona) for the investigation of EM wave propagation in streaming plasmas is very important. This is because the inhomogeneity and the magnetic field in the medium may lead to strong EM radiation due to instability of the waves (Field 1956; Tidman 1960; Tidman and Weiss 1961*a*, 1961*b*; Dumphy *et al.* 1967).

For such problems, our present mathematical procedure is not applicable. The WKB method is generally used for tackling the problem of wave propagation through an inhomogeneous plasma (Budden 1961; Chakraborty 1970; Khan and Paul 1977). For the actual astrophysical problem the effects of temperature and rotation are very important (Bandyopadhyay and Paul 1973; Bandyopadhyay 1972; Das *et al.* 1984). Inclusion of these parameters together with the effects of streaming ions and electrons will be a new contribution to the study of EM waves in plasmas. For the investigation of linear (Uberoi and Das 1970) and nonlinear (Sur *et al.* 1988; Paul *et al.* 1992; Kashyapi *et al.* 1992) propagation of EM waves, a consideration of streaming electrons will yield interesting results, particularly for the cutoffs and resonances, the width of the stop-band, instability of waves, etc.

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Manuscript received 13 April, accepted 14 September 1993