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Laser-modified Long-range Forces between Neutral Atoms

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Abstract

Modification of the long-range forces between atoms by laser light has been considered by Thirunamachandran (1980). More recently, Grossel and van Labeke (1994) treated the modified interaction between a conducting surface and an atom. In the present paper we establish the formulas for laser-modified Casimir forces between neutral atoms using a semiclassical formalism. These forces appear as a result of interaction of atomic dipoles with the background vacuum, in which spectral density is modified by the laser field. In particular, the interaction potential between two neutral atoms displays non-monotonic behaviour in an external field for certain polarisations. The averaging of this result over the mutual position of the atoms coincides with Thirunamachandran's result for the laser-modified Casimir forces. We give estimates for the value of this interaction, which may dominate the Casimir force for distances comparable to the wavelength. The validity of the semi-classical method is proven by comparison with the results of second-order perturbation theory. Furthermore, the method is used to calculate the external field modification of dispersive forces between a magnetically polarisable body and an electrically polarisable one and a chiral system coupled to both electric and magnetic fields.

1. Introduction

Several effects of cavity QED are well known and have been studied in recent years: modification of the spontaneous emission rate by an external cavity (Dehmelt 1975; Haroche and Raimond 1985; Gallas *et al.* 1985), collapse and revival in two-level systems (Narozhny *et al.* 1981; Rempe *et al.* 1987), and Casimir forces between atoms and surfaces (Hinds 1991). The effect which will be studied in this paper can be also addressed in a similar way, because it arises from the different spectral density of photons in an external field and in free space. A laser field acting on atoms is scattered by them. Because the spectral density of photons in a laser field is much greater than the quantum noise induced by zero-point fluctuations, a laser-induced interaction of the dipole moments can under certain conditions dominate the interaction of the dipoles induced by the scattering of the quantum noise, i.e. conventional dispersion forces. The wavelength of the laser field introduces a new linear scale in the problem of intermolecular interaction. Typically, the wavelength of laser radiation is four magnitudes higher than the characteristic linear dimension of atomic systems. The effects of retardation will be manifest at a distance comparable to the laser wavelength, so that the near-zone fields as well as the far-zone should be included

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in the formula for the dipole radiation, since the laser-induced effects will always occur in the region where long-range forces have already changed a van der Waals dependence into a Casimir form (this crossover happens at distances around $2\pi\cdot 137\cdot a_0$, where a_0 is the Bohr radius).

The classical period of development of the theory of dispersive forces has been reviewed, and more recently the formulation of dispersive forces within quantum electrodynamics has been developed (Andrews and Thirunamachandran 1978; Mahanty and Ninham 1986; Craig and Thirunamachandran 1984; Meath and Power 1989). This allows a deeper understanding of the origins of dispersive forces, as well as their connection with other phenomena, such as the Lamb shift (Milonni and Shih 1992).

In the present paper we provide a quantitative deduction of the laws governing long-range forces for the cases of electrically and magnetically polarised bodies, for the single-mode laser field of linear, circular and general elliptic polarisations, as well as for isotropic radiation. These expressions for the forces can be deduced from the laws of *classical* electrodynamics governing the propagation of dipole radiation, or from a second-order perturbation theory based on quantum electrodynamics. In the first case, the only quantum concept which enters into consideration is that each mode has intrinsic quantum noise with intensity equal to $\frac{1}{2}$ quanta per mode. If we do not want to reproduce crossover between laser-induced forces and the dispersion forces in the Casimir regime we can disregard the quantum nature of the entire problem! This result is even more surprising, because to obtain an expression for the shorter-range London force we seemingly need the energy of zero-point fluctuations as well (Born 1962).

2. Long-range Forces for the Single Polarised Mode and Static Atoms

Following Milonni and Shih (1992, see equations 9 and 76), we write the field acting on atom A in the form

$$\mathbf{E}_k(\mathbf{x}_A, t) = \mathbf{E}_{0,k}(\mathbf{x}_A, t) + \mathbf{E}_{B,k}(\mathbf{x}_A, t)\,, \tag{1}$$

where $\mathbf{E}_{0,k}$ is a source electric field in the mode $\mathbf{k}$ (laser field plus vacuum fluctuations), while $\mathbf{E}_{B,k}$ is the field scattered by atom B. The interaction energy between two atoms is given by the usual expression for the energy of interaction between induced dipoles

$$\begin{aligned} W_{AB} = &-\tfrac{1}{2}(\Sigma_{\mathbf{k}}\alpha_A(\omega_k)\langle \mathbf{E}_{0,k}(\mathbf{x}_A, t)\,.\,\mathbf{E}_{B,k}(\mathbf{x}_A, t)\rangle \\ &+ \alpha_B(\omega_k)\langle \mathbf{E}_{0,k}(\mathbf{x}_B, t)\,.\,\mathbf{E}_{A,k}(\mathbf{x}_B, t)\rangle)\,, \end{aligned} \tag{2}$$

where the indices A, B refer to the scattered field at the position of a given atom by the atoms B and A respectively and $\alpha_{A,B}(\omega)$ are the respective dynamical polarisabilities. The scattered fields $E_{A,k}$ and $E_{B,k}$ can be expressed in terms of the induced dipole by well-known formulas (Jackson 1975), which involves separation of the fields $\mathbf{E_0}$, E into positive-frequency and negative-frequency parts

$$\mathbf{E} = \mathbf{E}^{(+)} + \mathbf{E}^{(-)},$$

$$\mathbf{E}^{(+)}_{B,k} = \frac{1}{r^3}(1 - ikr)\left[3(\mu_B\hat{\mathbf{s}}\hat{\mathbf{s}} - \mu_B) + \frac{k^2}{r}(\hat{\mathbf{s}} \times \mu_B \times \hat{\mathbf{s}})\right]p_B\left(t - \frac{r}{c}\right),$$

$$\mathbf{E}^{(-)}_{B,k} = \frac{1}{r^3}(1+ikr)\left[3(\mu_B\hat{\mathbf{s}}\hat{\mathbf{s}} - \mu_B) + \frac{k^2}{r}(\hat{\mathbf{s}}\times\mu_B\times\hat{\mathbf{s}})\right]p_B\left(t-\frac{r}{c}\right), \tag{3}$$

where $r = \mid \mathbf{x}_A - \mathbf{x}_B \mid$ and $\hat{\mathbf{s}}$ is a unit vector pointing from atom B to atom A. Similar expressions can be immediately written for the scattered field at the location of the atom B as well. The induced dipole moment can be expressed through polarisability as

$$\mu_B p_B(t-r/c) = \Sigma_k \alpha_B(\omega_k)\left(\mathbf{E}^{(+)}_{0,k}(\mathbf{x}_B, t-r/c) + \mathbf{E}^{(-)}_{0,k}(\mathbf{x}_B, t-r/c)\right). \tag{4}$$

If we then express the electromagnetic field in the second-quantised form:

$$\mathbf{E}_{0,k} = i\left(\frac{2\pi\hbar\omega_k}{V}\right)^{\frac{1}{2}}[a_{\mathbf{k}\lambda}e^{-i\omega_k t}e^{i\mathbf{k}\cdot\mathbf{x}}\mathbf{e}^*{}_{\mathbf{k}\lambda} - a^\dagger{}_{\mathbf{k}\lambda}e^{i\omega_k t}e^{-i\mathbf{k}\cdot\mathbf{x}}\mathbf{e}_{\mathbf{k}\lambda}] \tag{5}$$

and insert this expression into equations (2)–(4) we obtain

$$\begin{aligned} W_{AB} \approx\ & -\frac{2\pi}{r^3 V} Re\Sigma_{\mathbf{k}\lambda}\alpha_A(\omega_k)\alpha_B(\omega_k) \\ & \times [3|\mathbf{e}_{\mathbf{k}\lambda}\cdot\hat{\mathbf{s}}|^2 - 1]\hbar\omega_k(1-ikr)\left[n_{\mathbf{k}\lambda}e^{-ikr} + (n_{\mathbf{k}\lambda}+1)e^{ikr}\right]\cos(\mathbf{k}\cdot\mathbf{r}) \\ & + (kr)^2[1-|\mathbf{e}_{\mathbf{k}\lambda}\hat{\mathbf{s}}|^2]\hbar\omega_k\left[n_{\mathbf{k}\lambda}e^{-ikr} + (n_{\mathbf{k}\lambda}+1)e^{ikr}\right]\cos(\mathbf{k}\cdot\mathbf{r}), \end{aligned} \tag{6}$$

where $n_{\mathbf{k}\lambda}$ is the expectation value for the photon number in the mode with wavevector $\mathbf{k}$ and polarisation λ and $\cos(\mathbf{k}\cdot\mathbf{r})$ reflects the overall symmetry between A and B. For the large population numbers typical of a laser field, one can omit unity in the square brackets of equation (6).

The far-field terms in (3), proportional to r^{-2} (induction-zone term), r^{-1} (wave-zone term) are also necessary to obtain the Casimir result for the interatomic potential in a vacuum without the laser field; in short, all terms are required to get the right answer. It is instructive to introduce explicitly the mean square of the field strength ${E_{\mathbf{k}\lambda}}^2 = (4\pi\hbar\omega_k/V)(n_{\mathbf{k}\lambda} + \frac{1}{2})$ for a given mode. The formula (6) can then be rewritten in the following form which clearly demonstrates the origin of the Casimir forces:

$$W_{AB} \approx -\Sigma_{\mathbf{k}\lambda}\frac{\alpha(\omega_k)\alpha(\omega_k){E_{\mathbf{k}\lambda}}^2}{2}F(r,\theta_{\mathbf{k}},\varphi_{\mathbf{k}}), \tag{7}$$

where $\theta_{\mathbf{k}}, \varphi_{\mathbf{k}}$ are the polar and azimutal angles of the vector $\mathbf{k}$ in a coordinate system where vector $\hat{\mathbf{s}}$ is taken as the direction of the z axis. For a single mode of the laser field we have

$$\begin{aligned} W_{AB} =\ & -\frac{\alpha_A(\omega_k)\alpha_B(\omega_k){E_{\mathbf{k}\lambda}}^2}{2}\left[\frac{3\sin^2\theta\sin^2\varphi - 1}{r^3}\right. \\ & \left.\times[\cos(kr) + kr\sin(kr)] - \frac{1-\sin^2\theta\sin^2\varphi}{r}\cos(kr)\right]\cos(kr\cos\theta). \end{aligned} \tag{8}$$

For a laser frequency well below the difference between the ground and the first excited state it is a good approximation to use the static polarisabilities $\alpha_A(0)$ and $\alpha_B(0)$ in (8). A discussion of this approximation is given in Section 3.

The result for a circularly polarised field is readily obtained from the previous one. For circularly polarised light we have

$$\mathbf{e}^{L/R} = 2^{-1/2}[\hat{x} \pm i \cdot \hat{y}]\,, \tag{9}$$

and

$$\begin{aligned} W_{AB} = &- \frac{\alpha_A(0)\alpha_B(0)E_{\mathbf{k}\lambda}{}^2}{2} \\ &\times \left[\frac{(1-3\cos^2\theta)}{r^3}[\cos(kr) + kr\sin(kr)]\right. \\ &\left. - \frac{k^2(1+\cos^2\theta)}{r}\cos(kr)\right]\cos(kr\cos\theta)\,. \end{aligned} \tag{10}$$

This formula coincides with that for the directional average of the relative position of the vector of interatomic separation in the case of linear polarisation. Indeed, for circular polarisation, the field defines only one spatial direction—the direction of the wavevector. Averaging over φ corresponds to averaging over all directions orthogonal to $\mathbf{k}$.

3. Estimates for the Laser-mediated Dispersion Forces

The interaction energy, which is given by the expression (8), is not monotonic for all angles θ, φ. As the term contributing to energy of order r^{-3} vanishes after averaging over all possible mutual positions, recovering the usual r^{-6} or r^{-7} asymptotics of the van der Waals–Casimir effect, it suggests that the angular distribution of the function $F(kr, \theta, \varphi)$ should have 'lobes' with negative values of the interaction potential (attraction), almost compensated by other regions with a positive value for this interaction (repulsion). This lobed structure is portrayed in Fig. 1 for a particular φ. Similar profiles for circular polarisation (or for linear polarisation after summation over φ) are given in Fig. 2. The overall pattern is very similar with the exclusion of the central lobe. Fig. 3 demonstrates that the change in φ provides relatively little qualitative difference on a large scale, while the central lobe of the potential surface is reconstructed completely by a change of φ. There are regions, in which the field-modified interaction potential behaves in a non-monotonic fashion. This behaviour is illustrated in Fig. 4 for fixed θ values of the polarisation angle and in Fig. 5 for a fixed kr.

Consideration of the field-modified potential has most relevance if the laser-induced dispersion forces sometimes dominate the Casimir forces. To estimate the effect, one should first evaluate characteristic lengths of the problem. The smallest is the atomic diameter, which for the most atoms, is close to the Bohr radius a_0. The crossover between van der Waals and Casimir regimes is encountered at typical distances of $2\pi \times 137a_0$, since the velocity of light, which defines the occurrence of retardation effects is 137 in atomic units (Berestetskii *et al.* 1982).

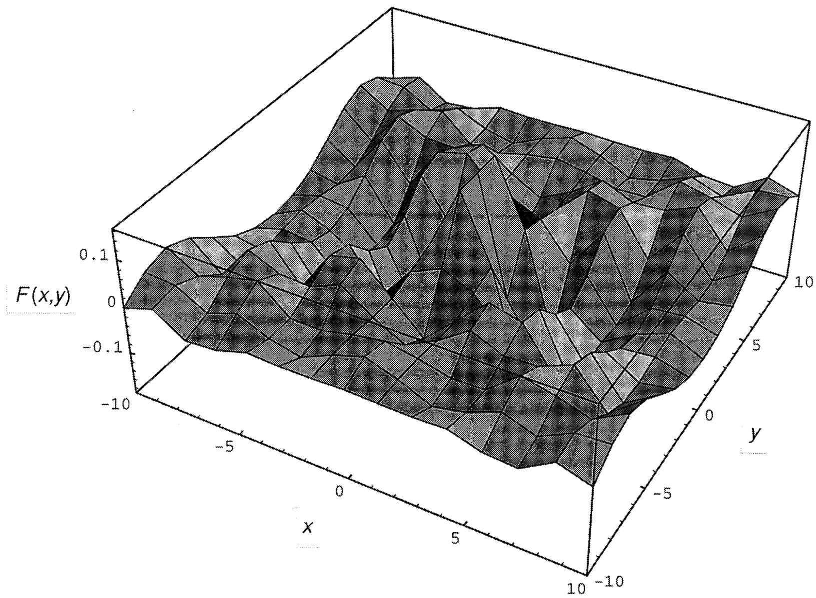


Fig. 1. Dimensionless energy of interaction $F(r, \theta, \phi)$ (in units of $0 \cdot 5\alpha_A \alpha_B k^3 E^2$, see equation 7). This function is plotted for $F = F(x{=}kr\cos\theta, y{=}kr\sin\theta, \varphi{=}3\pi/8)$ for linear polarisation of the laser light.

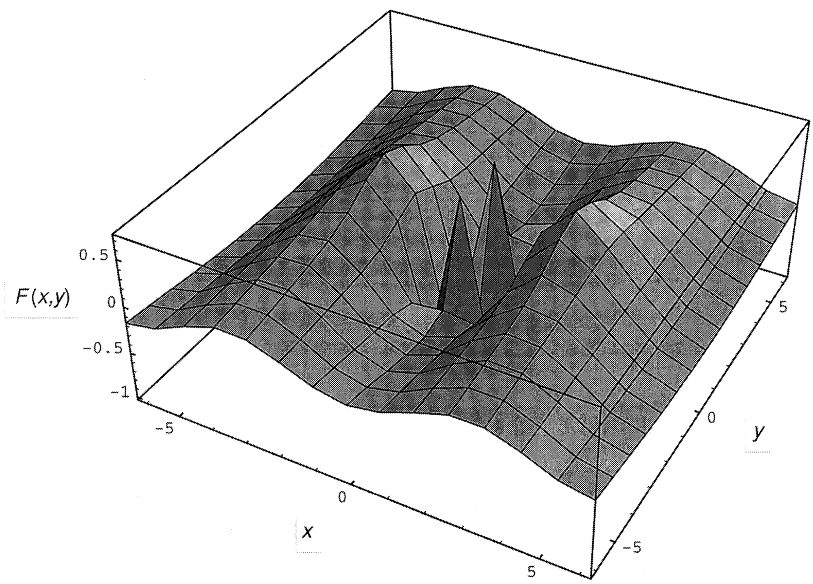
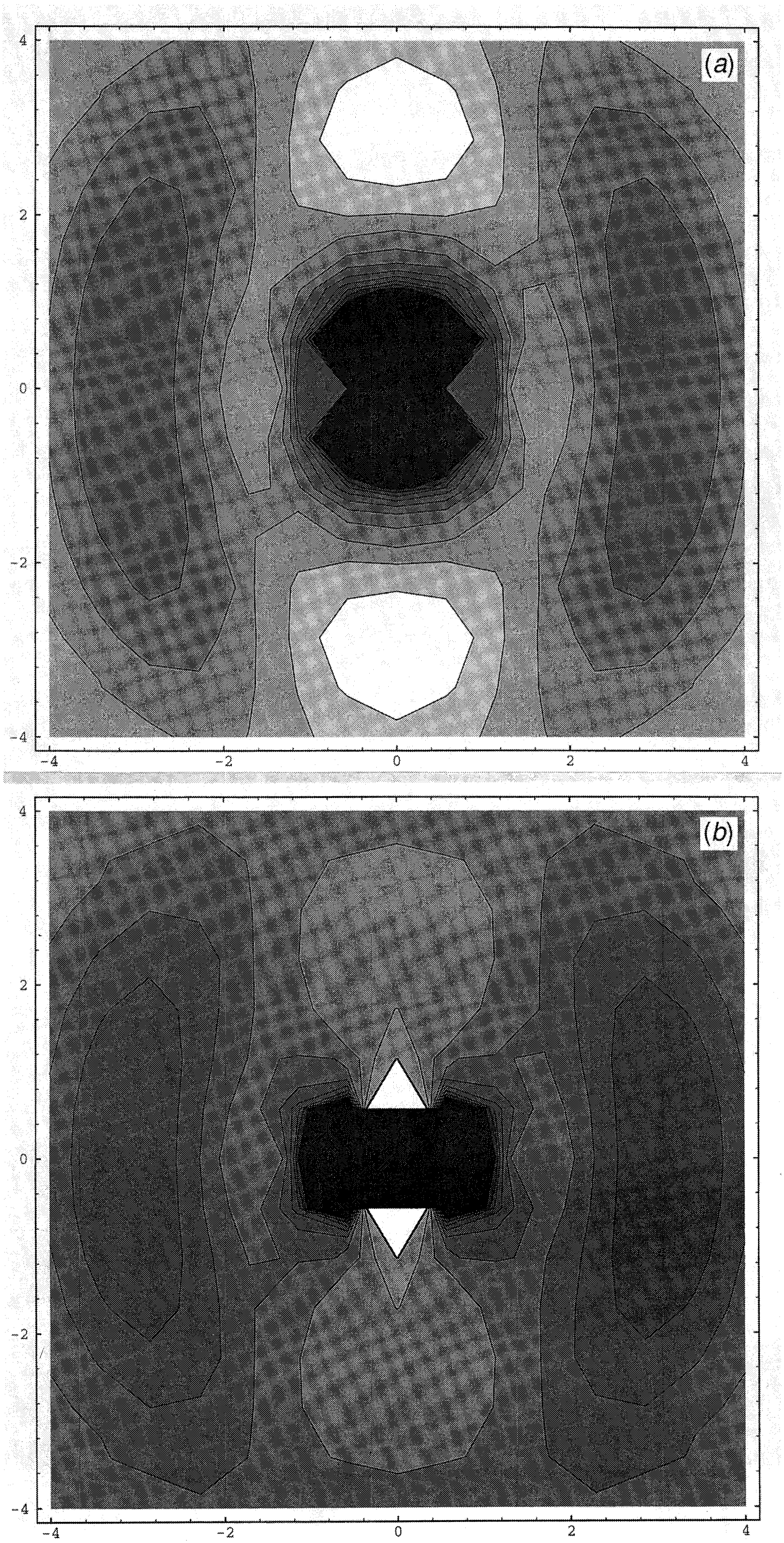


Fig. 2. Dimensionless energy $F = F(x{=}kr\cos\theta, y{=}kr\sin\theta)$ for circular polarisation. In this case F is independent of φ.



Figs 3a and 3b.

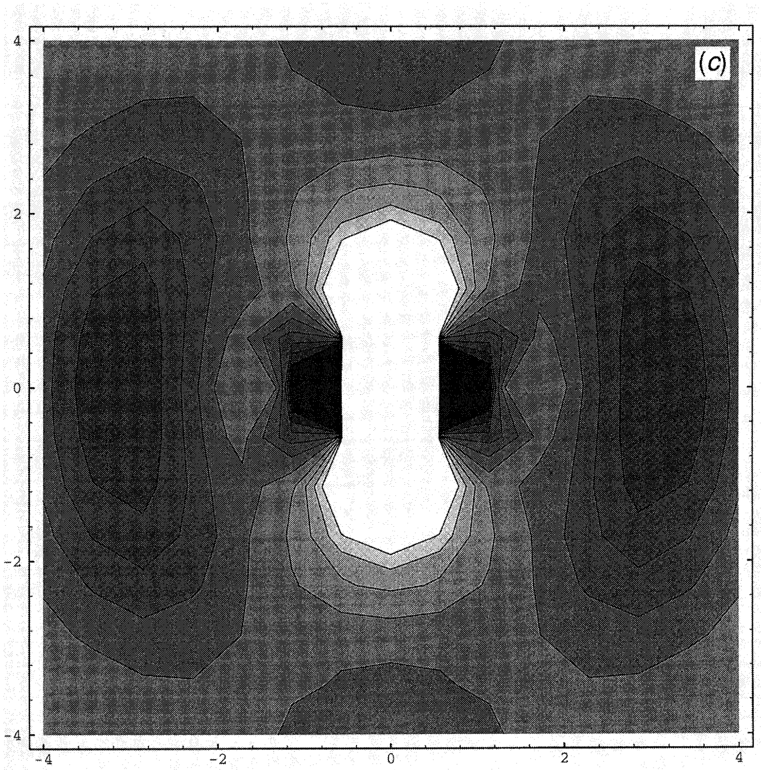


Fig. 3 Reconstruction of a central cell with a change in direction of laser polarisation with respect to the azimuth angle φ between $\mathbf{k}$, the direction of the wavevector and $\mathbf{s}$, the vector of interatomic separation: (a) $\varphi = \pi/8$; (b) $\varphi = \pi/4$; and (c) $\varphi = 3\pi/8$; same as in Fig. 1.

This typical length is half the wavelength of the transition from the ground state $4\pi \times 137 a_0 = 1243\,\text{Å}$, since this is the smallest discrete wavelength emitted by a valence electron. The wavelength should be larger than this value. The use of short-wavelength radiation for the testing of this effect cannot be justified, because it would cause one-photon ionisation of atoms into the continuum. This means that the laser wavelength is the largest linear scale in this problem: $a_0 \ll 4\pi \times 137 a_0 \ll 2\pi c/\omega_L$. As follows from (6) the real small parameter of the problem is, thus, $\omega_L r/c$ and the expression for the laser-modified force is the power of this parameter times the London force. This does not mean that the effect cannot be large, in comparison with the Casimir forces, because this parameter can be offset by large population numbers for quanta in the laser modes and a slower law of fall off of the interaction energy with distance (r^{-3} instead of r^{-7}).

Estimates for the ratio of the laser-modified force and the usual, vacuum-induced contribution easily follows from (6):

$$\text{(Casimir regime)}: \qquad W_{\text{laser}}/W_{\text{vacuum}} \approx E^2 (kr)^n r^4 / c\hbar\,,$$

$$\text{(van der Waals regime)}: \qquad W_{\text{laser}}/W_{\text{vacuum}} \approx E^2 (kr)^n r^3 / E_0\,,$$

where E_0 is the energy of the ground state and n is either 0 or 2, in the case of the ordered or spatially-averaged position of atomic dipoles (see Section 4).

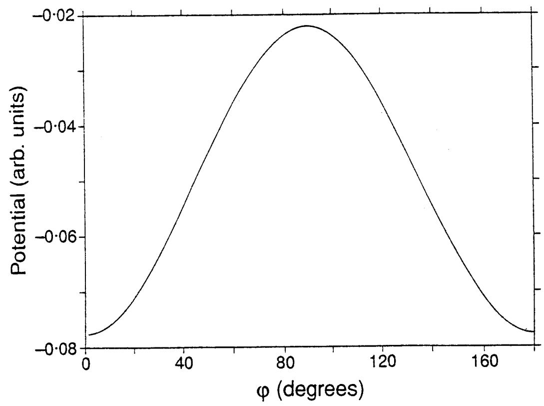


Fig. 4. Dependence of the dimensionless energy on angle φ for a fixed distance and fixed θ. Values chosen are at $kr = 10 \cdot 0$ and $\theta = 3\pi/8$ for case A (purely electric).

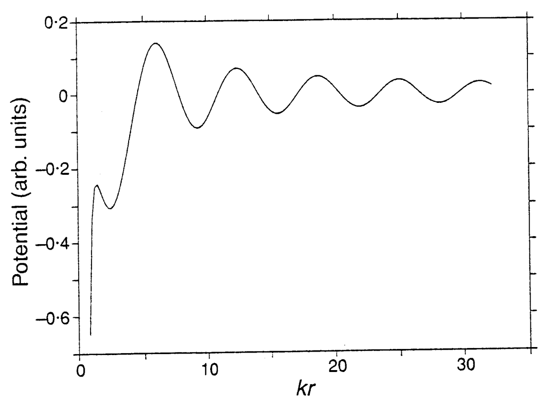


Fig. 5. Dependence of the dimensionless energy $F(kr, \theta{=}\pi/4, \varphi{=}2\pi \times 0 \cdot 085)$ on distance for fixed θ and φ.

For the van der Waals regime a typical value of kr is around 10^{-2} and the necessary laser fields in the case of isotropic radiation to overcome the usual van der Waals forces are of magnitude $10^{13} - 10^{14}\,\mathrm{W\,cm^{-2}}$ and they necessarily will cause ionisation. This example shows how strong the vacuum fluctuation forces are: because the typical frequencies of electromagnetic quanta which transmit van der Waals interaction are $\sim c/r$ and the radiation depends on the third power of the frequency, one needs large real fields with population numbers of order 10^{20} in the Fourier-limited gigawatt-range nanosecond pulse to counter this conventional dispersion force with a typical 1 photon per mode! Unlike real fields of high frequency which immediately result in photoionisation, zero-point fluctuations, by definition, cannot cause real transitions. Yet, in the case of dipoles with positions ordered with respect to the wavevector of the radiation, the critical intensity for which van der Waals and laser-field forces become equal is $10^8 - 10^9\,\mathrm{W\ cm^{-2}}$ (the population numbers are still enormous: $\sim 10^{16}$ under the same conditions). This is quite practical to allow an experiment without substantial ionisation. In the Casimir regime the picture is somewhat more optimistic. One can provide $(kr) \sim 0{\cdot}1 - 1$ and fields of intensity 10^4 W cm^{-2} would be sufficient to overcome the influence of fluctuation forces. The problem is that at those distances, of the order of 10^3 atomic units, both the laser-induced and fluctuation-induced long-range forces between single atoms are extremely small and provide no possibility of direct experimental observation. Nevertheless, in the large samples of interacting particles in the modern laser traps, these effects could be, in principle, within the practical limits of verification. However, as with the case of Casimir forces (Hinds 1991), the most hopeful possibility for the discovery of these forces appears not in the interactions between atoms, but in the interaction between atoms and solid bodies or between solid bodies.

4. Reproduction of the Casimir Law and the Case of Isotropic Radiation

Our starting point in this section will be expression (2) which is valid for the multimode case and arbitrary polarisations of the field. However, if the laser field is unpolarised we can sum over all directions of polarisation of the laser field. For this purpose one can make an angular integration over the angle φ of the laser field

$$\Sigma_\lambda(3|\mathbf{e}{\cdot}\hat{\mathbf{s}}|^2 - 1) = 3(\delta_{ij} - \hat{k}_i\hat{k}_j)\hat{s}_i\hat{s}_j - 2 = 1 - 3(\hat{\mathbf{k}}\cdot\hat{\mathbf{s}})^2, \tag{11}$$

where $\hat{\mathbf{k}}$ is a unit vector in the direction of the wavevector and similarly

$$\Sigma_\lambda(1 - |\mathbf{e}{\cdot}\hat{\mathbf{s}}|^2) = -(1 + (\hat{\mathbf{k}}{\cdot}\hat{\mathbf{s}})^2). \tag{12}$$

In the convention for angles described in Section 2 the product $(\hat{\mathbf{k}}{\cdot}\hat{\mathbf{s}})$ is just $\cos\theta$ and we obtain the formula (8), previously introduced for the field with a circular polarisation. This is quite understandable, since the average of all linear polarisations with given intensity is characterised only by the direction of the wavevector, as is a circularly polarised field. This is untrue, for instance, in the case of arbitrary elliptic polarisation, since there is another quantity of importance: the direction of the main axis of the polarisation ellipsis.

The simple limiting case immediately follows from equation (8). Namely, suppose that $kr \ll 1$. Then (8) reduces to

$$W_{AB} = -\frac{\mu_A\mu_B}{r^3}(1 - 3\cos^2\theta)\,, \tag{13}$$

where $\mu_{\mathbf{A},\mathbf{B}} = \alpha_{A,B}(0)\mathbf{E_0}$ are the induced dipole moments of the atoms. In this case we recover the dipole–dipole interaction, precisely what is expected in the near zone.

The averaging of equation (8) over all respective directions between the wavevector of the field and the atomic separation, assuming the occupation of modes is independent of wavevector $\hat{\mathbf{k}}$, with subsequent summation over all frequencies can be easily performed to give the following expression for the interaction energy (see equation 6):

$$\begin{aligned} W_{AB} = & -\frac{3\hbar}{\pi r^6}\int_0^\infty d\omega\alpha_A(\omega)\alpha_B(\omega)(2n(\omega)+1)\sin(2\omega r/c) \\ & +\frac{6\hbar}{\pi c r^5}\int_0^\infty d\omega\ \omega\alpha_A(\omega)\alpha_B(\omega)(2n(\omega)+1)\cos(2\omega r/c) \\ & +\frac{5\hbar}{\pi c^2 r^4}\int_0^\infty d\omega\ \omega^2\alpha_A(\omega)\alpha_B(\omega)(2n(\omega)+1)\sin(2\omega r/c) \\ & -\frac{2\hbar}{\pi c^3 r^3}\int_0^\infty d\omega\ \omega^3\alpha_A(\omega)\alpha_B(\omega)(2n(\omega)+1)\cos(2\omega r/c) \\ & -\frac{\hbar}{\pi c^4 r^2}\int_0^\infty d\omega\ \omega^4\alpha_A(\omega)\alpha_B(\omega)(2n(\omega)+1)\sin(2\omega r/c)\,. \end{aligned} \tag{14}$$

In this formula we restore the ω-dependence of atomic polarisability for generality, though unlike the usual van der Waals effect, laser-modified forces are basically influenced by static polarisability of the ground state. If one puts the population density of modes $n(\omega)$ equal to zero, this formula obviously reproduces the usual Casimir formula. This can be demonstrated by replacement of the integration contour by turning it through 90° in the complex plane and then simple regularisation: $r \longrightarrow r - i0$. If one separates the contribution of the vacuum fluctuations (which have small population number, but are distributed with spectral density 1 in all frequencies) and the extremely intense, but spectrally localised laser-induced contributions,

$$W_{AB} = W_{AB,v} + W_{AB,L}\,, \tag{15}$$

one can consider again that the field in $W_{AB,L}$ is quasi-monochromatic. In the region $2\pi \times 137a_0 = \lambda_{at} \leq r \leq \lambda$ where one can expect observation of the Casimir forces (since for distances larger than 10^4 atomic units, typical for laser wavelengths, the Casimir energies are smaller than 10^{-25} eV), the oscillatory factors in equation (14) can be expanded in a Taylor series providing the original result of Thirunamachandran (1980):

$$W_{AB,L} = -\frac{22}{15\pi}\hbar\omega_0\frac{{\omega_0}^4 n_{\omega_0}}{c^5 r}\alpha_A(0)\alpha_B(0)\,, \tag{16}$$

where n_{ω_0} is the spectral density of the photon field defined by $n(\omega) = n_{\omega_0}\,\delta(\omega - \omega_0)$. One sees that this r^{-1} law has nothing in common with the usual r^{-7} power law for Casimir forces. If one considers, as earlier, $(\omega r/c)$ to be a small parameter (this assumption we have already made in the deduction of equation 13), equation (16) arises in *fifth* order of perturbation over this small parameter. This is very unusual and happens only because a high symmetry of the problem for isotropic radiation leads to cancellation of all lower orders. We refer to Section 2 for the opposite case of single-mode radiation. Also, we note that because of the high order of expansion of the oscillatory terms, the Thirunamachandran formula is expected to work well up to the distances $kr \sim 1$ with an accuracy not worse than parts of a per cent (this is the accuracy of the power expansion up to the third term of sine and cosine). Because of the 'Coulomb' nature of the law equation (16), one is tempted to compare this force with gravity between atoms. A comparison of the gravitational force between two Rb atoms and the force induced by a 1 mW CO_2 laser sees the laser-modified Casimir forces dominate by eight orders of magnitude. This estimate presents a strong caution against the interpretation of any forces observed in precision experiments as a modification of gravitational interaction, until all laser modifications of intramolecular dispersive forces are ruled out.

5. Laser-modified Long-range Forces between Magnetisable Bodies

If one of the particles has a magnetic polarisability, the fluctuation forces will induce long-range magnetic interaction. The magnetic forces are *a priori* weaker than the electric forces, since they appear in some power of the small factor $\xi = v_{at}/c$. There are few possibilities for the Hamiltonian of such an interaction (compare with the classification of field-induced optical activity in Baranova *et al.* 1977). In the dipole approximation, there are two forms for the interaction Hamiltonian, quadratic in applied fields $\mathbf{E}$ and $\mathbf{B}$:

$$\text{(scalar coupling constant)}: \quad W_s = \frac{\alpha \mathbf{E}\cdot\mathbf{E} + \mu \mathbf{B}\cdot\mathbf{B}}{2},$$

$$\text{(pseudo-scalar coupling constant)}: \quad W_p = -\frac{g}{2}((\mathbf{E}\cdot\mathbf{B})_A + (\mathbf{E}\cdot\mathbf{B})_B)\,. \tag{17}$$

The scalar constant has approximate order $\xi^2 \sim 10^{-4}$ with respect to the electric field-induced effects, since this effect appears in the same order in c^{-1} as atomic diamagnetism. The last remark suggests that this effects always appears, for every atom.

The pseudo-scalar coupling manifests itself only with atoms and molecules, having permanent magnetic moment. The pseudo-scalar coupling constant has order $\mu_B H_0/E_0$, where H_0 and E_0 are atomic values for the magnetic field and energy. The comparison with the electric-field-induced effects provides the ratio $\xi^* = \lambda_c/a_B \sim 10^{-3}$, where λ_c is the Compton length for the electron and a_B is the Bohr radius, for their respective strength. This means that for molecules with intrinsic chirality the pseudo-scalar coupling dominates the scalar one.

Let us start with the somewhat more transparent case of the scalar coupling constant and long-range force arising between two polarisable bodies: one with electric and the other with magnetic polarisation. Of course, all combinations are possible, but they will be in the next order in ξ. We suppose that the induced magnetic and dipole moments are related to the corresponding field through

$$\mathbf{m} = \mu\mathbf{B}\,, \qquad \mathbf{p} = \alpha\mathbf{E}\,. \tag{18}$$

The formulas for *electric* field radiation by the induced *magnetic* moment and magnetic induction radiation by induced electric dipole are given in Thirunamachandran (1980):

$$\mathbf{E} = -k^2(\mathbf{s} \times \mathbf{m})\frac{e^{ikr}}{r}\left(1 - \frac{1}{ikr}\right), \qquad \mathbf{B} = k^2(\mathbf{s} \times \mathbf{p})\frac{e^{ikr}}{r}\left(1 - \frac{1}{ikr}\right). \quad (19)$$

The mutual interaction energy (which in first order in magnetic polarisability is additive to attraction of electric dipoles) is provided by the expression

$$\begin{aligned} W_{AB}^{M} &= -\frac{\alpha\mu k^2}{2}(\mathbf{s}\cdot(\mathbf{E}_{A,0} \times \mathbf{B}_{B,0}) + \mathbf{s}\cdot(\mathbf{B}_{A,0} \times \mathbf{E}_{B,0}))\mathrm{Re}\left\{\frac{e^{ikr}}{r}\left(1 - \frac{1}{ikr}\right)\right\} \\ &= -\frac{4\pi\alpha\mu k^2 S}{r}\left(\cos(kr) + \frac{\sin(kr)}{kr}\right)\sin(kr\cos\theta)\cos(\theta), \end{aligned} \quad (20)$$

where the index 0 refers to the strength of the source field and S is the absolute value of the field's Poynting vector.

Note that in this case the answer is independent of polarisation, whether it is circular or linear. This is true in the general case of elliptic polarisation (see Section 6). In the case of magnetically polarisable bodies, unlike the case of electric dipoles, the $1/r$ dependence appears without integration over position angles *and* asymptotic expansion. Indeed, fluctuations of the magnetic field fall off much slower than the fluctuations of the electric field, though they are suppressed by a ξ^* factor (Passante and Power 1987). We give a picture of the energy surface in Fig. 6.

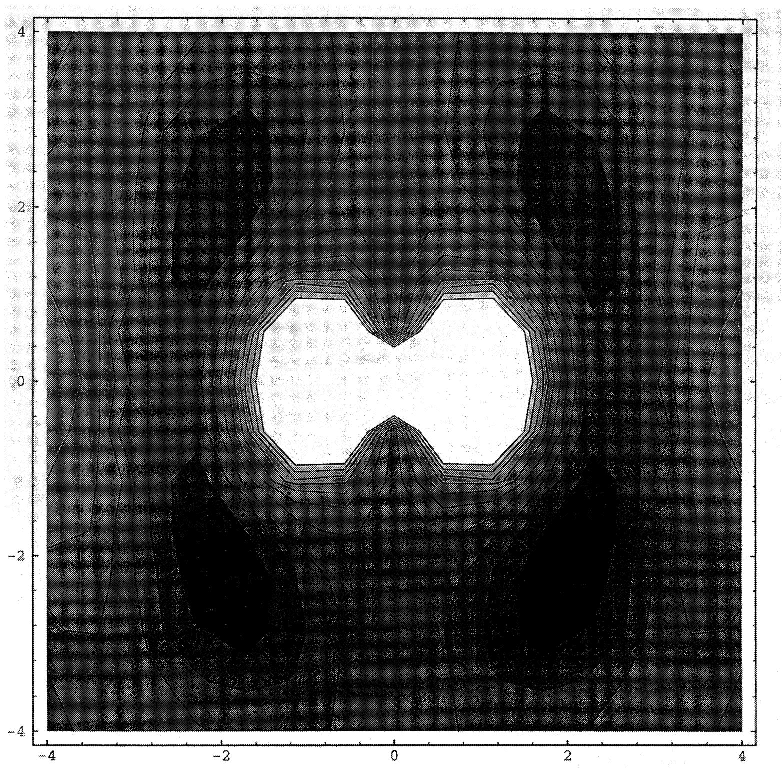


Fig. 6. Energy portrait for the case of magnetic polarisation and scalar interaction (case B in the text). The singularity for $r = 0$ is absent (see equation 20).

We recall that the case of a pseudo-scalar coupling constant (in Section 6 we shall cite the purely electric case as case A, the scalar magnetic case as B, and the pseudoscalar coupling case as C) or, equivalently, the intrinsic chirality of a molecule is supposed to produce stronger magnetic-related effects, though for most atoms this effect is identically zero. Now we approach a demonstration of the modification of the laws governing long-range forces in this last case. Since magnetic and electric fields enter symmetrically in the Hamiltonian we can consider this force as a result of an electric field scattered by an electric dipole induced by a magnetic field or, vice versa, a magnetic field scattered by an electrically induced magnetic moment. Note that the contribution of the source field for all transverse EM waves is identically zero, because of the scalar product $(\mathbf{E}\cdot\mathbf{B})$.

We shall adopt the first choice, i.e. calculate the effect by considering that the source field induces the magnetic moment (Lifschitz and Pitaevskii 1986):

$$\mathbf{p} = g\mathbf{B}\,, \tag{21}$$

which then acts as a scatterer of electric fields according to equations (3). For simplicity we consider the case of linear polarisation, with vector $\mathbf{b} = \mathbf{e}\times\mathbf{e}^*$ having a constant direction in space. The electric field produced by the magnetic field-induced electric dipole is thus

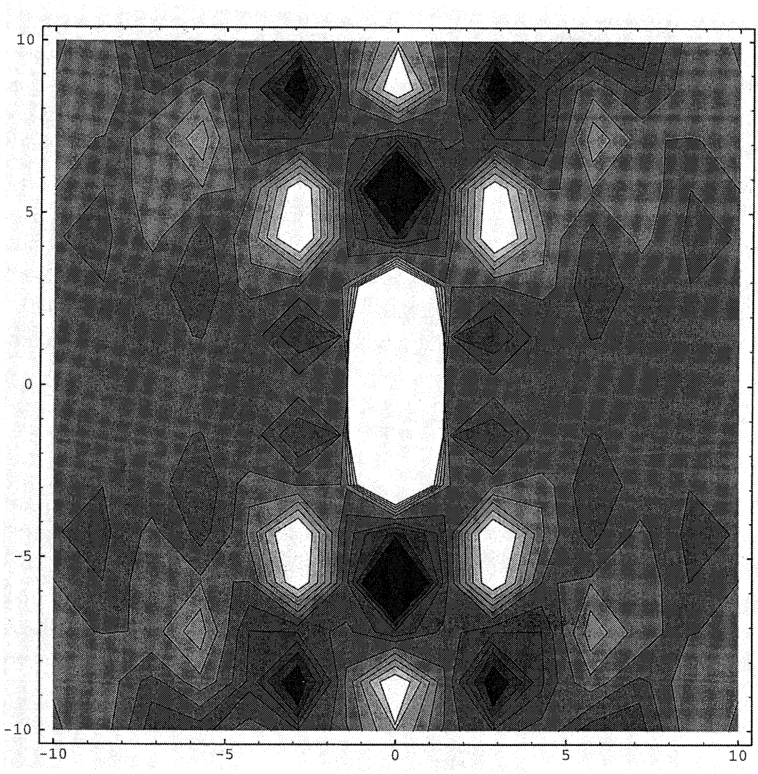


Fig. 7. Energy portrait for pseudo-scalar coupling (case C in the text) and angle $\varphi = 3\pi/8$.

$$\mathbf{E}^{(+)}_{B,k} = \frac{g}{r^3}(1-ikr)\left[3(\mathbf{b}\hat{\mathbf{s}}\hat{\mathbf{s}}-\mathbf{b}) + \frac{k^2}{r}(\hat{\mathbf{s}}\times\mathbf{b}\times\hat{\mathbf{s}})\right]B_0(\mathbf{x}_A, t-r/c)\,,$$

$$\mathbf{E}^{(-)}_{B,k} = \frac{g}{r^3}(1+ikr)\left[3(\mathbf{b}\hat{\mathbf{s}}\hat{\mathbf{s}}-\mathbf{b}) + \frac{k^2}{r}(\hat{\mathbf{s}}\times\mathbf{b}\times\hat{\mathbf{s}})\right]B_0(\mathbf{x}_A, t-r/c)\,, \qquad (22)$$

where B_0 is the amplitude of the source magnetic field. We obtain for the energy of the electric dipole induced by the source magnetic field

$$W^M_{AB} = -\frac{2\pi\alpha g S}{r^3}\sin^2\theta(\sin^2\varphi-\cos^2\varphi)\cos(kr\cos\theta)$$
$$\times\left[3(\cos(kr)+kr\sin(kr)) - (kr)^2\cos(kr)\right]. \qquad (23)$$

The general expression for elliptic polarisation will be given in the next section. Note that if we start from the induced magnetic moment

$$\mathbf{m} = g\mathbf{E}\,, \qquad (24)$$

we come to precisely the same formula as a result of the magnetic-dipole emission formula (Thirunamachandran 1980).

Also worth mentioning is the fact that Planck's constant does not enter explicitly either equations (8) and (10), (20) or (23). This can be considered as a demonstration of the *classical* nature of the effect, in spite of the fact that we can demonstrate crossover to the Casimir case! The appearance of Planck's constant in the Casimir law can be attributed to the quantisation of the electromagnetic field in the mode and then setting the energy density of the field equal to the energy density of zero-point fluctuations equation (14). The quantum nature of the Casimir effect can be thus ascribed to the explicit extraction of a discrete number of photons due to the 'one photon per mode' law for the quantum noise (Louisell 1964). This occurrence of $\hbar$ only through explicit introduction of the photon number, 'granularity of the photon field' (Cummings 1965) is typical of cavity quantum electrodynamics. The energy surface in this case is shown in Fig. 7.

6. Deduction of Laser-modified Long-range Forces by Explicit Perturbation-theory Method

In Sections 2–4 we provided an illustrative example of how the induced dipole moment produces long-range forces between atoms. Now we shall deduce these formulas out of second-order perturbation techniques.

The interaction of the spherical molecules in external field can be described by two diagrams (see Figs 8*a* and 8*b*) using the methods of Craig and Thirunamachandran 1984). The interaction Hamiltonian is the usual electrostatic Hamiltonian

$$H_{int} = -\tfrac{1}{2}\alpha(A)D^2(\mathbf{R}_A) - \tfrac{1}{2}\alpha(B)D^2(\mathbf{R}_B)\,, \qquad (25)$$

where $\mathbf{D(R)}$ is the electric polarisation vector at the point $\mathbf{R}$. The effect appears in second order perturbation theory, because the first-order perturbation provides

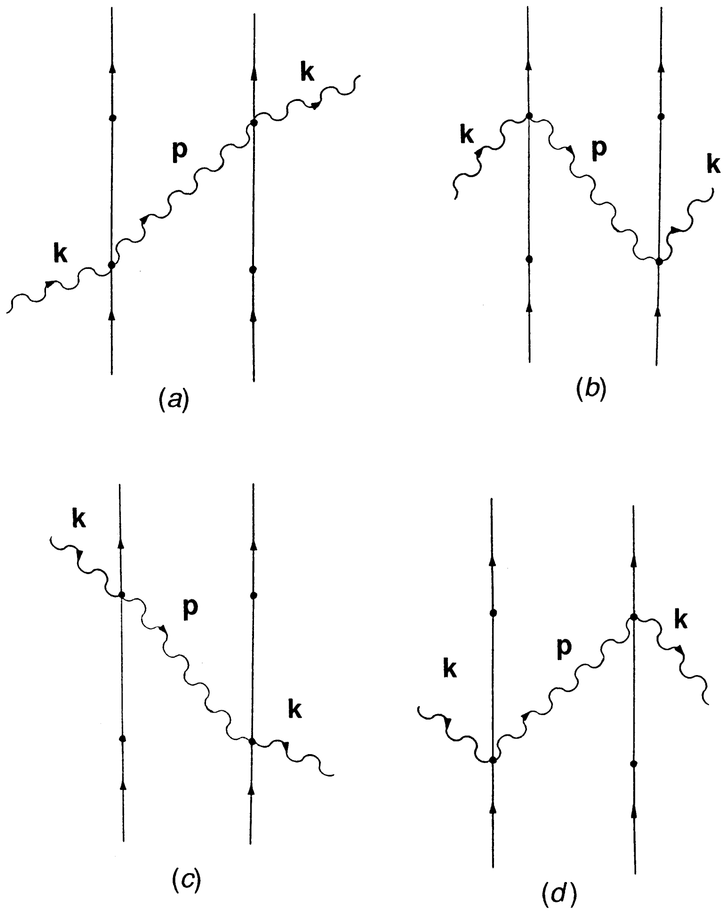


Fig. 8. Diagrams contributing to the dispersive forces (adapted from Craig and Thirunamachandran 1984), owing its origin to the interaction Hamiltonians $H_I^{int} = -\frac{1}{2}\alpha D^2$ and $H_{II}^{int} = -\frac{1}{2}\chi FG$ (equation 26 in the text). Straight lines correspond to atoms and the wavy lines to photons.

only for the processes with real excitation of atoms, which we try to avoid. For the magnetisable bodies these two diagrams are not enough. The contribution of all diagrams in Fig. 8 should be included our treatment. The form of the Hamiltonian would then more general with field operators **F** and **G**, representing any of the functions **D** and **B**:

$$H_{int} = -\tfrac{1}{2}\alpha(A)D^2 - \tfrac{1}{2}\chi(B)\mathbf{F}\cdot\mathbf{G} = H_I + H_{II}\,. \tag{26}$$

The correction to the ground-state energy of atoms is given by the expression

$$\langle i|H_{II}|1\rangle\frac{1}{\hbar ck - \hbar cp}\langle 1|H_I|i\rangle + \langle i|H_I|2\rangle\frac{-1}{\hbar ck + \hbar cp}\langle 2|H_{II}|i\rangle$$

$$+\langle i|H_I|3\rangle\frac{1}{\hbar ck - \hbar cp}\langle 3|H_{II}|i\rangle + \langle i|H_{II}|4\rangle\frac{-1}{\hbar ck + \hbar cp}\langle 4|H_I|i\rangle\,, \tag{27}$$

where $|1\rangle, |2\rangle, |3\rangle, |4\rangle$ are the wavefunctions describing absorption and emission of the photon with wavevector p by the atoms, such as the first atom emits the photon and the second one absorbs or, vice versa, respectively for $|1\rangle\,, |3\rangle$ and $|2\rangle, |4\rangle$.

We adapt the conventional expression for electric induction vectors $\mathbf{F}$ and $\mathbf{G}$ (compare equation 5):

$$\mathbf{F} = i\left(\frac{2\pi\hbar\omega_k}{V}\right)^{\frac{1}{2}}[\mathbf{f}^*_{\mathbf{k}\lambda}a_{\mathbf{k}\lambda}e^{-i\omega_k t}e^{i\mathbf{k}\cdot\mathbf{x}} - \mathbf{f}_{\mathbf{k}\lambda}a^\dagger{}_{\mathbf{k}\lambda}e^{i\omega_k t}e^{-i\mathbf{k}\cdot\mathbf{x}}],$$

$$\mathbf{G} = i\left(\frac{2\pi\hbar\omega_k}{V}\right)^{\frac{1}{2}}[\mathbf{g}^*_{\mathbf{k}\lambda}a_{\mathbf{k}\lambda}e^{-i\omega_k t}e^{i\mathbf{k}\cdot\mathbf{x}} - \mathbf{g}_{k\lambda}a^\dagger_{\mathbf{k}\lambda}e^{i\omega_k t}e^{-i\mathbf{k}\cdot\mathbf{x}}]. \tag{28}$$

The matrix elements of the interaction Hamiltonian H_I are given by

$$\langle i|D^2(R)|1\rangle = 2\left(\frac{2\pi ck}{V}\right)^{\frac{1}{2}}\left(\frac{2\pi cp}{V}\right)^{\frac{1}{2}}\mathbf{e}^*_{i\mathbf{p}}\mathbf{e}_{i\mathbf{k}}e^{-i(\mathbf{p}-\mathbf{k})\cdot\mathbf{R}}n_{\mathbf{k}}^{\frac{1}{2}},$$

$$\langle i|D^2(R)|2\rangle = 2\left(\frac{2\pi ck}{V}\right)^{\frac{1}{2}}\left(\frac{2\pi cp}{V}\right)^{\frac{1}{2}}\mathbf{e}_{i\mathbf{p}}\mathbf{e}_{i\mathbf{k}}e^{i(\mathbf{p}+\mathbf{k})\cdot\mathbf{R}}(n_{\mathbf{k}}+1)^{\frac{1}{2}},$$

$$\langle i|D^2(R)|3\rangle = 2\left(\frac{2\pi ck}{V}\right)^{\frac{1}{2}}\left(\frac{2\pi cp}{V}\right)^{\frac{1}{2}}\mathbf{e}_{i\mathbf{p}}\mathbf{e}^*_{i\mathbf{k}}e^{i(\mathbf{p}-\mathbf{k})\cdot\mathbf{R}}n_{\mathbf{k}}^{\frac{1}{2}},$$

$$\langle i|D^2(R)|4\rangle = 2\left(\frac{2\pi ck}{V}\right)^{\frac{1}{2}}\left(\frac{2\pi cp}{V}\right)^{\frac{1}{2}}\mathbf{e}^*_{i\mathbf{p}}\mathbf{e}^*_{i\mathbf{k}}e^{-i(\mathbf{p}+\mathbf{k})\cdot\mathbf{R}}(n_{\mathbf{k}}+1)^{\frac{1}{2}},$$

and matrix elements of the Hamiltonian H_{II} are given by

$$\langle i|\mathbf{F}\cdot\mathbf{G}|1\rangle = 2\left(\frac{2\pi ck}{V}\right)^{\frac{1}{2}}\left(\frac{2\pi cp}{V}\right)^{\frac{1}{2}}(\mathbf{f}^*_{j\mathbf{k}}\mathbf{g}_{j\mathbf{p}} + \mathbf{f}_{j\mathbf{p}}\mathbf{g}^*_{j\mathbf{k}})\cdot e^{-i(\mathbf{p}-\mathbf{k})\cdot\mathbf{R}}n_{\mathbf{k}}^{\frac{1}{2}},$$

$$\langle i|\mathbf{F}\cdot\mathbf{G}|2\rangle = 2\left(\frac{2\pi ck}{V}\right)^{\frac{1}{2}}\left(\frac{2\pi cp}{V}\right)^{\frac{1}{2}}(\mathbf{f}^*_{j\mathbf{k}}\mathbf{g}^*_{j\mathbf{p}} + \mathbf{f}_{j\mathbf{p}}\mathbf{g}_{j\mathbf{k}})\cdot e^{i(\mathbf{p}+\mathbf{k})\cdot\mathbf{R}}(n_{\mathbf{k}}+1)^{\frac{1}{2}},$$

$$\langle i|\mathbf{F}\cdot\mathbf{G}|3\rangle = 2\left(\frac{2\pi ck}{V}\right)^{\frac{1}{2}}\left(\frac{2\pi cp}{V}\right)^{\frac{1}{2}}(\mathbf{f}_{j\mathbf{k}}\mathbf{g}^*_{j\mathbf{p}} + \mathbf{f}^*_{j\mathbf{p}}\mathbf{g}_{j\mathbf{k}})\cdot e^{i(\mathbf{p}-\mathbf{k})\cdot\mathbf{R}}n_{\mathbf{k}}^{\frac{1}{2}},$$

$$\langle i|\mathbf{F}\cdot\mathbf{G}|4\rangle = 2\left(\frac{2\pi ck}{V}\right)^{\frac{1}{2}}\left(\frac{2\pi cp}{V}\right)^{\frac{1}{2}}(\mathbf{f}_{j\mathbf{k}}\mathbf{g}_{j\mathbf{p}} + \mathbf{f}_{j\mathbf{p}}\mathbf{g}_{j\mathbf{k}})\cdot e^{-i(\mathbf{p}+\mathbf{k})\cdot\mathbf{R}}(n_{\mathbf{k}}+1)^{\frac{1}{2}}.$$

We now assume the polarisation vector for the general elliptic polarisation of the laser field to be in the form

$$\mathbf{e}^I = e^{i\alpha/2}\cos\frac{\beta}{2}\mathbf{e}^L + e^{-i\alpha/2}\sin\frac{\beta}{2}\mathbf{e}^R, \tag{29}$$

where $\mathbf{e}^L, \mathbf{e}^R$ are the polarisation vectors for right- and left-circularly polarised light respectively. The substitution of these matrix elements into expression (26) for the energy gives us the formula (following Thirunamachandram we suppose $n \gg 1$ for the only mode we are interested in)

$$
\begin{aligned}
E^{(2)} = {} & \tfrac{1}{4}\alpha_A\chi_B \frac{2\pi\hbar ck}{V} n \frac{2\pi\hbar cp}{V} \\
& \times \left[(\mathbf{f}^*_{j\mathbf{k}}\mathbf{g}_{j\mathbf{p}} + \mathbf{f}_{j\mathbf{p}}\mathbf{g}^*_{j\mathbf{k}})\mathbf{e}_{i\mathbf{k}}\mathbf{e}^*_{i\mathbf{p}} \frac{e^{i(\mathbf{p}-\mathbf{k})\cdot(\mathbf{R_A}-\mathbf{R_B})}}{\hbar ck - \hbar cp} \right. \\
& + (\mathbf{f}^*_{j\mathbf{k}}\mathbf{g}^*_{j\mathbf{p}} + \mathbf{f}_{j\mathbf{p}}\mathbf{g}_{j\mathbf{k}})\mathbf{e}_{j\mathbf{p}}\mathbf{e}_{j\mathbf{k}} \frac{e^{-i(\mathbf{p}+\mathbf{k})\cdot(\mathbf{R_A}-\mathbf{R_B})}}{-\hbar ck - \hbar cp} \\
& + (\mathbf{f}^*_{j\mathbf{k}}\mathbf{g}^*_{j\mathbf{p}} + \mathbf{f}_{j\mathbf{p}}\mathbf{g}_{j\mathbf{k}})\mathbf{e}_{j\mathbf{p}}\mathbf{e}^*_{j\mathbf{k}} \frac{e^{-i(\mathbf{p}-\mathbf{k})\cdot(\mathbf{R_A}-\mathbf{R_B})}}{\hbar ck - \hbar cp} \\
& \left. + (\mathbf{f}^*_{j\mathbf{k}}\mathbf{g}^*_{j\mathbf{p}} + \mathbf{f}_{j\mathbf{p}}\mathbf{g}_{j\mathbf{k}})\mathbf{e}^*_{j\mathbf{p}}\mathbf{e}^*_{j\mathbf{k}} \frac{e^{i(\mathbf{p}+\mathbf{k})\cdot(\mathbf{R_A}-\mathbf{R_B})}}{-\hbar ck - \hbar cp} \right]. \qquad (30)
\end{aligned}
$$

Summation over all values of the momentum transfer is accomplished through the formula (here $r = |\mathbf{R}_A - \mathbf{R}_B|$)

$$
\Sigma_{\mathbf{p}} \frac{2\pi p}{V}(\delta_{ij} - \hat{p}_i \cdot \hat{p}_j) \left[\frac{e^{i\mathbf{p}\cdot\mathbf{R}}}{k-p} - \frac{e^{-i\mathbf{p}\cdot\mathbf{R}}}{k+p} \right] \mathbf{e}_{i,\mathbf{p}}\mathbf{e}_{j,\mathbf{p}} = -(\nabla^2\delta_{ij} + \nabla_i\nabla_j)\frac{\cos(kr)}{r}, \qquad (31)
$$

$$
\Sigma_{\mathbf{p}} \frac{2\pi p}{V}\epsilon_{ijk}\hat{p}_k \left[\frac{e^{i\mathbf{p}\cdot\mathbf{R}}}{k-p} + \frac{e^{-i\mathbf{p}\cdot\mathbf{R}}}{k+p} \right] \mathbf{e}_{i,\mathbf{p}}\mathbf{e}_{j,\mathbf{p}} = -k\epsilon_{ijk}\nabla_k \frac{\cos(kr)}{r}. \qquad (32)
$$

Straightforward differentiation leads to the formula

$$
\begin{aligned}
V_{ij} = {} & -(\nabla^2\delta_{ij} + \nabla_i\nabla_j)\frac{\cos(kr)}{r} \\
= {} & \beta_{ij}\frac{\cos(kr)}{r^3} + \beta_{ij}\frac{k\cos(kr)}{r^2} - \alpha_{ij}\frac{k^2\cos(kr)}{r}, \qquad (33)
\end{aligned}
$$

where

$$
\beta_{ij} = \delta_{ij} - 3\hat{s}_i \cdot \hat{s}_j, \qquad \alpha_{ij} = \delta_{ij} - \hat{s}_i \cdot \hat{s}_j. \qquad (34)
$$

To convert the expressions for the angular part into a more conventional form it is useful to introduce the following notation:

$$
\begin{aligned}
& e_i e_j^* \delta_{ij} = 1, \quad e_i e_j^* \hat{s}_i \hat{s}_j = \Xi, \quad e_i e_j^* \epsilon_{ijk}\hat{s}_k = \Sigma, \\
& e_i b_j^* \delta_{ij} = \Pi, \quad e_i b_j^* \hat{s}_i \hat{s}_j = Z, \quad e_i b_j^* \epsilon_{ijk}\hat{s}_k = \cos\theta. \qquad (35)
\end{aligned}
$$

The first and the last expressions are, of course, simple geometric equalities. The factors Ξ, Σ, Π, Z can be expressed through components of the electric field and vector $\mathbf{s}$ as

$$\Xi = \frac{1}{E_x^2 + E_y^2}\{E_x^2 s_x^2 + E_y^2 s_y^2 + 2E_x E_y s_x s_y \cos\delta\}\,, \tag{36}$$

$$\Sigma = i\frac{2E_x E_y s_z \sin\delta}{E_x^2 + E_y^2}\,, \tag{37}$$

$$\Pi = i\cos\beta\,, \tag{38}$$

$$Z = \frac{1}{E_x^2 + E_y^2}\{(E_x^2 - E_y^2)s_x s_y - E_x E_y(s_x^2 - s_y^2)\cos\delta - iE_x E_y(s_x^2 + s_y^2)\sin\delta\}\,, \tag{39}$$

with δ defined by

$$\tan\delta = -\sec\alpha\ \cot\beta \tag{40}$$

and α, β from equation (29).

Finally, for case A, we get the expression, similar to equation (6) of the Section 2,

$$W_{AB} = -\alpha(A)\alpha(B)\mathbf{E}^2_{0,\mathbf{k}}\mathbf{e}_{i\mathbf{k}}\mathbf{e}^*_{j\mathbf{k}}V_{ij}\cos(\mathbf{k}\cdot\mathbf{r})\,. \tag{41}$$

In this section we use the following geometry. Vector $\mathbf{k}$ is taken along the z-axis. The angles θ and ϕ are the polar and the azimuth vectors of $\hat{s}$ in the coordinate system formed by a triple $(\mathbf{e}, \mathbf{b}, \mathbf{k})$.

Straightforward, yet tedious calculations, provide the following expression for a geometric factor:

$$\mathbf{e}_i\mathbf{e}^*_j V_{ij} = (1 - 3\Xi)\left(\frac{\cos kr}{r^3} + \frac{k\sin kr}{r^2}\right) - (1 - \Xi)\frac{k^2\cos kr}{r}\,. \tag{42}$$

For the cases A and B we shall write down the equation explicitly, in terms of angles θ and ϕ. For the case C we shall restrict ourselves to the aggregate expression through Π and Z. The mutual interaction energy in the case A is given by

$$\begin{aligned} W_{AB} = &- \alpha(A)\alpha(B)\cos(kr\hat{s}_3) \\ &\times\Big\{[E_x^2 + E_y^2 - 3(\hat{s}_1^2 E_x^2 + \hat{s}_2^2 E_y^2 + 2\hat{s}_1\hat{s}_2 E_x E_y \cos\delta)]\left(\frac{\cos kr}{r^3} + \frac{k\sin kr}{r^2}\right) \\ &-[E_x^2 + E_y^2 - (\hat{s}_1^2 E_x^2 + \hat{s}_2^2 E_y^2 + 2\hat{s}_1\hat{s}_2 E_x E_y \cos\delta)]\frac{k^2\cos kr}{r}\Big\}\,. \end{aligned} \tag{43}$$

Expressing the previous formula through the angles θ, ϕ, we obtain for the energy

$$W_{AB} = \alpha(A)\alpha(B)\Big[(E_x^2(1 - -3\sin^2\theta\cos^2\phi) + E_y^2(1 - -3\sin^2\theta\cos^2\phi)$$
$$-6E_xE_y\cos\delta\sin^2\theta\cos\phi\sin\phi)\left(\frac{\cos kr}{r^3} + \frac{k\sin kr}{r^2}\right)$$
$$-(E_x^2(1-\sin^2\theta\cos^2\phi) + E_y^2(1-\sin^2\theta\sin^2\phi)$$
$$-2E_xE_y\cos\delta\sin^2\theta\cos\phi\sin\phi)\frac{k^2\cos kr}{r}\Big]\cos(kr\cos\theta)\,. \tag{44}$$

If we average the previous expression over the angle ϕ the term which contains δ will also average out to zero:

$$W_{AB} = \alpha(A)\alpha(B)\cos(kr\cos\theta)$$
$$\times\Big[(E_x^2(1-\tfrac{3}{2}\sin^2\theta) + E_y^2(1-\tfrac{3}{2}\sin^2\theta))\left(\frac{\cos kr}{r^3} + \frac{k\sin kr}{r^2}\right)$$
$$-(E_x^2(1-\tfrac{1}{2}\sin^2\theta) + E_y^2(1-\tfrac{1}{2}\sin^2\theta))\frac{k^2\cos kr}{r}\Big]\,. \tag{45}$$

Obviously, for $E_x^2 = E_y^2$ (circular polarisation), we return to equation (10). Additional averaging over the angle θ applied to the case of general elliptic polarisation gives the same result as equation (10) with the replacement of E_0^2 to $(E_x^2 + E_y^2)$ which is expected, i.e. in the case of a nonresonant effect with isotropic atoms, the result should depend only on the total intensity.

For the case B the solution, similar to (20), will be

$$W_A^B = -2k\alpha_A\chi_B I_{\mathbf{k}} e_{i\mathbf{k}} b^*_{j\mathbf{k}} \sin(\mathbf{k}\cdot\mathbf{r})\epsilon_{ijk}\nabla_k\frac{\cos(kr)}{kr}\,. \tag{46}$$

In the case C the solution is given by the equation

$$W_A^C = -\alpha_A\chi_B I_{\mathbf{k}}\{k e_{i\mathbf{k}} e^*_{j\mathbf{k}} \sin(\mathbf{k}\cdot\mathbf{r})\epsilon_{ijk}\nabla_k\frac{\cos(kr)}{kr} + e_{i\mathbf{k}} b^*_{j\mathbf{k}}\cos(\mathbf{k}\cdot\mathbf{r})V_{ij}\}\,, \tag{47}$$

where V_{ij} is defined by equation (43).

The spatial dependence in equation (47) and in the first term of equation (48) can simplified using

$$e_{i\mathbf{k}} b^*_{j\mathbf{k}}\epsilon_{ijk}\nabla_k\frac{\cos(kr)}{kr} = \cos\theta\left(-\frac{k\cos kr}{r} - \frac{\cos kr}{r^2}\right). \tag{48}$$

The final solution for W_{AB} is given by adding the complex conjugate to the expressions cited above. Thus, the solution for the general case of elliptic polarisation can be then expressed in the form

Case B

$$W_{AB}^{B} = 2\alpha_A \chi_B I_{\mathbf{k}} \sin(kr\cos\theta)\cos(\theta)\left(\frac{k\sin kr}{r} + \frac{\cos kr}{r^2}\right), \tag{49}$$

Case C

$$\begin{aligned} W_{AB}^{C} = \alpha_A \chi_B I_{\mathbf{k}} \Bigg\{ & k\sin(kr\cos\theta)\Sigma\left(\frac{k\sin kr}{r} + \frac{\cos kr}{r^2}\right) \\ & - \cos(kr\cos\theta)\left[(\Pi - Z)\frac{k^2\cos kr}{r}\right. \\ & \left. -(\Pi - 3Z)\left(\frac{k\sin kr}{r^2} + \frac{\cos kr}{r}\right)\right]\Bigg\} + c.c. \end{aligned} \tag{50}$$

Note, that Ξ is real, Σ, Π are imaginary and Z is complex, and this produces the complex expression only in the case C.

The formula for case C for general elliptic polarisation is given by

$$\begin{aligned} W_{AB}^{M} = & -\frac{4\pi\alpha_A\chi_B S}{r^3}\cos(kr\cos\theta) \\ & \times[3(\cos(kr) + kr\sin(kr)) - (kr)^2\cos(kr)] \\ & \times\frac{0\cdot 5(E_x^2 - E_y^2)\sin^2\theta\sin 2\phi - \cos\delta E_x E_y \sin^2\theta\cos 2\phi}{E_x^2 + E_y^2}\,. \end{aligned} \tag{51}$$

This concludes the derivation of the laser-modified long-range forces for arbitrary elliptical polarisation for the most general Hamiltonian (27), quadratic in the fields **E** and **B**, which does not depend on derivatives of the field.

We recapitulate the physical differences between our cases A, B and C. They result from a different symmetry of interaction. Unlike the case A, both cases B and C involve the reaction of a *magnetic dipole* induced by an *electric field*. Because the electric field is a polar vector, while the magnetic field is an axial one, this results in the fact that different spatial components of the field give rise to the dispersive forces. The scalar (case B) and pseudo-scalar (case C) character of the coupling constant (equation 17) modifies the spatial symmetry of the interaction as well. As a result, the long-range forces obtained in cases A, B and C all have distinct functional forms. Thus, a distinction between the cases A, B and C bears its origin in *classical* electrodynamics.

7. Conclusion

In this paper we have established the quantitative laws of the laser-modified long-range forces between neutral atoms. We pointed out that these forces can be influenced by magnetic, as well as an electric, component of the laser field. This effect is particularly important in the case of chiral molecules because it represents a long-range force, which has a different impact on the chiral isomers.

Laser-modified long-range forces can overcome conventional long-range (van der Waals and Casimir) forces, arising from fluctuations of the electromagnetic field of the atoms in the ground state, though under very stringent conditions. We express hope that swift progress in obtaining ultra-cold samples of neutral particles in traps will make feasible an experimental observation of laser modification of long-range forces. It may manifest itself as a small additional nonlinear refraction in the samples of the neutral, symmetric particles.

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References

Andrews, D. L., and Thirunamachandran, T. (1978). *Proc. R. Soc. London* A **358**, 297.
Baranova, N. B., *et al.* (1977). *Sov. Phys. Uspechi* **20**, 870.
Berestetskii, V. B., Lifshiz, E. M., and Pitaevskii, L. P. (1982). 'Quantum Electrodynamics' (Pergamon: New York).
Born, M. (1962). 'Atomic Physics', Appendix 39 (Hafner: New York).
Craig, D., and Thirunamachandran, T. (1984). 'Molecular Quantum Electrodynamics' (Academic: New York).
Cummings, F. W. (1965). *Phys. Rev.* A **140**, 1094.
Dehmelt, H. (1975). *Bull. Am. Phys. Soc.* **20**, 60.
Gallas, J. A. C., Leuchs, G., Walther, H., and Figger, H. (1985). *In* 'Advances in Atomic and Molecular Physics' (Eds R. Bates and B. Bederson), Vol. 20, p. 413 (Academic: New York).
Grossel, P., and van Labeke, D. (1994). *Phys. Rev.* A **49**, 1177.
Haroche, S., and Raimond, J. M. (1985). *In* 'Advances in Atomic and Molecular Physics' (Eds R. Bates and B. Bederson), Vol. 20, p. 347 (Academic: New York).
Hinds, E. A. (1991). *In* 'Advances in Atomic and Molecular Physics' (Eds D. Bates and B. Bederson), Vol. 28, p. 235 (Academic: New York).
Jackson, J. D. (1975). 'Classical Electrodynamics' (Wiley: New York).
Lifschitz, E. M., and Pitaevskii, L. P. (1986). 'Electrodynamics of Continuous Media' (Pergamon: New York).
Louisell, R. (1964). 'Radiation and Noise in Quantum Electronics' (McGraw–Hill: New York).
Mahanty, J., and Ninham, B. (1986). 'Dispersive Forces' (Academic: New York).
Meath, W. J., and Power, E. A., (1989). *J. Mod. Opt.* **36**, 977.
Milonni, P. W., and Shih M.-L. (1992). *Phys. Rev.* A **45**, 4241.
Narozhny, N. B., Sanchez-Mondragon, J. J., and Eberly, J. H. (1981). *Phys. Rev.* A **23**, 236.
Passante, R., and Power, E. A. (1987). *Phys. Rev.* A **35**, 188.
Rempe, G., Walther, H., and Klein, N. (1987). *Phys. Rev. Lett.* **58**, 353.
Thirunamachandran, T. (1980). *Mol. Phys.* **40**, 393.
Yoo, H.-I., and Eberly, J. H. (1985). *Phys. Rep.* **118**, 235.

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