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# Phase Variation Effect on Proton–Nucleus Elastic Scattering

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*Abstract*

Proton–deuteron elastic scattering at intermediate and high energies illustrates the importance of the phase variation of the nucleon–nucleon elastic scattering amplitude as in previous work. Two kinds of phase variation are examined. The first is the usual one which is related to free nucleon–free nucleon collisions as suggested by Franco. The second is assumed to be related to the time ordering of multi-scattering processes. The two kinds play similar roles in improving the results. The contributions of both kinds lead to a good fit to the experimental data at the energies considered. The value of the phase variation parameter in each kind (where they are considered together), which gives a good fit to the experimental data, are approximately the same and of order 4 $(\mathrm{GeV}/c)^{-2}$. Also, with relatively small values of the phase variation parameter, the phase variation effect improves the agreement with the experimental data for the p–$^4$He elastic scattering differential cross section at the minimum region in the energy range 97–393 GeV.

## 1. Introduction

The phase variation of the nucleon–nucleon elastic scattering amplitude with the momentum transfer squared $q^2$ was used earlier to improve agreement of the hadron–nucleus (Bassel and Wilkin 1968; Michael and Wilkin 1969; Lombard and Maillet 1990) and nucleus–nucleus (Franco and Yin 1985, 1986; Zhen *et al.* 1990) calculations with the experimental data, especially with increasing $q^2$. Agreement with the experimental data has been obtained by different workers, at the same and different energies, for different values of the phase parameter $\gamma$ which takes the values 5–16 $(\mathrm{GeV}/c)^{-2}$ with positive and negative sign, where the phase factor in the nucleon–nucleon amplitude is taken as $\mathrm{e}^{-i\gamma q^2/2}$. The value of $\gamma$ at a specific energy must be independent of the target nucleus or colliding nuclei and the difference between the results of positive and negative values can be neglected (Franco and Yin 1985, 1986). At the same time, the study of nucleon–nucleon scattering in the range 100–2080 GeV leads to indistinguishable results with and without the introduction of phase variation (Kundrát *et al.* 1987). Also, using the nucleon–nucleon optical potential at 1 GeV, Ahamed and Alvi (1993) obtained a value of $\gamma{\sim}1$ $(\mathrm{GeV}/c)^{-2}$.

These different results and conclusions mean that the phase variation effect still needs more study and discussion. Therefore, one purpose of this work is to

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discuss the values of $\gamma$, obtained earlier, using the proton–deuteron scattering case at different energies.

Another purpose of this work is to study the origin of the phase variation. The effect of the phase variation phenomenon is interpreted as a result of the change of the difference in the phase between the different multi-scattering terms. This leads to the correct interference resultant between these terms (Franco and Yin 1985, 1986). All that we know about the origin of this phenomenon is that it is related to many-nucleon processes. Some authors (Lombard and Maillet 1990) investigated relating this variation to non-eikonal propagation, but they concluded that this is not true. Therefore, we suggest the effect of the first scattering occurring on the following scattering (i.e. the time-ordering) as a cause of the phase variation. In this case we assume that $\Gamma_i \Gamma_j \neq \Gamma_j \Gamma_i$ where $\Gamma_i$ is the profile function operator in the Glauber (1959) formalism and $\Gamma_i \Gamma_j$ means that the incident particle scatters with the $j$th target nucleon first and then with the $i$th target nucleon. We try to formulate this ordering effect using the phase variation factor for the proton–deutron scattering case in the framework of the Glauber approximation. We begin with this formulation, since the usual formalism of the phase variation effect can be obtained from it. Also, the p–$^4$He elastic scattering differential cross section in the range 97–393 GeV (Bujak *et al.* 1981) is calculated using our assumption on the phase variation origin to improve the Glauber results at the minimum region.

## 2. Proton–Deuteron Elastic Scattering

The proton–deuteron elastic scattering amplitude in the Glauber (1959) approximation is given by

$$F_{\mathrm{d}}(\boldsymbol{q}) = \frac{\mathrm{i}k}{2\pi} \int \int \mathrm{e}^{\mathrm{i}\boldsymbol{q} \cdot \boldsymbol{b}} \phi^*(\boldsymbol{r}) \, \Gamma(\boldsymbol{b}, \boldsymbol{s}) \, \phi(\boldsymbol{r}) \, \mathrm{d}^2\boldsymbol{b} \, \mathrm{d}^3\boldsymbol{r} \,, \tag{1}$$

where $k$ is the incident momentum, $\boldsymbol{q}$ is the momentum transfer vector, $\boldsymbol{b}$ is the impact parameter vector, $\phi(\boldsymbol{r})$ is the deuteron ground state wave function, $\boldsymbol{r}$ is the relative position vector between the two nucleons in the deuteron and $\boldsymbol{s}$ is the projection of $\boldsymbol{r}$ on the impact plane. The profile function $\Gamma(\boldsymbol{b}, \boldsymbol{s})$ of the hadron–deuteron interaction, considering $\Gamma_i \Gamma_j \neq \Gamma_j \Gamma_i$, is written as (Franco 1968)

$$\Gamma(\boldsymbol{b}, \boldsymbol{s}) = \Gamma_1(\boldsymbol{b} - \tfrac{1}{2}\boldsymbol{s}) + \Gamma_2(\boldsymbol{b} + \tfrac{1}{2}\boldsymbol{s}) - \tfrac{1}{2}\Big\{\Gamma_1(\boldsymbol{b} - \tfrac{1}{2}\boldsymbol{s})\Gamma_2(\boldsymbol{b} + \tfrac{1}{2}\boldsymbol{s}) + \Gamma_2(\boldsymbol{b} + \tfrac{1}{2}\boldsymbol{s})\Gamma_1(\boldsymbol{b} - \tfrac{1}{2}\boldsymbol{s})\Big\} . \tag{2}$$

The first two terms represent the single scattering processes, where the incident particle scatters from only one particle in the target deuteron. The last two terms represent the double scattering processes, where there are two possibilities. The first is where the incident hadron interacts with particle 1 and then with particle 2; the second is where the incident hadron interacts with particle 2 and then with particle 1.

The profile function $\Gamma_i(\boldsymbol{b})$ is related to the particle–particle elastic scattering amplitude $f_i(\boldsymbol{q})$ by the Fourier transformation (Glauber 1959)

$$f_i(\boldsymbol{q}) = \frac{ik}{2\pi} \int \mathrm{e}^{\mathrm{i}\boldsymbol{q} \cdot \boldsymbol{b}} \Gamma_i(\boldsymbol{b}) \, \mathrm{d}^2\boldsymbol{b} . \tag{3}$$

The inverse transformation is

$$\Gamma_i(\boldsymbol{b}) = \frac{1}{2\pi \mathrm{i}k} \int \mathrm{e}^{-\mathrm{i}\boldsymbol{q} \cdot \boldsymbol{b}} f_i(\boldsymbol{q}) \, \mathrm{d}^2\boldsymbol{q} . \tag{4}$$

For a more accurate description of the proton–deuteron elastic scattering amplitude, we use the notation $f_i$ for scattering of the incident proton only on the $i$th nucleon and $f_{ij}$ with $i \neq j$ for the scattering of the incident proton on the $i$th nucleon after scattering on the $j$th nucleon, where $i, j$ take the values 1 and 2 in the deuteron case. In general, we consider that

$$f_i(\boldsymbol{q}) \neq f_{ij}(\boldsymbol{q}), \qquad i \neq j; \quad i,\, j = 1,\, 2 . \tag{5}$$

The difference between them is represented by a phase factor in the form $\mathrm{e}^{-\mathrm{i}\gamma_{ij} q^2/2}$, such that

$$f_{ij}(\boldsymbol{q}) = f_i(\boldsymbol{q}) \, \mathrm{e}^{-\mathrm{i}\gamma_{ij} q^2/2} , \tag{6}$$

where $-\gamma_{ij}\, q^2/2$ is considered as a phase shift due to scattering on the $j$th nucleon at first, and this is our representation for the origin of the phase variation.

From equation (1), using (2), (4) and the notation (5), we get

$$\begin{aligned} F_{\mathrm{d}}(\boldsymbol{q}) = \frac{ik}{2\pi} \int \int \mathrm{e}^{\mathrm{i}\boldsymbol{q} \cdot \boldsymbol{b}} |\phi(\boldsymbol{r})|^2 \bigg\{ & \frac{1}{2\pi \mathrm{i}k} \bigg\{ \int \exp[-\mathrm{i}\boldsymbol{q}' \cdot (\boldsymbol{b} - \tfrac{1}{2}\boldsymbol{s})] \, f_1(\boldsymbol{q}') \, \mathrm{d}^2\boldsymbol{q}' \\ & + \int \exp[-\mathrm{i}\boldsymbol{q}' \cdot (\boldsymbol{b} + \tfrac{1}{2}\boldsymbol{s})] \, f_2(\boldsymbol{q}') \, \mathrm{d}^2\boldsymbol{q}' \bigg\} \\ & - \frac{1}{2(2\pi \mathrm{i}k)^2} \bigg\{ \int \exp[-\mathrm{i}\boldsymbol{q}' \cdot (\boldsymbol{b} - \tfrac{1}{2}\boldsymbol{s})] \, f_{12}(\boldsymbol{q}') \, \mathrm{d}^2\boldsymbol{q}' \\ & \times \int \exp[-\mathrm{i}\boldsymbol{q}'' \cdot (\boldsymbol{b} + \tfrac{1}{2}\boldsymbol{s})] \, f_2(\boldsymbol{q}'') \, \mathrm{d}^2\boldsymbol{q}'' \\ & + \int \exp[-\mathrm{i}\boldsymbol{q}' \cdot (\boldsymbol{b} + \tfrac{1}{2}\boldsymbol{s})] \, f_{21}(\boldsymbol{q}') \, \mathrm{d}^2\boldsymbol{q}' \\ & \times \int \exp[-\mathrm{i}\boldsymbol{q}'' \cdot (\boldsymbol{b} - \tfrac{1}{2}\boldsymbol{s})] \, f_1(\boldsymbol{q}'') \mathrm{d}^2\boldsymbol{q}'' \bigg\} \, \mathrm{d}^2\boldsymbol{b} \, \mathrm{d}^3\boldsymbol{r} . \end{aligned} \tag{7}$$

We consider two cases: the first is the usual one without any difference between $f_i(\boldsymbol{q})$ and $f_{ij}(\boldsymbol{q})$, i.e. the case where $\gamma_{ij} = 0$. The second is where $\gamma_{ij} \neq 0$. This case takes into account the time-ordering effect. In single scattering time ordering is meaningless. In each of these two cases we consider a further two cases. The first is if we consider the phase variation of the nucleon–nucleon elastic scattering amplitude $f_i(\boldsymbol{q})$ as a proper property, then its appearance must be in the scattering amplitude of the two free colliding nucleons, i.e. it is not related to the target nucleus as suggested by Franco (Bassel and Wilkin 1968). The other case is when we neglect this kind of phase variation. We represent these latter two cases by the following two nucleon–nucleon elastic scattering amplitudes (Franco and Yin 1985):

$$f_i(\boldsymbol{q}) = \frac{k\sigma_i}{4\pi}(\mathrm{i}+\alpha_i)\mathrm{e}^{-\beta_i^2 q^2/2}\mathrm{e}^{-\mathrm{i}\gamma_i q^2/2}\,, \tag{8}$$

$$f_i(\boldsymbol{q}) = \frac{k\sigma_i}{4\pi}(\mathrm{i}+\alpha_i)\mathrm{e}^{-\beta_i^2 q^2/2}\,, \tag{9}$$

where $\sigma_i$ is the nucleon–nucleon total cross section, $\alpha_i$ is the ratio of real to imaginary parts of the amplitude, $\beta_i$ is the slope parameter and $\gamma_i$ in equation (8) is a phase variation parameter.

In all cases we use the deuteron wave function in the form (Franco and Varma 1977)

$$\phi(\boldsymbol{r}) = \left(\sum_{j=1}^{3} c_j(4\pi d_j)^{-3/2}\exp(-r^2/4d_j)\right)^{\frac{1}{2}}, \tag{10}$$

where

$$c_1 = 0\cdot 34, \qquad c_2 = 0\cdot 58, \qquad c_3 = 0\cdot 08,$$

$$d_1 = 141\cdot 5\ (\mathrm{GeV}/c)^{-2}, \qquad d_2 = 26\cdot 1\ (\mathrm{GeV}/c)^{-2}, \qquad d_3 = 15\cdot 5\ (\mathrm{GeV}/c)^{-2}.$$

Using the relation

$$\delta(\boldsymbol{q}) = \frac{1}{(2\pi)^2}\int \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{b}}\,\mathrm{d}^2\boldsymbol{b}\,, \tag{11}$$

we can write (7) in the form

$$\begin{aligned} F_{\mathrm{d}}(\boldsymbol{q}) &= f_1(\boldsymbol{q})S(\tfrac{1}{2}\boldsymbol{q}) + f_2(\boldsymbol{q})S(-\tfrac{1}{2}\boldsymbol{q}) \\ &\quad + \frac{\mathrm{i}}{2\pi k}\left\{\int S(\boldsymbol{q}'-\tfrac{1}{2}\boldsymbol{q})f_{12}(\boldsymbol{q}')\,f_2(\boldsymbol{q}-\boldsymbol{q}')\,\mathrm{d}^2\boldsymbol{q}'\right. \\ &\quad \left. + \int S(\boldsymbol{q}'-\tfrac{1}{2}\boldsymbol{q})\,f_{21}(\boldsymbol{q}-\boldsymbol{q}')\,f_1(\boldsymbol{q}')\,\mathrm{d}^2\boldsymbol{q}'\right\}, \end{aligned} \tag{12}$$

where

$$S(\boldsymbol{q}) = \int \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}} |\phi(\boldsymbol{r})|^2 \, \mathrm{d}^3\boldsymbol{r} \tag{13}$$

is the deuteron form factor. For the wave function (10), this form factor is

$$S(\boldsymbol{q}) = \sum_{j=1}^{3} c_j \, \mathrm{e}^{-d_j q^2} \, . \tag{14}$$

Therefore, the final form for the nucleon–deuteron elastic scattering amplitude in the general case, where $\gamma_i$ and $\gamma_{ij}$ are not equal to zero, is given by

$$\begin{aligned}
F_{\mathrm{d}}(\boldsymbol{q}) = \sum_{j=1}^{3} \frac{kc_j}{4\pi} \mathrm{e}^{-d_j q^2/4} \Bigg\{ & \sigma_1(\mathrm{i}+\alpha_1)\exp[-(\beta_1^2+\mathrm{i}\gamma_1)q^2/2] \\
& + \sigma_2(\mathrm{i}+\alpha_2)\exp[-(\beta_2^2+\mathrm{i}\gamma_2)q^2/2] \\
& + \frac{\mathrm{i}\sigma_1\sigma_2}{16\pi^2}(\mathrm{i}+\alpha_1)(\mathrm{i}+\alpha_2)\Bigg\{ \exp[-(\beta_2^2+\mathrm{i}\gamma_2)q^2/2] \\
& \times \sum_{r=0}^{\infty} \frac{\pi}{2^{r-1} r\,!} \, \frac{(\beta_2^2+d_j-\mathrm{i}\gamma_2)^{2r} q^{2r}}{[(\beta_1^2+\beta_2^2+2d_j)^2+(\gamma_{12}+\gamma_2)^2]^{(r+1)/2}} \\
& \times \left( \cos\left[(r+1)\tan^{-1}\left(\frac{\gamma_{12}+\gamma_2}{\beta_1^2+\beta_2^2+2d_j}\right)\right] \right. \\
& \left. - \mathrm{i}\sin\left[(r+1)\tan^{-1}\left(\frac{\gamma_{12}+\gamma_2}{\beta_1^2+\beta_2^2+2d_j}\right)\right] \right) \\
& + \exp[-(\beta_2^2+\mathrm{i}\gamma_{21})q^2/2] \sum_{r=0}^{\infty} \frac{\pi}{2^{r-1} r\,!} \\
& \times \frac{(\beta_2^2+d_j-\mathrm{i}\gamma_{21})^{2r} q^{2r}}{[(\beta_1^2+\beta_2^2+2d_j)^2+(\gamma_1+\gamma_{21})^2]^{(r+1)/2}} \\
& \times \left( \cos\left[(r+1)\tan^{-1}\left(\frac{\gamma_1+\gamma_{21}}{\beta_1^2+\beta_2^2+2d_j}\right)\right] \right. \\
& \left. - \mathrm{i}\sin\left[(r+1)\tan^{-1}\left(\frac{\gamma_1+\gamma_{21}}{\beta_1^2+\beta_2^2+2d_j}\right)\right] \right) \Bigg\}\Bigg\} .
\end{aligned} \tag{15}$$

## 3. Results for p–d Scattering and Discussion

Using equation (15), with $\gamma_{ij} = 0$ and $\gamma_i = \gamma \neq 0$, the calculated results for the p–d elastic scattering differential cross section at 1 and $11 \cdot 9$ GeV are presented in the Figs 1*a* and 1*b* respectively. The nucleon–nucleon parameters are given in Table 1. The value 8 $(\mathrm{GeV}/c)^{-2}$ for $\gamma$ gives the best agreement with the experimental data. This value of $\gamma$ can be considered as the mean of those used [$\gamma = 5$, 10 $(\mathrm{GeV}/c)^{-2}$] by Lombard and Maillet (1990) in p–$^4$He elastic scattering at $1 \cdot 05$ GeV. This mean value is used to obtain a good fit with the data for $\alpha$–$^4$He elastic scattering at $5 \cdot 07$ GeV/$c$ (Franco and Yin 1986). We note that Bassel and Wilkin (1968) obtained a good fit to the p–d elastic scattering data at 1 GeV using $\gamma = -12 \cdot 84$ or $-15 \cdot 408$ $(\mathrm{GeV}/c)^{-2}$. These values are compared with $\gamma$ obtained from the nucleus–nucleus case (Franco and Yin 1985, 1986), where the multi-scattering effect is larger than that in the nucleon–nucleus case.

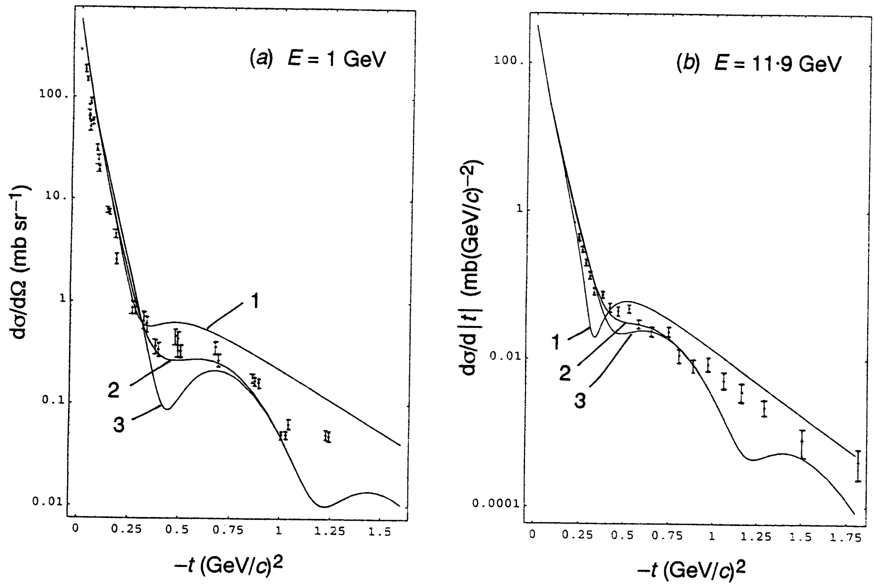


**Fig. 1.** The p–d elastic scattering differential cross section at 1 and $11 \cdot 9$ GeV, where the experimental data are taken from Igo *et al.* (1967), Friedes (1967) and Coleman *et al.* (1968) at 1 GeV and from Bradamante *et al.* (1969, 1970) at $11 \cdot 9$ GeV. Curves 1, 2 and 3 correspond to 0, 8 and $-8$ $(\mathrm{GeV}/c)^{-2}$ for $\gamma_i$, respectively, and $\gamma_{ij} = 0$.

**Table 1. Nucleon–nucleon parameters at 1 and 11 · 9 GeV**

| $E$ (GeV) | $\beta^2_{\mathrm{pp}}$ $(\mathrm{GeV}/c)^{-2}$ | $\beta^2_{\mathrm{pn}}$ $(\mathrm{GeV}/c)^{-2}$ | $\alpha_{\mathrm{pp}}$ | $\alpha_{\mathrm{pn}}$ | $\sigma_{\mathrm{pp}}$ (mb) | $\sigma_{\mathrm{pn}}$ (mb) | Reference |
|---|---|---|---|---|---|---|---|
| $1 \cdot 0$ | $5 \cdot 5985$ | $5 \cdot 5985$ | $-0 \cdot 60$ | $-1 \cdot 200$ | $47 \cdot 50$ | $40 \cdot 4$ | (Fäldt 1970) |
| $11 \cdot 9$ | $8 \cdot 1600$ | $8 \cdot 1600$ | $-0 \cdot 27$ | $-0 \cdot 383$ | $39 \cdot 61$ | $39 \cdot 0$ | (Goggi *et al.* 1979) |

On the other hand, agreement with the experimental data, at $E = 1$ and $11 \cdot 9$ GeV, in the ranges $0 \cdot 2$–1 $(\mathrm{GeV}/c)^2$ and $0 \cdot 45$–1 $(\mathrm{GeV}/c)^2$ of momentum transfer squared, respectively, is obtained with $\gamma = \pm 2$ (GeV/$c$)$^{-2}$; see Figs 2*a* and 2*b*. These $\gamma$ agree with the value obtained by Ahamed and Alvi (1993) using a nucleon–nucleon interaction at 1 GeV, where $\gamma \sim 1$ (GeV/$c$)$^{-2}$.

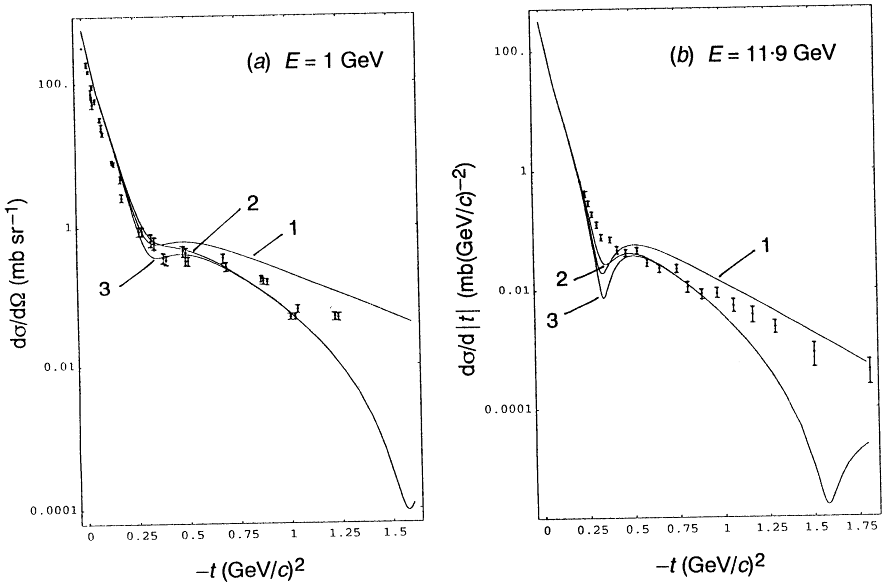


**Fig. 2.** Same as in Fig. 1 except curves 1, 2 and 3 correspond to 0, $-2$ and 2 (GeV/$c$)$^{-2}$ for $\gamma_i$, respectively, and $\gamma_{ij} = 0$.

The difference between the results with positive and negative values is clear only in the region of the first minimum, $q^2 \sim 0 \cdot 2$–$0 \cdot 6$ (GeV/$c$)$^2$. That is because this is an interference region of the single and double scattering terms, where the change of the phase sign leads to a change in the phase differences between the two terms and then to different interference results. However, in the regions where only single or double scattering is dominant, there are not serious differences between the positive and negative sign results. Finally, the value of $\gamma$, from the results, is independent of the energies used.

Thus, what is the correct value of $\gamma$? To answer this question we need to determine the origin of the phase variation of the nucleon–nucleon elastic scattering amplitude. Therefore, we discuss time ordering of multi-scattering processes as a cause of this phase variation. Therefore, we calculated the p–d elastic scattering differential cross section at the previously used energies on the basis of equation (15) with $\gamma_i = 0$ and $\gamma_{ij} \neq 0$. Better agreement is obtained at the two values $E = 1$ and $11 \cdot 9$ GeV with $\gamma_{ij} = -1 \cdot 5$ (GeV/$c$)$^{-2}$; see Figs 3*a* and 3*b*. The results obtained in this case mean that the phase variation of nucleon–nucleon scattering amplitude can come, partially at least, from the time ordering of multi-scattering processes. We note that, in the usual case where $\gamma_i \neq 0$ and $\gamma_{ij} = 0$, we can, taking the phase factor of the single scattering terms as a common factor from all terms, write the p–d elastic scattering amplitude in a form similar (approximately) to this amplitude in the case where $\gamma_i = 0$ and $\gamma_{ij} \neq 0$, and then interpret the similar results in the two cases with, of course, different $\gamma$.

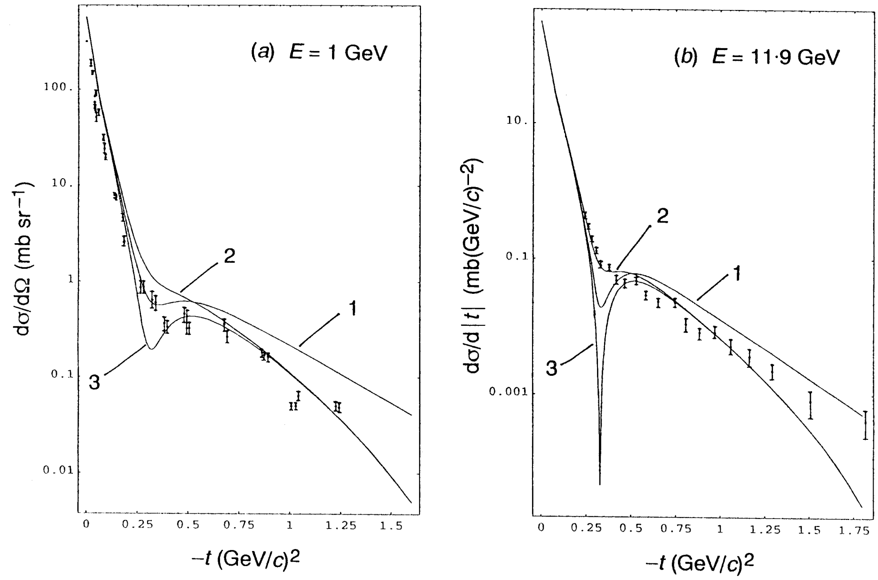


**Fig. 3.** Same as in Fig. 1 except curves 1, 2 and 3 correspond to 0, $-1{\cdot}5$ and $1{\cdot}5$ $(\mathrm{GeV}/c)^{-2}$ for $\gamma_{ij}$, respectively, and $\gamma_i = 0$.

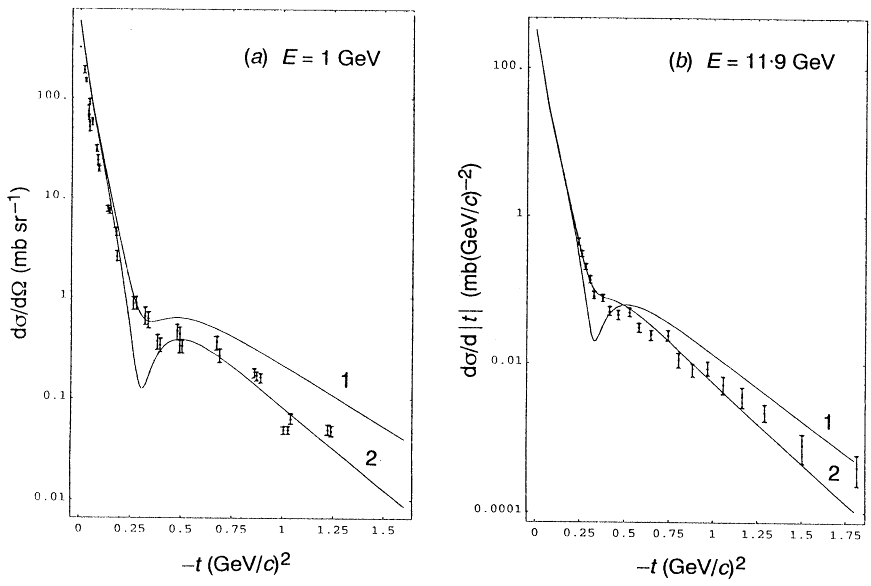


**Fig. 4.** Same as in Fig. 1 except curves 1 and 2 correspond to the values $(0, 0)$ and $(4, 4)$ $(\mathrm{GeV}/c)^{-2}$ of $(\gamma_i, \gamma_{ij})$, respectively.

What happens if we consider the above two cases together, i.e. the case where $\gamma_i \neq 0$ and $\gamma_{ij} \neq 0$? The new representation of the phase variation of the nucleon–nucleon scattering amplitude with two different sources of variation, one the proper nucleon–nucleon interaction and the other the time ordering of multi-scattering processes, gives a good fit at 1 and 11·9 GeV using $(\gamma_i, \gamma_{ij})$ equal to $(4, 4)$ $(\mathrm{GeV}/c)^{-2}$, see Figs 4*a* and 4*b*. We note that a good fit is obtained with the same sign for $\gamma_i$ and $\gamma_{ij}$. If the signs of $\gamma_i$ and $\gamma_{ij}$ are different the discrepancy between the theoretical and experimental data is clear; for example, see Figs 5*a* and 5*b* where $\gamma_i$ and $\gamma_{ij}$ with different sign are used. Also, the $\gamma_i$ and $\gamma_{ij}$, which give a good fit to the data, have approximately the same values at the given energies and, also are approximately independent of energy.

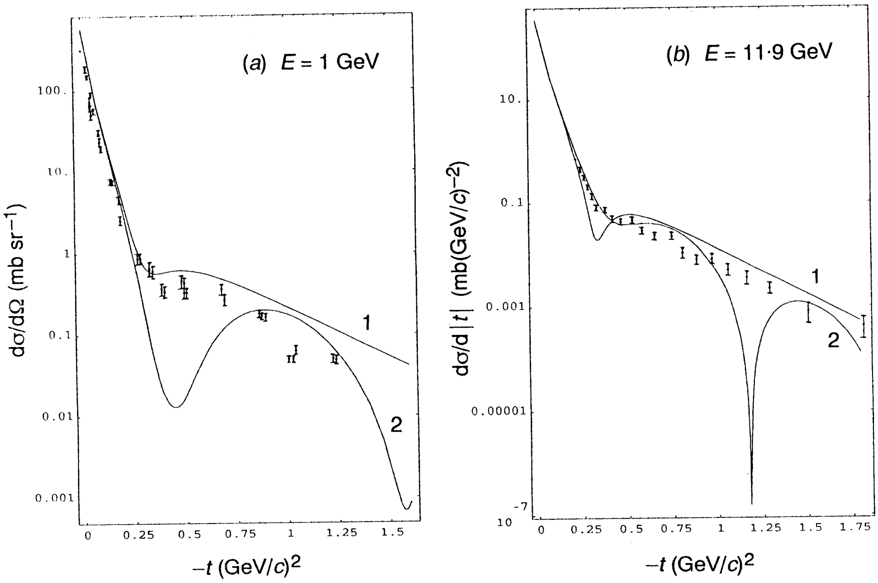


**Fig. 5.** Same as in Fig. 1 except curves 1 and 2 correspond to $(0, 0)$ and $(-2, 4)$ $(\mathrm{GeV}/c)^{-2}$ for 1 GeV and $(0, 0)$ and $(-4, 4)$ $(\mathrm{GeV}/c)^{-2}$ for 11·9 GeV for $(\gamma_i, \gamma_{ij})$ respectively.

In conclusion, our results for p–d elastic scattering ensure the importance of the phase variation of the nucleon–nucleon elastic scattering amplitude as in previous work. Two kinds of phase variation are examined, the usual one which is related to free nucleon–free nucleon collisions and was suggested by Franco (Bassel and Wilkin 1968; Michael and Wilkin 1969), and the second which is our suggestion to interpret the source of the phase variation of the nucleon–nucleon elastic scattering amplitude in nucleon–nucleus and nucleus–nucleus scattering. This kind is assumed to be related to time ordering of multi-scattering processes. We can say that the two kinds play similar roles in improving the results. The contributions of both kinds lead to a good fit with the experimental data at the energies considered. Also, both kinds have similar effects in the forward direction $q = 0$ where they play unimportant roles. The value of the phase variation parameter in each case (where they are considered together), which gives a good fit to the experimental data, is the same of order 4 $(\mathrm{GeV}/c)^{-2}$. Since the phase factor in the nucleon–nucleon elastic scattering amplitude takes the form $\exp(-\mathrm{i}\gamma_i q^2/2 - \mathrm{i}\gamma_{ij} q^2/2)$, then with $\gamma_i = \gamma_{ij} = \gamma$ we get $\exp(-\mathrm{i}\gamma_i q^2/2 - \mathrm{i}\gamma_{ij} q^2/2) = \exp(-\mathrm{i}\gamma q^2)$. Thus, the value $\gamma = 4$ $(\mathrm{GeV}/c)^{-2}$ in this case is consistent with that used by Franco and Yin (1985, 1986) in nucleus–nucleus scattering and by Lombard and Maillet (1990) in nucleon-nucleus scattering, where the phase factor is $e^{i\gamma t/2}$ and $\gamma \not< 10$ $(\mathrm{GeV}/c)^{-2}$. Also, the value $\gamma = 4$ $(\mathrm{GeV}/c)^{-2}$ used by Lombard *et al.* (1991) in proton–nucleus scattering is described by a complex optical potential, which is calculated using the nucleon–nucleon amplitude and the nucleus form factor. Thus, this value is related, in some way, to the target nucleus as is the parameter $\gamma_{ij}$ [=4 $(\mathrm{GeV}/c)^{-2}$] in our calculations.

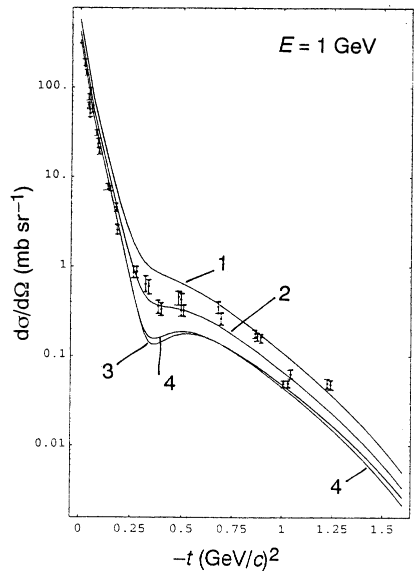


**Fig. 6.** The p–d elastic scattering differential cross section at 1 GeV for different sets of nucleon–nucleon parameters. Curves 1, 2, 3 and 4 correspond to the four sets of parameters in Table 2. The value of $(\gamma_i, \gamma_{ij})$ is $(0, -1\cdot5)$ $(\mathrm{GeV}/c)^{-2}$. The experimental data are from Igo *et al.* (1967), Friedes (1967) and Coleman *et al.* (1968).

Finally, any discrepancy between the data and our results in the forward direction is not related to the phase variation effect. This is due to the minor role of the phase variation in this region and it may be due to some uncertainty in the other nucleon–nucleon parameters, as in the case of 1 GeV, which leads also to a discrepancy at the first minimum (see Fig. 6) where $\gamma_i = 0$ and $\gamma_{ij} = -1\cdot5$ $(\mathrm{GeV}/c)^{-2}$ and different sets of nucleon–nucleon parameters (Table 2) are used. Thus, as is well known, the determination of the nucleon–nucleon parameters is very important. However, this conclusion does not contradict our previous conclusion about the phase variation, because all results using different sets of nucleon–nucleon parameters are consistent in shape, due to the phase variation effect, with the experimental data apart from a small shift up or down.

**Table 2. Sets of nucleon–nucleon parameters at 1 GeV**

| $\beta^2_{\mathrm{pp}}$ $(\mathrm{GeV}/c)^{-2}$ | $\beta^2_{\mathrm{pn}}$ $(\mathrm{GeV}/c)^{-2}$ | $\alpha_{\mathrm{pp}}$ | $\alpha_{\mathrm{pn}}$ | $\sigma_{\mathrm{pp}}$ (mb) | $\sigma_{\mathrm{pn}}$ (mb) | Reference |
|---|---|---|---|---|---|---|
| 5·5985 | 5·5985 | −0·60 | −1·2 | 47·5 | 40·4 | (Fäldt 1970) |
| 5·8807 | 4·9819 | −0·35 | −0·8 | 47·5 | 40·4 | (Goggi *et al.* 1979) |
| 6·1623 | 4·3656 | −0·10 | −0·4 | 47·5 | 40·4 | (Alkhazov *et al.* 1979) |
| 5·4500 | 5·4500 | −0·05 | −0·5 | 47·5 | 40·4 | (Bassel and Wilkin 1968) |

## 4. p–$^4$He Elastic Scattering Amplitude

The p–$^4$He elastic scattering amplitude is given by (Glauber and Matthiae 1970)

$$F(\boldsymbol{q})=\frac{\mathrm{i}k}{2\pi}\int \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{b}}\,\mathrm{d}^2\boldsymbol{b}\int|\psi(\boldsymbol{r}_1,\,\dots,\,\boldsymbol{r}_4)|^2\Gamma(\boldsymbol{b},\,\boldsymbol{s}_1,\,\boldsymbol{s}_2,\,\boldsymbol{s}_3,\,\boldsymbol{s}_4)$$

$$\times\,\delta\bigg(\tfrac{1}{4}\sum_{i=1}^{4}\boldsymbol{r}_i\bigg)\prod_{j=1}^{4}\mathrm{d}\boldsymbol{r}_j\,, \tag{16}$$

where $\psi$ is the ground state wave function of the $^4$He nucleus, $\boldsymbol{r}_i$, $i=1,2,3,4$ are the position vectors of nucleons in the target nucleus and $\Gamma(\boldsymbol{b},\,\boldsymbol{s}_1,\,\boldsymbol{s}_2,\,\boldsymbol{s}_3,\,\boldsymbol{s}_4)$ is the total profile function which describes the proton–$^4$He interaction. If $\Gamma_i\,\Gamma_j\neq\Gamma_j\,\Gamma_i$ this profile function takes the form

$$\Gamma(\boldsymbol{b},\,\boldsymbol{s}_1,\,\boldsymbol{s}_2,\,\boldsymbol{s}_3,\,\boldsymbol{s}_4)=\sum_{i=1}^{4}\Gamma_i(\boldsymbol{b},\,\boldsymbol{s}_i)-\tfrac{1}{2}\sum_{\substack{i,j\\ i\neq j}}\Gamma_i(\boldsymbol{b},\,\boldsymbol{s}_i)\,\Gamma_j(\boldsymbol{b},\,\boldsymbol{s}_j)$$

$$+\tfrac{1}{6}\sum_{\substack{i,j,k\\ i\neq j\neq k}}\Gamma_i(\boldsymbol{b},\,\boldsymbol{s}_i)\,\Gamma_j(\boldsymbol{b},\,\boldsymbol{s}_j)\,\Gamma_k(\boldsymbol{b},\,\boldsymbol{s}_k)$$

$$-\tfrac{1}{24}\sum_{\substack{i,j,k,\ell\\ i\neq j\neq k\neq \ell}}\Gamma_i(\boldsymbol{b},\,\boldsymbol{s}_i)\,\Gamma_j(\boldsymbol{b},\,\boldsymbol{s}_j)\,\Gamma_k(\boldsymbol{b},\,\boldsymbol{s}_k)\,\Gamma_\ell(\boldsymbol{b},\,\boldsymbol{s}_\ell)\,. \tag{17}$$

The ground state wave function of $^4$He is (Sherif 1963)

$$\psi(\boldsymbol{r}_1,\,\boldsymbol{r}_2,\,\boldsymbol{r}_3,\,\boldsymbol{r}_4)=N\exp\bigg(-\alpha^2\sum_{i<j}r_{ij}^2\bigg)\,, \tag{18}$$

where $N$ is the normalisation constant, $\boldsymbol{r}_{ij}=\boldsymbol{r}_i-\boldsymbol{r}_j$ and the nuclear structure parameter $\alpha$ is related to the rms radius $r_{\mathrm{rms}}$ of the $^4$He nucleus by the relation $r_{\mathrm{rms}}=3/8\alpha$, where $\alpha$ takes the value $0\cdot078\ \mathrm{mb}^{-1/2}$ which corresponds to $r_{\mathrm{rms}}=1\cdot52$ fm.

We only consider, for the p–$^4$He case, the effect of the phase variation which comes from time ordering of multi-scattering processes. The nucleon–nucleon scattering amplitudes $f_i(\boldsymbol{q})$ and $f_{ij}(\boldsymbol{q})$ which appear in the single and double scattering terms are defined in Section 2. However, we also used in the triple and quadruple scattering terms the amplitudes

$$f_{ijk}(\boldsymbol{q})=\frac{k\sigma_i}{4\pi}(\mathrm{i}+\alpha_i)\mathrm{e}^{-\beta_i^2q^2/2}\mathrm{e}^{-\mathrm{i}\gamma_{ijk}q^2/2}\,, \tag{19}$$

$$f_{ijk\ell}(\boldsymbol{q})=\frac{k\sigma_i}{4\pi}(\mathrm{i}+\alpha_i)\mathrm{e}^{-\beta_i^2q^2/2}\mathrm{e}^{-\mathrm{i}\gamma_{ijkl}q^2/2}\,, \tag{20}$$

where, for example, $f_{ijk}(\boldsymbol{q})$ is the $i$th nucleon elastic scattering amplitude after scattering on the $k$th and $j$th nucleons. In our calculations we assumed, for simplicity, that $\gamma_{ij} = \gamma_{ijk} = \gamma_{ijkl} = \gamma$, where $\gamma_i$ is taken to be zero. The final form of the p–$^4$He elastic scattering amplitude is very detailed, and is not reproduced here.

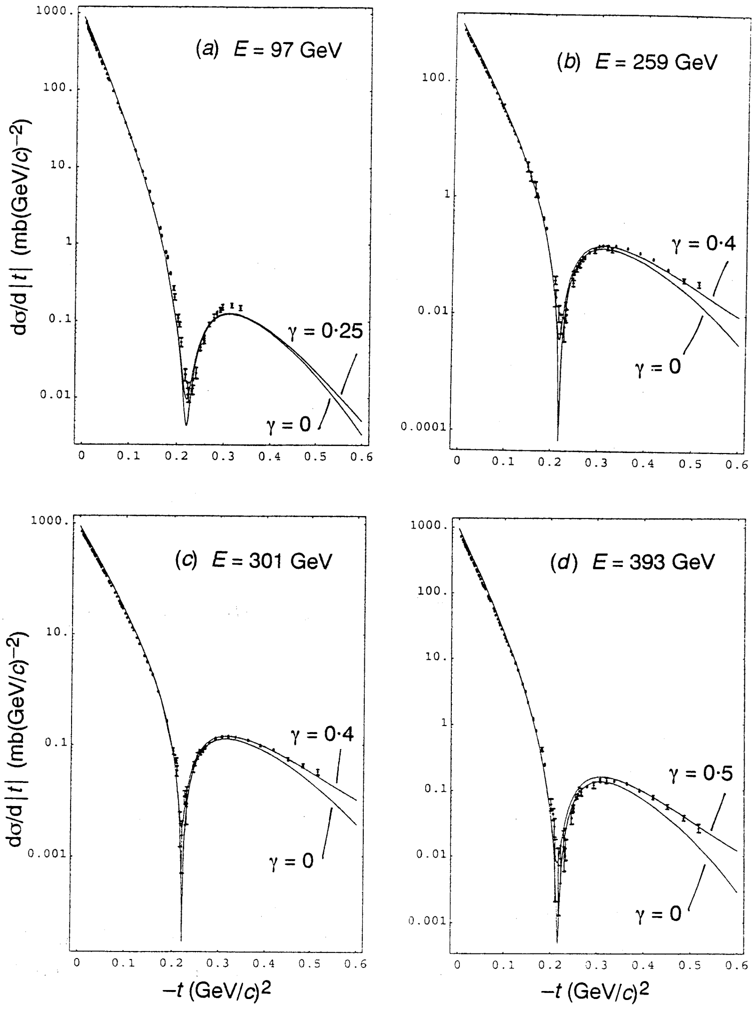


**Fig. 7.** The p–$^4$He elastic scattering differential cross section at energies 97, 259, 301 and 393 GeV, where values of $\gamma_i = 0$ and $\gamma_{ij} = \gamma_{ijk} = \gamma_{ijk\ell} = \gamma$ are given on the curves. The experimental data are taken from Bujak *et al.* (1981).

## 5. Results for p–$^4$He Scattering and Discussion

Under the above assumptions, the p–$^4$He elastic scattering differential cross section was calculated in the range 45–393 GeV where the experimental data were taken from Bujak *et al.* (1981). We attempted to obtain a good fit between our results and the experimental data at the first minimum, where a discrepancy between the usual Glauber calculations and the experimental data is clear. The results are presented at $E = 97$, 259, 301 and 393 GeV in Fig. 7.

The nucleon–nucleon parameters are given in Table 3. It is clear that the phase variations in the representation considered improves the agreement with experimental data. However, to obtain good agreement, $\gamma$ must lie in the range $0\cdot25$–$0\cdot5\,(\mathrm{GeV}/c)^{-2}$. It slowly increases with increasing energy. This value is small compared with the values of $\gamma_{ij}$ used before in the deuteron case in the range 1–12 GeV, where $\gamma_{ij}{\sim}4\,(\mathrm{GeV}/c)^{-2}$. Also, this value of $\gamma_{ij}$ is very small compared with the values of $\gamma_i$ used before in hadron–nucleus scattering (Bassel and Wilkin 1968; Michael and Wilkin 1969; Lombard and Maillet 1990) and nucleus–nucleus scattering (Franco and Yin 1985, 1986; Zhen *et al.* 1990), where $\gamma_i{\sim}5$–$15\,(\mathrm{GeV}/c)^{-2}$.

**Table 3. Nucleon–nucleon parameters at 97, 259, 301 and 393 GeV (Bujak *et al.* 1981)**

| $E$ (GeV) | $\beta_{\rm p}^2$ (fm$^2$) | $\alpha_{\rm p}$ | $\sigma_{\rm p}$ (mb) |
|---|---|---|---|
| 97 | 0·4171 | −0·0702 | 38·57 |
| 259 | 0·4226 | −0·0074 | 39·57 |
| 301 | 0·4053 | −0·0056 | 38·75 |
| 393 | 0·4318 | −0·0223 | 40·00 |

In conclusion, the phase variation of the nucleon–nucleon elastic scattering amplitude, which is used with relatively large values of the phase variation parameter in proton–deuteron elastic scattering at intermediate energies to remove the deep minimum of the theoretical results, can be used in proton–$^4$He elastic scattering with relatively small values of the parameter at higher energies to improve the results at this mimimum. This means that the nucleon–nucleon phase changes rapidly at intermediate energies and slowly at high energies, which is in agreement with the asymptotic case $E\to\infty$, where the nucleon–nucleon amplitude tends to an imaginary quantity (Van Hove 1963), i.e. with a constant phase.

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