

# Lattice Dynamics of the High $T_c$ Superconductor $\text{ErBa}_2\text{Cu}_3\text{O}_7$

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## *Abstract*

The lattice dynamics of the high  $T_c$  superconductor  $\text{ErBa}_2\text{Cu}_3\text{O}_7$  have been investigated in detail with a modified three-body force shell model. The model accounts for the effect of many-body interactions in the lattice potential. The aim of the present work is to treat the various interactions between the ions in generalised way without making them numerically equal. The values of the phonon frequencies calculated at the zone centre by this new approach are in good agreement with the available Raman and infrared data.

## 1. Introduction

In general, the lattice dynamics of a high  $T_c$  superconductor are described by a frequency–wavevector dispersion relation about the zone centre. Several lattice-dynamical models have been developed to explain the theoretical phonon dispersion curves of high  $T_c$  superconductors. These phonon dispersion curves about the zone centre give a better insight into the role of phonons for high transition temperatures, and also throw some light on the electron–phonon pairing mechanism. In this paper we present a modified shell model for the calculation of phonon frequencies at the  $\Gamma$  point of the Brillouin zone. The proposed model accounts for charge transfer between neighbouring ions in addition to the dipoles induced on the ions, i.e. the electronic polarisability.

Superconducting properties in  $\text{ErBa}_2\text{Cu}_3\text{O}_7$  prepared by the method of powder calcination were studied by Mohan *et al.* (1990). The compounds showed a sharp superconducting transition width  $\Delta T$ , less than 1 K. The pressure shift of  $T_c$  and the a.c. susceptibility measurements were also studied. Recently, we have also done a normal coordinate analysis of the zero-wavevector vibrations of  $\text{ErBa}_2\text{Cu}_3\text{O}_7$ . In continuation of our work on lattice dynamics, here we present a complete calculation of the lattice vibrations of  $\text{ErBa}_2\text{Cu}_3\text{O}_7$ , based on the modified three-body force shell model (TSM). Like the shell model, the TSM is also based mainly on the interionic potential consisting of a long-range Coulomb part, for which the potential is exactly known, and a short-range potential of Born–Mayer form.

## 2. Theoretical Considerations

The high  $T_c$  superconductor  $\text{ErBa}_2\text{Cu}_3\text{O}_7$  crystallises in the orthorhombic system which belongs to the space group  $P_{mmm}(D_{2h})$ . Lehner *et al.* (1982) pointed out that the short-range interionic potentials could be used for the calculation of force constants. Phonon frequencies can then be calculated using the force constants derived from the interionic potentials. These interionic potentials for short-range interactions can be transferred from one structure to another in similar environments. Following Lehner *et al.* (1982), our interionic pair potentials for oxidic perovskites and metal oxides were taken from known alkali halides and fluoridic perovskites. Surface dynamics studies of alkali halides (Kress *et al.* 1987) and perovskites (Reiger *et al.* 1987) also confirm this idea of transferability of these potentials from bulk to surface studies, despite the change in distance between interacting ions by surface relaxation.

## 3. Lattice-dynamical Calculations

Calculation of the lattice-dynamical vibrational frequencies of  $\text{ErBa}_2\text{Cu}_3\text{O}_7$  is performed using the TSM developed by Verma and Singh (1969) and Verma and Agarwal (1973), and which is a descendent of early work by Lundqvist (1955). In the TSM, the electronic charge density of an ion in the crystal is taken to be a function of the instantaneous relative positions of that ion's first nearest neighbours. It is assumed that the neighbours produce an isotropic scalar deformation of the charge density of the reference ion. The net effects of these deformations on the cohesive energy and the equations of motion are a renormalisation of the ionic charges, additional short-range two-body forces that can be absorbed into the central force repulsive interactions, and long-range three-body interactions between the vibrating ions (Mahan and Mostoller 1986). The shell model equations are given in the matrix form by

$$\begin{aligned}\omega^2 MU &= (R + ZCZ)U + (T + ZCY)W, \\ 0 &= (T' + YCZ)U + (S + K + YCY)W.\end{aligned}$$

Elimination of  $W$  leads to the dynamical matrix

$$D = (R + ZCZ) - (T + ZCY)(S + K + YCY)^{-1}(T' + YCZ).$$

The second term in this equation represents the three-body part of the lattice potential. Here  $M$ ,  $Z$  and  $Y$  are diagonal matrices representing the mass, ionic charge and the charge on the shell, while  $R$ ,  $S$  and  $T$  are matrices specifying the short-range core-core, shell-shell and core-shell interactions respectively. Further,  $U$  is the ionic displacement from the equilibrium position and  $C$  represents the Coulomb term. In the earlier approaches, the  $R$ ,  $S$ ,  $T$  elements were considered to be equal to one another, but we expect that there should be some difference in those values (Mohan *et al.* 1986). In the present investigation, we have started with a more general approach such that  $R \neq S \neq T$ . The  $T$  matrix elements are calculated using the relation connecting the parameters  $A_I, A_{II}, B_I$  and  $B_{II}$  (Verma and Singh 1971). Then,  $S$  matrix elements are calculated using the relation  $2T - R = S$ . The dynamical matrix of the model consists of the long-range

Coulomb and three-body interactions and the short-range overlap repulsions. The off-diagonal elements of this matrix along the symmetry directions contain a completely new term having a significant contribution for unequal  $R$ ,  $S$  and  $T$ . A qualitative analysis shows that this modification is important since they relate the optical and mechanical polarisabilities.

The force constants for the short-range interionic potentials between Cu and O, Ba and O, Er and O and between the neighbouring oxygen atoms are calculated using the Born-Mayer potential

$$V_{ij}(r) = a_{ij} \exp(-b_{ij} r),$$

where  $i, j$  label the ions and  $r$  their separation. The force constants evaluated by this method are in agreement with values in the literature (Kress *et al.* 1988). The parameters  $a_{ij}$  and  $b_{ij}$  of the pair potentials and the parameters  $Y_i$  and  $K_i$  which determine the electronic polarisabilities were computed and agree well with those of Kress *et al.* (1988).

The ionic charges are +3, +2 and +2, for Er, Ba and Cu, while for oxygen the value is determined using the charge neutrality condition. Lattice-dynamical calculations were performed with these values as model parameters.

**Table 1. Vibration frequencies obtained by the TSM calculation on  $\text{ErBa}_2\text{Cu}_3\text{O}_7$  with the observed Raman and IR frequencies in parentheses (Cardona *et al.* 1988)**

| Raman-active modes ( $\text{cm}^{-1}$ ) |          |          | Infrared-active modes ( $\text{cm}^{-1}$ ) |          |          |          |          |          |
|---|----------|----------|--|----------|----------|----------|----------|----------|
| $A_g$                                   | $B_{2g}$ | $B_{3g}$ | $B_{1u}$                                   |          | $B_{2u}$ |          | $B_{3u}$ |          |
|   |          |          | TO   | LO       | TO       | LO       | TO       | LO       |
| 562                                     | 622      | 641(640) | 622  | 631      | 638      | 642      | 640      | 652(630) |
| 449(442)                                | 591(580) | 591      | 552  | 563(550) | 618      | 628(630) | 582      | 591      |
| 418                                     | 471      | 584      | 468  | 484      | 574      | 589(580) | 515      | 524      |
| 333(320)                                | 312(330) | 283      | 410  | 420      | 555      | 564      | 414      | 428      |
| 158(150)                                | 160      | 111      | 241  | 262(210) | 181      | 188      | 169      | 175      |
|   |          |          | 184  | 198      | 161      | 168      | 159      | 162(165) |
|   |          |          | 101  | 111      | 105      | 111      | 102      | 108      |

#### 4. Results and Discussion

The model parameters obtained by the TSM are in close agreement with the reported values (Kress *et al.* 1988). The calculations reproduce very well the observed frequencies of Raman- and infrared-active modes (Cardona *et al.* 1988). The lattice vibration frequencies at the  $\Gamma$  point are obtained at  $q = 0$ . The calculated frequencies of the zone-centre phonon modes of  $\text{ErBa}_2\text{Cu}_3\text{O}_7$  and the observed Raman and infrared frequencies are presented in Table 1.

$\text{YBa}_2\text{Cu}_3\text{O}_7$  and  $\text{ErBa}_2\text{Cu}_3\text{O}_7$  possess similar phonon structures. Hence the vibrational assignments available for  $\text{YBa}_2\text{Cu}_3\text{O}_7$  can be applied to  $\text{ErBa}_2\text{Cu}_3\text{O}_7$ . The evaluated frequencies were assigned to different modes of vibration on the basis of the orthorhombic system and lattice-dynamical calculations. This system yields a total of 36 optic modes distributed as  $\Gamma = 5A_g + 5B_{2g} + 7B_{1u} + 5B_{3g} + 7B_{2u} + 7B_{3u}$ . The species belonging to  $A_g$ ,  $B_{2g}$  and  $B_{3g}$  are Raman-active modes, while

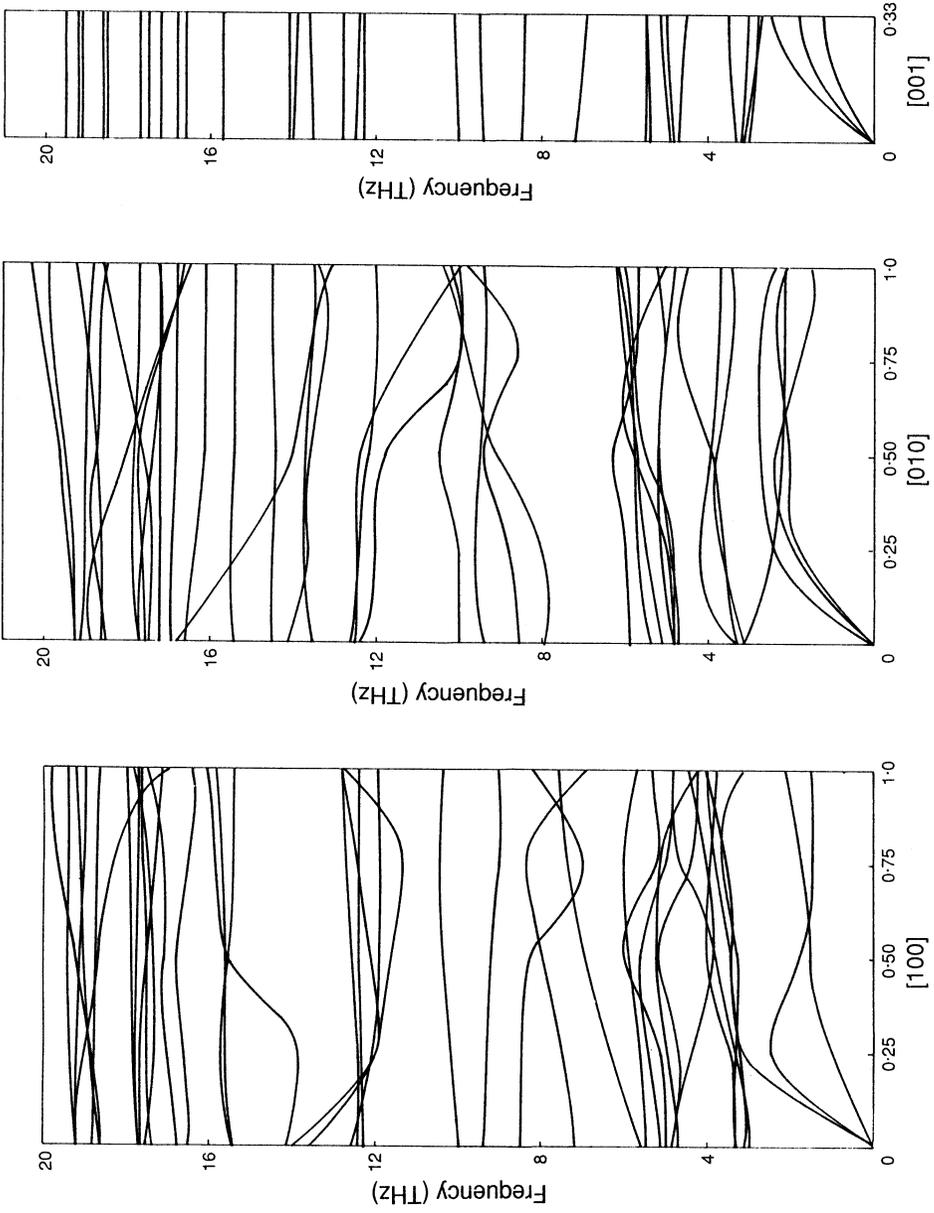


Fig. 1. Dispersion curves of  $\text{ErBa}_2\text{Cu}_3\text{O}_7$  in the [100], [010] and [001] directions.

$B_{1u}$ ,  $B_{2u}$  and  $B_{3u}$  are infrared-active modes. Although a number of papers have appeared in the literature concerning experimental and theoretical studies on phonons, they contradict each other. Bengang and Katiyar (1992) resolved the discrepancies in their work on a lattice-dynamical formalism using a rigid ion model for the superconductor  $\text{YBa}_2\text{Cu}_3\text{O}_7$ . Their conclusions are taken as a guide in assigning the modes of vibration of  $\text{ErBa}_2\text{Cu}_3\text{O}_7$  in the present work.

The Raman-active  $A_g$  mode of  $158\text{ cm}^{-1}$  (Cu–Ba–O motion) is the lowest calculated wavenumber which agrees well with the observed value. Vibrational modes in the region  $400$  to  $500\text{ cm}^{-1}$  are assigned to Ba–O or Er–O stretches. The observed frequency of  $320\text{ cm}^{-1}$  due to oxygen motion agrees reasonably well with the calculated frequency of  $333\text{ cm}^{-1}$ . The evaluated frequency of  $591\text{ cm}^{-1}$  due to the motion of the barium atoms against O(III) is close to the observed frequency of  $580\text{ cm}^{-1}$ .

The frequency  $312\text{ cm}^{-1}$  is assigned to the bond bending vibrations of O(II)–Cu(II)–O(III), in good agreement with the experimental frequency of  $330\text{ cm}^{-1}$ . The lowest phonon frequency of  $111\text{ cm}^{-1}$  belongs to  $B_{3g}$  symmetry. The observed modes for  $B_{1u}$  species are the  $210$  [Cu(I)–Ba–Er–O(III) vibration] and  $550$  [Cu–Er–O atom vibration] wavenumbers. The calculated values corresponding to TO and LO modes are at  $241$  and  $262\text{ cm}^{-1}$  wavenumbers and at  $552$  and  $563\text{ cm}^{-1}$ . The calculated frequencies of  $184$  and  $198\text{ cm}^{-1}$  obtained in the present investigation may be due to the motion of barium and erbium atoms against the Cu(I)–O(III)–Cu(II) chain.

The calculated TO and LO frequencies for the infrared-active  $B_{2u}$  species are  $105$ ,  $161$ ,  $181$ ,  $555$ ,  $574$ ,  $618$  and  $638\text{ cm}^{-1}$  and  $111$ ,  $168$ ,  $188$ ,  $564$ ,  $589$ ,  $628$  and  $642\text{ cm}^{-1}$  respectively. The experimentally observed frequencies of  $630$  and  $580\text{ cm}^{-1}$  agree very well with the calculated frequencies. The frequencies of  $161$  and  $181\text{ cm}^{-1}$  are due to vibrational motion of copper, barium and erbium atoms. As in  $\text{YBa}_2\text{Cu}_3\text{O}_7$ , the  $555\text{ cm}^{-1}$  frequency is due to the motion of oxygen atoms.

For the  $B_{3u}$  normal modes, the experimental frequency of  $165\text{ cm}^{-1}$  [vibration of copper, barium and erbium atoms against O(III)] and of  $630\text{ cm}^{-1}$  agree very well with the evaluated frequency in the present work. The calculated frequency of  $515\text{ cm}^{-1}$  in TO and  $524\text{ cm}^{-1}$  in LO belong to O(IV) and O(V) vibrations as pointed out by Bengang and Katiyar (1992). The observed frequency of  $630\text{ cm}^{-1}$  [640 for TO and 652 for LO] is due to the motion of O(I) against O(III).

Each of the 12 reliable observed frequencies reported in the literature is well accounted for by the calculated frequencies in the present work. The phonon dispersion curves of  $\text{ErBa}_2\text{Cu}_3\text{O}_7$  in the three symmetry directions obtained in this work are depicted in Fig. 1.

## 5. Conclusion

The shell model parameters for  $\text{ErBa}_2\text{Cu}_3\text{O}_7$  have been obtained and they provide reasonable agreement with the measured infrared and Raman frequencies. Our lattice-dynamical calculation provided not only the phonon frequency at  $q = 0$ , but also the phonon dispersion curves in the three main symmetry directions. The theoretical phonon frequency values obtained in this work comply with the available experimental values.

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