# Inelastic Proton Scattering Analyses and Ambiguities in Spin-Parity Assignments of States in ${ }^{12} \mathrm{C}$ 

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#### Abstract

Data from the excitation of the $13 \cdot 35,18 \cdot 35,19 \cdot 20$ and $19 \cdot 40 \mathrm{MeV}$ states in ${ }^{12} \mathrm{C}$ by the inelastic scattering of protons have been analysed using a fully microscopic distorted wave approximation to determine optimal assignments from amongst the current and ambiguous $J^{\pi} ; T$ values of each state involved. While not conclusive, the present analysis suggests the assignments of $4_{1}^{-} ; 0,2_{2}^{-} ; 0,4_{2}^{-} ; 0$ and $2_{2}^{-} ; 1$ respectively.


## 1. Introduction

The assignments of spin, parity and isospin of the $13 \cdot 35 \mathrm{MeV}, 18 \cdot 35 \mathrm{MeV}$, 19.20 MeV and 19.40 MeV states in ${ }^{12} \mathrm{C}$ remain in doubt. In the latest compilation (Ajzenberg-Selove 1990), these states are listed as $13 \cdot 35-\left(2^{-}\right) ; 0$, $18 \cdot 35-$ both $3^{-} ; 1$ and $2^{-} ; 0+1,19 \cdot 20-\left(1^{-} ; 1\right)$, and $19 \cdot 40-\left(2^{-} ; 1\right)$. Millener (Ajzenberg-Selove 1990; Millener 1995) has suggested, however, that the $13 \cdot 35$ MeV state is $4^{-} ; 0$. Its assignment of $2^{-} ; 0$ was based on the measurement of the $\alpha$-decay width of this 13.35 MeV state being larger than would be consistent with an assignment of $4^{-}$. However, the measured $\alpha$-decay width was reproduced by an $\alpha$-cluster model calculation of ${ }^{12} \mathrm{C}$ (Uegaki et al. 1977) assuming that the assignment was $4^{-} ; 0$. Also, as a shell model calculation using the $S U_{3}$ basis also predicted a $4^{-} ; 0$ state to lie near $13 \cdot 35 \mathrm{MeV}$ (Millener 1995) and there was no corresponding $2^{-} ; 0$ state in that spectrum, Millener surmised that the $13 \cdot 35 \mathrm{MeV}$ state more rightly should be listed as $\left(4^{-}\right) ; 0$.

The $18 \cdot 35 \mathrm{MeV}$ state is listed as both a $3^{-} ; 1$ and a $2^{-} ; 0+1$ state. The $2^{-}$assignment is supported by analyses of pion and proton inelastic scattering data (Neuschaefer et al. 1983; Cottingame et al. 1987; Jones et al. 1983), while the assignment of $3^{-}$; 1 was based on an analysis of inelastic electron scattering data (Neuschaefer et al. 1983; Yamaguchi et al. 1971). However, in that analysis of the (longitudinal) electron scattering form factor, Yamaguchi et al. gave the energy of the state they observed as 18.60 MeV , on the basis of which it has been included in the compilation (Ajzenberg-Selove 1990). Also, a more recent measurement (Hicks et al. 1984) of transverse M2 electron scattering form factors gave evidence for a $2^{-} ; 0$ state at $18 \cdot 20 \mathrm{MeV}$. Conversely, a measurement of the $\gamma$-decay of a state at 18.35 MeV excitation (Hanna et al. 1982) gave a unique assignment of $3^{-} ; 1$. Also the presence of a $3^{-} ; 1$ state at $3 \cdot 389 \mathrm{MeV}$ excitation
in ${ }^{12} \mathrm{~B}$ (Ajzenberg-Selove 1990) suggests that an isobaric analogue should lie in the 15 to 20 MeV region of excitation of ${ }^{12} \mathrm{C}$. However, the inelastic proton scattering data to the specific state at 18.35 MeV excitation we have analysed does not reflect the $3^{-} ; 1$ character at all. Therefore, the 18.35 MeV state is more likely to have the $2^{-} ; 0$ assignment.

The purity of the isospin of the $18 \cdot 20 \mathrm{MeV}\left(2^{-}\right)$state remains unclear. There is evidence for isospin mixing in this state (Ajzenberg-Selove 1990; Millener 1995). Inelastic proton scattering cannot be used to observe isospin mixing as the interaction promoting it is assumed to be charge independent. This was the case with analyses of the transitions to the $1^{+} ; 0(12 \cdot 71 \mathrm{MeV})$ and $1^{+} ; 1(15 \cdot 11 \mathrm{MeV})$ states (Karataglidis et al. 1995a). For those cases, isospin mixing was observed in the analyses of the electron scattering form factors but, and for the reasons stated above, was not observed in the inelastic proton scattering observables.

Only recently have large basis shell model calculations of light nuclei been made that, for ${ }^{12} \mathrm{C}$, may specify states in the 12 to 20 MeV excitation range. One such calculation, a full $(0+2) \hbar \omega$ shell model calculation for positive parity states (in ${ }^{12} \mathrm{C}$ ) and a restricted $(1+3) \hbar \omega$ shell model calculation for negative parity states, was made recently (Karataglidis et al. 1995a). The associated one body density matrix elements (Karataglidis 1995a) (OBDME) were used in analyses of electron scattering form factors (Karataglidis et al. 1995b) and in a fully microscopic distorted wave approximation (DWA) calculation of differential cross sections and analysing powers from the elastic and inelastic scattering of 200 MeV protons from ${ }^{12} \mathrm{C}$ (Karataglidis et al. 1995a). The only restriction placed on the $(1+3) \hbar \omega$ study was to neglect single particle excitations out of the $0 p$ shell into the ( $0 g 1 d 2 s$ ) shell. This restriction is not severe and all known negative parity states to 20 MeV excitation lie within 2 MeV of experimental candidates. There was also very good agreement between data and analyses when the OBDME for the excitation of the $3_{1}^{-} ; 0(9 \cdot 64 \mathrm{MeV})$ and $1_{1}^{-} ; 0(10 \cdot 85 \mathrm{MeV})$ states were used to analyse the electron scattering form factors and differential cross section, and analysing powers from the inelastic scattering of 200 MeV protons (Karataglidis et al. 1995a).

That shell model study gave a good indication of the $J^{\pi} ; T$ assignments in the 12 to 20 MeV excitation region. The first $2^{-} ; 0$ state was predicted to be at 13.09 MeV , for which the measured energy is 11.83 MeV , and the second at 17.02 MeV . This same shell model study also gave the energy of the $4_{1}^{-} ; 0$ state as $12 \cdot 37 \mathrm{MeV}$, as did a large basis particle-hole model (PHM) (Amos et al. 1981) that was based upon a Hartree-Fock ground state specification where the predicted energy was $12 \cdot 28 \mathrm{MeV}$.

The negative parity states of ${ }^{12} \mathrm{C}$ considered in this study are listed in Table 1. The structure calculations suggest that the $18 \cdot 35 \mathrm{MeV}$ state has a $2^{-}$assignment, but both the $T=0$ and $T=1$ cases are possible. A $1^{-} ; 1$ state is predicted in the vicinity of 19.20 MeV by both the (large space) shell model and the PHM calculations. This is by no means the only candidate assignment for the observed state, as a $4^{-} ; 0$ state is predicted to lie near to this energy also. The shell model predicts a $2^{-} ; 1$ state at $19 \cdot 28 \mathrm{MeV}$ suggesting another assignment which may be reasonable for the observed $19 \cdot 40 \mathrm{MeV}$ state.

Herein, we report on the results of DWA analyses of inelastic proton scattering data seeking with them clarification of the $J^{\pi} ; T$ assignments of these four states. Both cross section and analysing power data from the scattering of 200 MeV
(Comfort et al. 1982) and 398 MeV (Jones et al. 1983, 1994) protons have been studied using the fully microscopic DWA approach reported recently (Karataglidis et al. 1995a). The required effective nucleon-nucleon ( $N N$ ) interaction (in coordinate space and having central, tensor and two-body spin-orbit terms) at both energies was specified by optimally mapping $N N g$-matrix elements obtained from solution of the Bethe-Goldstone infinite matter equations. Pauli blocking and the mean background field were incorporated therein under appropriate averages (Haftel and Tabakin 1970). These matrix elements are given in unpublished reports (Dortmans and Amos 1995). The resultant effective $N N$ interactions are density dependent and are identified as DD interactions hereafter. We have also made (fully microscopic DWA) calculations starting with the Love-Franey (LF) (1985) interaction.

Table 1. States considered in the present analysis, together with energies predicted from the $(1+3) \hbar \omega$ shell model calculation (Karataglidis et al. 1995a) and from the PHM (Amos et al. 1981)

| Experiment |  | Candidates |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Energy (MeV) | $J^{\pi} ; T$ | $J^{\pi} ; T$ | Energy (MeV) |  |
|  |  |  | $(1+3) \hbar \omega$ | PHM |
| $13 \cdot 35$ | $\left(2^{-}\right) ; 0$ | $4_{1}^{-} ; 0$ | $12 \cdot 37$ | $12 \cdot 28$ |
|  |  | $2_{2}^{-1} ; 0$ | $17 \cdot 02$ | $15 \cdot 78$ |
| $18 \cdot 35$ | $3^{-} ; 1,2^{-} ; 0+1$ | $2_{2}^{-} ; 0$ | $17 \cdot 02$ | $15 \cdot 78$ |
|  |  | $2_{3}^{-} ; 0$ | $19 \cdot 03$ | $18 \cdot 27$ |
|  |  | $2_{2}^{-} ; 1$ | $19 \cdot 28$ | $18 \cdot 00$ |
|  |  | $3_{1}^{-} ; 1$ | $18 \cdot 77$ | $17 \cdot 82$ |
| (18.60) | $\left(3^{-}\right)$ | $3_{1}^{-} ; 1$ | $18 \cdot 77$ | $17 \cdot 82$ |
| $19 \cdot 20$ | $\left(1^{-} ; 1\right)$ | $1_{2}^{-} ; 1$ | 19.67 | $18 \cdot 33$ |
|  |  | $4-2$ | 19.04 | $18 \cdot 69$ |
| $19 \cdot 40$ | $\left(2^{-} ; 1\right)$ | $2-\frac{1}{2}$ | $19 \cdot 28$ | 18.00 |

The distorted wave functions required in the DWA calculations were generated from the non-local interactions obtained by folding the DD or LF force as is appropriate with the $(0+2) \hbar \omega$ and $(1+3) \hbar \omega$ shell model density matrix elements of the ground and residual states of ${ }^{12} \mathrm{C}$. By so doing in analyses of the elastic scattering data at 200 MeV (Karataglidis et al. 1995a), DD calculations resulted in quite exceptional fits to both cross section and analysing power data (to $\sim 50^{\circ}$ ).

## 2. Discussion of Results

We first consider a single electron scattering form factor. Specifically, we analyse the longitudinal $E 3$ electron scattering form factor for the $0^{+} \rightarrow 3_{1}^{-} ; 1$ transition. This result is displayed in Fig. 1, and was obtained by a calculation made using the OBDME from the $(1+3) \hbar \omega$ wave functions (Karataglidis 1995b). It is compared with the data of Yamaguchi et al. (1971), who noted that the data came from a state at 18.60 MeV . Clearly there is reasonable agreement between those data and the results of our calculation. This indicates that the preferred assignment for the 18.35 MeV state of those listed in the compilation (Ajzenberg-Selove 1990) is $2_{2}^{-} ; 0$ (the $3_{1}^{-} ; 1$ assignment being assumed for the $18 \cdot 6 \mathrm{MeV}$ state).


Fig. 1. Longitudinal electron scattering form factor for the $0^{+} \rightarrow 3_{1}^{-} ; 1$ transition. The 18.60 MeV excitation data of Yamaguchi et al. (1971) are compared with the result of the calculation made using the $(1+3) \hbar \omega$ wave functions.


Fig. 2. Differential cross section for the inelastic scattering of 200 and 398 MeV protons from ${ }^{12} \mathrm{C}$. The data from scattering to the $13 \cdot 35,18 \cdot 35,19 \cdot 20$ and 19.40 MeV states are shown from top to bottom in sequence. The 200 MeV data of Comfort et al. (1982) and the 398 MeV data of Jones et al. $(1983,1994)$ are compared with the results of the DD calculations using the OBDME from the $(1+3) \hbar \omega$ shell model calculation.

The results of the calculations of the differential cross sections for the inelastic scattering of protons from ${ }^{12} \mathrm{C}$ to these states using both the DD and LF forces are shown in Figs 2 and 3 and at the energies shown and for the diverse, possible choices of $J^{\pi} ; T$ that are listed in Table 1. The associated analysing powers are displayed in Figs 4 and 5. In all four diagrams, the $13 \cdot 35,18 \cdot 35,19 \cdot 20$ and 19.40 MeV transitions are displayed from top to bottom. The preferred set of assignments, based on a comparison between the measured and the predicted spectra, and on the results of the analysis of the longitudinal form factor for the $18 \cdot 60 \mathrm{MeV}$ state, contains the assignments $4_{1}^{-} ; 0(13 \cdot 35 \mathrm{MeV}), 2_{2}^{-} ; 0$ $(18 \cdot 35 \mathrm{MeV}), 4_{2}^{-} ; 0(19 \cdot 20 \mathrm{MeV})$ and $2_{2}^{-} ; 1(19.40 \mathrm{MeV})$, and is given by the solid curves in these figures.


Fig. 3. As for Fig. 2, but showing the results of calculations made using the LF force.
There are other possibilities. If the 13.35 MeV state were the $2_{2}^{-} ; 0$ state, then the higher excitation assignments could be $2_{3}^{-} ; 0(18 \cdot 35 \mathrm{MeV})$ and $4_{1}^{-} ; 0$ $(19 \cdot 20 \mathrm{MeV})$. Using that specification we obtained the DWA results that are displayed in the figures by the dashed curves. A third possibility assigns $2_{2}^{-} ; 1$ to the 18.35 MeV state and $2_{3}^{-} ; 1$ to the 19.40 MeV state. The choice of $1_{2}^{-} ; 1$ for the 19.20 MeV state is independent of all other choices but is included in this third set. The results of this set are displayed by the dot-dash curves.

The results of the calculations of the differential cross sections made using both the DD and LF interactions exhibit similar features. For the 13.35 MeV
excitation by 200 MeV protons, the assignment of $4_{1}^{-} ; 0$ is preferred, but the data indicate the possibility of there being a minimum at $30^{\circ}$. The presence of such a minimum would favour the $2_{2}^{-} ; 0$ assignment and so such a choice is not discounted by this analysis. More data are required at small angles to uniquely identify the spin of this state, as at small momentum transfers the two calculations differ by up to 3 orders of magnitude. This is so irrespective of the choice of interaction. The results at 398 MeV are less clear. The calculation based on the assignment of $2^{-}$reproduces the magnitude of the limited data set at large angles but that which is based on the assignment of $4^{-}$reproduces the shape, if not the magnitude. Again, this is evident in the results obtained using either force, but as this data set comprises just 5 points, more data are required to make a firm conclusion.


Fig. 4. As for Fig. 2, but for the analysing powers.
The choice regarding the 13.35 MeV state affects the choice of assignment for the 18.35 MeV state. The preferred candidate is the $2_{2}^{-} ; 0$ state (Jones et al. 1983), a choice supported by both calculations at 200 MeV . Recall that this required the assignment of $4_{1}^{-} ; 0$ for the 13.35 MeV state. However, at 398 MeV the choice is not as definite, for while the results of the calculations of the differential cross section favour the preferred $2_{2}^{-} ; 0$ state, consistent with the analysis at 200 MeV , as shall be noted later, the results of the calculations of the analysing power favour the other possible assignments. The results of the
calculations of the differential cross section at 18.35 MeV assuming that this state is the $3_{1}^{-} ; 1$ state are given in Fig. 6, wherein the data are compared to those results (dashed curves), as well as those obtained for the preferred assignment of $2_{2}^{-} ; 0$ (solid curves). The data clearly support the choice of $2_{2}^{-} ; 0$ for the 18.35 MeV state, consistent with the assignment of $3_{1}^{-} ; 1$ for the state at 18.60 MeV .


Fig. 5. As for Fig. 3, but for the analysing powers.
It is clear from the analysis of the $19 \cdot 20 \mathrm{MeV}$ differential cross-section data at both 200 and 398 MeV that this excitation is not of a $1^{-} ; 1$ state, as the shape of the data at both proton energies favours an assignment of $4^{-}$. If the preferred scheme is correct, then this state would be the $4_{2}^{-} ; 0$ state, although calculations based on this assumption consistently overestimate the magnitude of the cross-section data. The alternative choice, making this the $4_{1}^{-} ; 0$ state, results in calculations predicting the correct shape and magnitude, for both energies. However, such a choice must be questioned on the basis of the calculated energies. The preferred assignments are not discounted as the magnitude of the 19.20 MeV data is higher than that for the 13.35 MeV data, suggesting there is more strength required for the excitation of the 19.20 MeV state. This is reflected also in the results of the calculations.

The $J^{\pi} ; T$ assignment of the state at $19 \cdot 40 \mathrm{MeV}$ has been determined to be $2^{-} ; 1$ (Jones et al. 1983). In that analysis of the data at 398 MeV , the
correspondence with the $2_{2}^{-} ; 1$ state is favoured, which is consistent with our preferred set of states. However, while confirming the assignment of $2^{-} ; 1$ to this state, the present analysis cannot distinguish between the second or the third $2^{-} ; 1$ state, calculated to lie at 21.41 MeV by the shell model. If it were the second state, this would confirm the choice of the isoscalar transition for the excitation of the 18.35 MeV state. More data at higher momentum transfer may allow a unique conclusion to be drawn.


Fig. 6. Differential cross section for the inelastic scattering of 200 and 398 MeV protons to the 18.35 MeV state in ${ }^{12} \mathrm{C}$. The results of the calculations, assuming that this state is the $2_{2}^{-} ; 0$ state, are given by the solid curves, while the results obtained assuming that this is the $3_{1}^{-} ; 1$ state are shown by the dashed curves.

The analysing powers calculated using the DD and LF forces are displayed in Figs 4 and 5, respectively. The notation is consistent with that in Figs 2 and 3. As from the analyses of the cross section, at 200 MeV , the 13.35 MeV transition analysing power data are best represented by assuming that it is an excitation of the $4^{-} ; 0$ state. Unfortunately, there are just too few data points at 398 MeV to be instructive. The 18.35 MeV excitation analysing power data is best described by the LF calculation and assuming the $2_{3}^{-} ; 0$ assignment. However, the associated cross sections are not very good and this would need the 13.35 MeV state to be $2_{2}^{-} ; 0$. The DD cross section, if we assume the $2_{3}^{-} ; 0$ assignment, is good as is the analysing power at 200 MeV . The 398 MeV analysing powers with any assignment and the DD force do not match the data. The analysing powers from the excitation of the states at $19 \cdot 20$ and $19 \cdot 40 \mathrm{MeV}$ are very small. As with the cross section fits, they discount a $1^{-} ; 1$ assignment.

A key feature in setting our preferred assignment to the levels is the choice of $4_{1}^{-} ; 0$ for the 13.35 MeV state. At 200 and 398 MeV , the inelastic scattering cross
sections for that assignment differed by up to 3 orders of magnitude (smaller) from predictions based upon the alternate $2_{2}^{-} ; 0$ specification. This difference persists at lower energies, down to 120 MeV , for which our prescription scheme for the $N N$ effective interaction remains applicable. The results of calculations made for 160 MeV protons are shown in Fig. 7. In this case the results obtained with the LF and DD forces are shown on the left and right respectively with the calculations made assuming the $4_{1}^{-} ; 0$ and $2_{2}^{-} ; 0$ assignments shown by the solid and dashed curves respectively. The marked differences between the cross section in particular, observed at 200 and 398 MeV , persist.


Fig. 7. Differential cross sections and analysing powers from excitation of the two candidates for the $13 \cdot 35 \mathrm{MeV}$ state by 160 MeV protons. The $4_{1}^{-} ; 0$ excitation result is shown by the solid curves while the $2_{2}^{-} ; 0$ excitation is displayed by the dashed curve.

## 3. Conclusions

The assignments of $4_{1}^{-} ; 0,2_{2}^{-} ; 0,4_{2}^{-} ; 0$ and $2_{2}^{-} ; 1$ for the $13 \cdot 35,18 \cdot 35$, $19 \cdot 20$ and $19 \cdot 40 \mathrm{MeV}$ states in ${ }^{12} \mathrm{C}$, respectively, are the preferred ones assessed by the present analyses of the inelastic proton scattering data to these states. Also, the $3_{1}^{-} ; 1$ state is identified as that at 18.60 MeV . However, there still remains some doubt as the current data do not include values at those momentum transfers at which the diverse calculations show the greatest differences. Such data are required before a conclusive statement concerning the $J^{\pi} ; T$ assignment of these states can be made. We stress that the 13.35 MeV state assignment is crucial since, if new data were to help make the preferred choice more definite, there is a 'shake down' in the assignments possible for the higher excited states. With the data set currently available, we can only suggest that the preferred assignments are correct. An alternative would be to perform much larger shell model calculations than are currently practicable seeking thereby, directly, a more accurate specification of the spectrum of ${ }^{12} \mathrm{C}$.

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