10.1071/PVv2015n176p41

Feature Paper

Preface

Phil Schmidt
president@aseg.org.au

Which of the magnetic flux (induction, B) and the magnetic intensity (H) is fundamental and which is derived has been argued about in the literature for some time. Another dilemma is whether magnetisation (M) should take the units of B (Tesla) or H (Amps/metre). These issues are both dealt with in a recent article published by the Institute for Rock Magnetism (University of Minnesota) in the *IRM Quarterly* **24**, 4. The article goes one step further and proposes a new unit for magnetisation: the Néel. This is an important initiative and the article is reprinted, with permission, in this issue of *Preview*.

As there is a very small overlap of readership between the *IRM Quarterly* and *Preview,* reprinting was enthusiastically endorsed by the authors and the IRM, for which they are gratefully acknowledged. For those a bit rusty on B, H and M, a very readable article by Mike Jackson of the IRM can be found in *IRM Quarterly* **18**, 1. If I were to give a course on magnetism I would start with Mike's primer.

The *IRM Quarterly*, which is freely available at the address below, is always full of interesting 'hot off the press' research results, biographies, histories and essays on all kinds of magnetic phenomena: http://www.irm.umn.edu/IRM/quarterly.html

A new basis for the SI system of units: occasion to reconsider the presentation and teaching of magnetism

Frank Stacey
frank.stacey@csiro.au

Frank Stacey[1], Bruce Moskowitz[2], Mike Jackson[2], David Dunlop[3], Özden Özdemir[3] and Subir Banerjee[2]

[1]*CSIRO Energy and Resources, Brisbane, Australia*
frank.stacey@csiro.au
[2]*Institute for Rock Magnetism, University of Minnesota, Minneapolis, USA*
[3]*Geophysics Department of Physics, University of Toronto, Toronto, Ontario, Canada*

The metric system of physical units, now formalised as the SI system (Système International d'Unités) began with the definition of the metre as one ten millionth of the distance from the pole to the equator along the meridian through Paris. Then, with the metre specified, the kilogram was defined as the mass of a cubic decimetre of pure water at 0°C and, with the second established as a specific fraction of the assumed constant duration of the solar day, the MKS (metre-kilogram-second) system became the basis of an international agreement on units. There have been numerous revisions of the system and its definitions, driven by demands for reproducibility and accuracy, incorporation of units for electricity and magnetism and making use of improvements in measurement techniques. For some time there were platinum standards for both the metre and kilogram, but now there is only one remaining material artefact, the standard kilogram kept in Paris. The need to supersede it has been recognised for many years and a change is imminent. A forewarning was recently published by the chairman of the CODATA Task Force on Fundamental Constants (Newell, 2014). It will be more dramatic than the earlier redefinitions of standards. Four fundamental physical constants, Planck's constant, h, Boltzmann's constant, k, the elementary electric charge, e, and Avogadro's number, N_A, will no longer be parameters with measurement uncertainties, but will become constants with defined values. A consequence is that some presently defined constants will be treated as measured parameters with attendant uncertainties. One of them is the permeability of free space, μ_0, presently defined as $4\pi \times 10^{-7}$ H m^{-1}, and we need to consider the implications of this for the magnetism community.

μ_0 is the coefficient relating the magnetic intensity, or flux density B, to the field strength, H, in a vacuum: $B = \mu_0 H$. Historically, the Gaussian electromagnetic system of units was used, with $\mu_0 = 1$ by definition and the numerical values of B and H equal in a vacuum, although their units were recognised

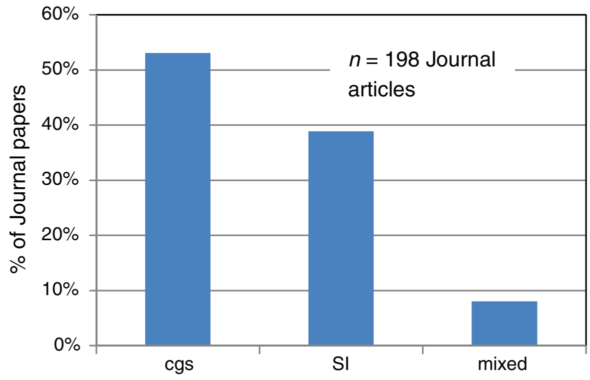

Figure 1. *Magnetism units used in several recent Physics and Engineering Journals. Data collected from the following journals: J. Magnetism and Magnetic Materials (v 384, 15-June-2015; vol. 382, 15-May 2015); Phys. Rev. B (vol. 91, no. 2, 1 Jan 2015, no. 6, 1 Feb 2015), Phys. Rev. B Condensed Matter/ material physics (91, No1 1 Jan 2015), J. Appl. Physics (vol. 117, Issue 17, 07 May 2015), and IEEE Transitions on Magnetics (Jan 2015).*

to be different, B in gauss and H in oersted. The difference becomes obvious when materials are involved and a value of permeability, μ, differing from μ_0 is required. The ratio, μ/μ_0, could be either slightly less than unity (for diamagnetic materials), slightly greater than unity (for paramagnetic materials) or, in the most interesting cases of ferromagnetic materials, much greater. A value for μ, or susceptibility $\chi = (\mu/\mu_0 - 1)$, was an immediately obvious indication of how strongly magnetic a material was. To retain that simple indication without the inconvenient and non-intuitive numerical values of μ in the SI system, some authors (e.g. Harnwell, 1938) wrote permeability as a product ($\mu\,\mu_0$), with μ, the relative permeability, coinciding with the definition of permeability in the Gaussian system. But many practitioners of material magnetism avoid these problems altogether by continuing to use the Gaussian electromagnetic units, oersted and gauss, which remain the practical units of the subject, sometimes with conversion to SI units for political correctness in publication. A brief survey of magnetic units used in 198 peer-reviewed papers in 6 physics and engineering journals published in 2015 shows that Gaussian units are still preferred over SI by magnetists outside the GP community (Figure 1). In addition, Table 1 shows the variety of units used in figures of hysteresis loops (M-H, B-H) within the same group of publications.

The electromagnetic unit system (emu) worked well in the restricted sphere of magnetic and electromagnetic studies, but did not include phenomena involving electric fields, for which a separate system of electrostatic units (esu) was used. The logical advantage of the SI system is that both are combined in a single comprehensive system in which μ_0 and the permittivity of free space, ε_0, are related by $(\mu_0\varepsilon_0) = 1/c^2$, where c is the speed of light, which, in both the present and proposed revised SI systems is a defined constant (as h, k, e and N_A will become in the revised system). This will give the individual parameters, μ_0 and ε_0, anticorrelated observational uncertainties, but for most purposes those uncertainties will be inconsequentially small (0.32 ppb, Newell, 2014). However, the formal uncertainty in μ_0, with the vacuum condition $B = \mu_0 H$, re-opens the contentious debate about the roles of the H and B fields in presentations of the magnetic properties of materials in general and rocks and minerals in particular.

When the rock magnetism community became constrained by the general adoption of SI units for all science, a quasi-political

Table 1. Labelling of hysteresis loop axes from published figures in several recent Physics and Engineering Journals

Magnetization axis Label	Field axis Label
Am^2/kg	T
Am^2/kg	A/m
emu/g	T
emu	T
emu oe/mole	T
A/m	kOe
J(T)	kOe
J(T)	$\mu_0 H$ (T)
$\mu_0 M$ (T)	A/m
moment/μ_B	T
moment/μ_B	Oe
G	kOe
Arbitrary	T, kOe, A/m

μ_0 = permeability of free space; μ_B = Bohr magneton; moment = not specified but presumably in the same units as μ_B; J = magnetic polarization, G(gauss), Oe(oersted), emu (electromagnetic unit), T(tesla), A (ampere)

division developed between H-fundamentalists and B-fundamentalists. To many of us who came into the subject from a Physics base, H is primary and B is a material dependent consequence, but others took an opposite view, treating B as fundamental. A third, agnostic, stance was to argue that, as long as μ_0 was regarded as a fixed constant of nature, with the vacuum relationship $B = \mu_0 H$, there is really no difference between the approaches, but that argument fails with μ_0 relegated to the status of an observed parameter with attendant uncertainty, however small that may be. A historical review of the B and H problem appeared in IRMQ **18**(1) (2008) and now is a good time to revisit it and initiate a discussion that may lead to a resolution of the problem of units applied to the magnetic properties of solids.

The philosophical significance of the change in unit definitions is summarised by Ampere's theorem, one of the fundamental bases of electromagnetism. It considers a loop l enclosing a total current i which is equated to the integral of the magnetic field around the loop

$$\int \boldsymbol{H}.\mathbf{d}\boldsymbol{l} = i \qquad (1)$$

This equation is independent of the medium and variations in it on the path of the integral. In a vacuum it can be rewritten

$$\int \boldsymbol{B}.\mathbf{d}\boldsymbol{l} = \mu_0 i \qquad (2)$$

but if the medium is not a vacuum, then a value of permeability differing from μ_0 is required. The simple case of homogeneous media represented by these equations makes it clear that the current causes H and that B is a consequence that depends on the medium. Eq. (1) is definitive for H, but it has not been used as such, because there is an independent definition of B and with $H = B/\mu_0$, and μ_0 a fixed constant, H could not have an independent definition. Definitions aside, Eq. (1) makes it difficult to avoid fixing the unit of H as A m^{-1} but this is rarely used. In the conventional SI presentation of magnetic properties the inconvenience of this unit, and its awkward conversion to the practical units (oersteds) by the factor $4\pi\times10^{-3}$, has been a stumbling block to recognition that H is a primary field and has contributed to attempts to write it out of magnetism altogether in introductory physics textbooks (e.g. Tippler and Mosca, 2008; Halliday, Resnick and Walker, 2014) and to lose sight of the underlying basic physics. Crangle and Gibbs (1994) have proposed a variation of SI magnetism units that eliminates the usage of the H-field entirely (see Table 2).

B has been defined in terms of the force exerted by a field on an electric current or moving charge. A charge q moving at speed v in a direction perpendicular to B experiences a force F, in a direction perpendicular to the B-v plane, of magnitude given by

$$F = qvB \tag{3}$$

As the defining equation for B, Eq. (3) can be rewritten in an equivalent form in terms of a current instead of moving charge without affecting the definition. This means that the dimensions of B prescribe its unit as newtons per ampere metre (N A^{-1} m^{-1} or kg A^{-1} s^{-2}), and named the tesla. But this is not the conventional interpretation. Rather, the tesla is seen as the unit of magnetic flux density, Wb m^{-2}, with the weber, the unit of flux, being the quantity of fundamental interest. The reason why B appears in Eq. (3) and not H can be seen by considering the force between two currents as the variation in their mutual potential energy with separation. Each current produces a field H, but its potential energy in the field of the other one depends on the magnetic flux crossed as it moves and therefore on a product of H and B fields. We return to this point below, in considering the definition of the ampere.

The conclusion that magnetic energy is a product of H and B is useful to an understanding of the nature of our units problem. To confirm its validity we can check the dimensions of the product $H\times B$, (A m^{-1})×(N A^{-1} m^{-1}) = N m^{-2} or J m^{-3}, that is, energy per unit volume. Conventionally magnetic energy per unit volume has been written as $B^2/2\,\mu_0$, but this is unhelpful to its application to magnetic materials and it is better recognised as $H\times B/2$, with the factor ½ invoking an assumption of linearity in the $B-H$ relationship, that is

Table 2. Comparison of Magnetism Units, Expressions and Values for Different Unit Systems

Symbol	Kennelly	Kennelly (Neel unit)	Sommerfeld	Crangle-Gibbs
	$B = \mu_0 H + M$		$B = \mu_0 (H + M)$	$B = B_0 + \mu_0 M$
H	[A/m]		[A/m]	none
B	[Tesla] ([weber/m²])		[Tesla] ([weber/m²])	[Tesla] ([weber/m²])
B_o	---		---	[T]
μ_0	$4\pi\times10^{-7}$ H/m		$4\pi\times10^{-7}$ H/m	$4\pi\times10^{-7}$ H/m
m(dipole moment)	[Wb m]	[Neel m³]	[A m²] ([J/T])	[J/T]
M(magnetization)	[T] ([Wb/m²])	[Neel]	[A/m]	[J/T m³]
σ (magnetization/mass)	[Wb m/kg]	[Neel m³/kg]	[A m²/kg]	[J/T kg]
χ (by volume)	[T m/A], [H/m] or [Wb/m A]	[Neel m/A]	dimensionless	[J/T²m³] or [m/H]
χ (by mass)	[Wb m²/A kg] or [H m²/kg]	[Neel m⁴/kg A]	[m³/kg]	[J/T² kg]
Saturation Mag. Magnetite (by volume) Magnetite (by mass)	0.6 T (0.6 Wb/m²) 1.15×10^{-4} Wb m/ kg	0.6 Nl 1.15×10^{-4} Nl m³/kg	480 kA/m 92 Am²/kg	480 J/T m³ 92 J/T kg
Dipole moment of Earth	1×10^{17} Wb-m	1×10^{17} Nl m³	8×10^{22} Am²	8×10^{22} J/T
Bohr magneton	1.165×10^{-29} Wb m	1.165×10^{-29} Nl m³	0.927×10^{-23} A m²	0.927×10^{-23} J/T
NRM (basalt, by volume) NRM (limestone)	1.26×10^{-6} T (1.26 μT) 1.26×10^{-10} T (126 pT)	1.26 μNl 126 pNl	1 A/m 10^{-4} A/m	1 J/T m³ 10^{-4} J/T m³
χ_0 (MD magnetite, by vol) χ_0 (basalt)	$3\mu_0$ (3.77×10^{-6} H/m) $10^{-3}\mu_0$ (1.26×10^{-9} H/m)	3.77 μNl m/A 1.26 nNl m/A	3 10^{-3}	$3/\mu_0$ (2.39×10^{6} J/T²m³) $10^{-3}/\mu_0$ (796 J/T²m³)
χ_0 (MD magnetite, by mass)	7.25×10^{-10} Wb m²/A kg	7.25×10^{-10} Nl m⁴/kg A	5.8×10^{-4} m³/kg	$10^{-4}/\mu_0$ (79.6 J/T² kg)
N (demagnetizing factor)	$0\le N\le 1/\mu_0$ [m/H]	$0\le N\le 1/\mu_0$] ([A/Nl m])	$0\le N\le 1$ (dimensionless)	$0\le N\le \mu_0$ [H/m]
μ (permeability)	$\chi+\mu_0$ ([H/m])	$\chi+\mu_0$ [Nl m/A]	$\mu_0(1+\chi)$ [H/m]	$1+\mu_0\chi$ (dimensionless)
Energy of dipole	MH [J/m³]	MH [J/m³]	μ_0MH [J/m³]	MB_o [J/m³]
Demagnetizing Energy	$(1/2\mu_0)NM^2$ [J/m³]	$(1/2\mu_0)NM^2$ [J/m³]	$(\mu_0/2)NM^2$ [J/m³]	$(\mu_0/2)NM^2$ [J/m³]
Néel Relaxation time	$\tau = \frac{1}{f_0}\exp\left[\frac{vM_sH_c}{2kT}\right]$		$\tau = \frac{1}{f_0}\exp\left[\frac{v\mu_0 M_sH_c}{2kT}\right]$	$\tau = \frac{1}{f_0}\exp\left[\frac{vM_sB_c}{2kT}\right]$

Nl (Neel), Wb (weber), T (tesla), A (ampere), H (henry), J (joule), B_0=magnetic induction in free space (=$\mu_0 H$)

$$\text{Magnetic field energy} = \int B.\mathrm{d}H \quad (4)$$

with the assumption $B \propto H$. In dealing with magnetic materials, we need this integral but must abandon the linearity assumption and consider the more general situation of a hysteresis loop. This is a plot of B vs H, so that energy is represented by area in the diagram and the area enclosed by a loop is the energy dissipation per cycle. This basic relationship is lost in the now common, but fundamentally and dimensionally invalid, practice of plotting two different versions of B. The phenomenon of hysteresis introduces an irrefutable argument that, at least in dealing with magnetic materials, H is the primary, causative field. The principle of causality disallows any effect that precedes its cause. As we commonly observe, B lags H. B is not causal but a consequence and the same applies to magnetization, M, which is a contribution to B, additional to $\mu_0 H$, a point that we return to. Table 1 lists the combinations of axis labelling for hysteresis loops found in a survey of recent papers. Confusion reigns!

There is another question arising from the energy argument implied by Eq. (3) that needs to be resolved in selecting units for magnetisation, demagnetising fields and demagnetising factors. The force on a current-carrying conductor depends on B and therefore so does the torque on a current loop. This means that the report by Whitworth and Stopes-Roe (1971), that the torque on a permanent magnet depends on H not B, appears as a paradox. Their magnet was not physically equivalent to a current loop. It means that magnetisation does not respond to a field in the same way as a current loop and must be recognised as a B field, interacting with H of the external field and not as an internal H field interacting with B of the external field. The unit of magnetisation must reflect this, with corresponding demagnetisation factors. It means that the conventional SI presentation (Sommerfeld system) of the relationship between B, H and magnetisation, M, that is

$$B = \mu_0 (H + M) \quad (5)$$

is fundamentally flawed and the system needs to recognise the validity of the Kennelly system in which

$$B = \mu_0 H + M \quad (6)$$

The point is that M is an addition to B and not an addition to H, as implied by Eq. (5). For hysteresis to make sense, a M vs H loop must represent energy, with M having the same dimensions as B. This is recognised in two major books on magnetism (Chikazumi, 1997, and Cullity, 1972[1]), although rather pointedly most of their data are presented in oersteds and gauss anyway. This leads us directly to a suggestion about the units for M. Although it is dimensionally the same as B, it needs its own unit. In recognition of Louis Néel (1904-2000), who was awarded the 1970 Nobel prize in Physics for fundamental contributions to the magnetism of materials, we propose the Neel (Nl) as the unit for M. It is crucial to avoid writing the unit of M as A m^{-1}. Our choice of units and corresponding conversion factors are given in Tables 2 and 3.

Now we face the possibility of a circular argument involving the definition of the ampere, which is specified by the force between a pair of infinitely long parallel currents. If the currents, i, are equal and separated by a distance d then the force between them per unit length is

$$F/\,l = \mu_0\, i^2/\, 2\pi d \quad (7)$$

with μ_0 necessarily involved because this force is the variation with d of the magnetic field energy ($H \times B$). In the revised SI system circularity of the argument will be avoided by referencing everything to fundamental constants, but this means that a dramatically new, simpler system of units could be developed. The revised SI units system will still be a patched up arrangement loaded with historical compromises. We will have 7 fundamental constants, including c, h, k, e, N_A, with values defined by what they happen to be in the existing system. They will each have 8 or more digits with high positive or negative powers of 10. Instead of having fundamental constants that are consequences of history we could produce a new set, redefined from scratch, to yield a system of units that have practical values, perhaps unrelated to existing units, that solve the problem of magnetic units and avoid residual illogicalities. In particular the mass unit, kilogram, is an admission that the primary unit is the gram with the mole in its wake and the prefixes micro-, milli-, mega- etc. thrown out of kilter. If such a new system becomes possible it will a very long term prospect and cannot be seriously addressed here. Our immediate aim is a minimalist resolution of the disruption to magnetism studies that has resulted from introduction of the SI system. We recommend the following:

- Rejection of Eq. (5) in favour of Eq. (6)
- Adoption of the Neel as the unit of magnetisation
- Consistency in plotting hysteresis loops (M-H or B-H) with the x-axis in units of the H-field (in A/m) and the y-axis in units of M (in Neel) or B (in Tesla). This means that measures of coercivity (H_c, H_{cr}, MDF) should be in units of A/m and not T or mT.
- Acceptance of self-demagnetising factors, $N = H_{demag}/M$, with $N_1 + N_2 + N_3 = 1/\mu_0$ for three orthogonal directions.

Table 3. New SI units and their Sommerfeld and cgs equivalents

Symbol	Sommerfeld	Conversion Factor[1]	Kennelly	Cgs unit[2]
	$B = \mu_0 (H + M)$		$B = \mu_0 H + M$	$B = H + 4\pi M$
H	A/m	1	A/m	$4\pi\ 10^{-3}$ Oe
B	Tesla	1	Tesla	10^4 G
m (dipole moment)	A m^2	μ_0	Neel m^3	$10^3/\mu_0$ G cm^3
M (magnetization)	A/m	μ_0	Neel	$10^{-3}/\mu_0$ emu/cm^3
σ (magnetization/mass)	A m^2/kg	μ_0	Neel m^3/kg	$1/\mu_0$ emu/g
χ (by volume)	--	μ_0	Neel m/A	$1/4\pi\mu_0$ emu/cm^3 Oe
χ (by mass)	m^3/kg	μ_0	Neel m^4/kg A	$10^3/4\pi\mu_0$ emu/g Oe

[1] *Multiply a number in Sommerfeld units by conversion factor ($\mu_0 = 4\pi\ 10^{-7}$) to covert to Kennelly units (e.g., 1 Am^2/kg = μ_0 Neel m^3/kg).*
[2] *Cgs unit conversion to Kennelly units (e.g., 1 Neel = ($10^{-3}/\mu_0$) emu/cm^3).*

[1] *It should be noted that in the second edition of Cullity (Cullity and Graham, 2008), the conventional SI system (based equation 5) is used.*

Difficulties in presenting magnetism in the SI system have been aired for many years (Stacey and Banerjee, 1974; Crangle and Gibbs, 1994; Moskowitz, 1995; Dunlop and Özdemir, 1997), but a solution to the problem has not been obvious, or not sufficiently obvious to lead to a generally acceptable resolution. It is essentially a question of units and the planned SI revision makes a revisit opportune. This note aims to provoke a clarifying discussion. To facilitate the process we have set up an online forum, which may be accessed through the IRM web site (www.irm.umn.edu), or directly at https://groups.google.com/a/umn.edu/forum/#!categories/magmeasure-peat/units.

References

Chikazumi, S. 1997, *Physics of Magnetism,* 2nd edition, Volume 94 of International Series of Monographs on Physics, Oxford University Press.

Crangle, J., and Gibbs, M., 1994, Units and unity in magnetism: a call for consistency, Physics World, Nov. 1994.

Cullity, B. D. 1972, *Introduction to Magnetic Materials,* John Wiley & Sons.

Cullity, B. D., and Graham, C. D., (Eds), 2008, *Introduction to Magnetic Materials*, 2nd edition, Wiley IEEE Press.

Dunlop, D. J., and Özdemir, Ö., 1997, *Rock Magnetism: Fundamentals and Frontiers*. Cambridge University Press.

Halliday, D., Resnick, R., and Walker, J. 2014, Fundamentals of Physics, 10th edition, Wiley.

Harnwell, G. P. 1938, *Principles of electricity and electromagnetism*. McGraw-Hill.

Moskowitz, B. M. 1995, Fundamental physical constants and conversion factors. In T. J. Ahrens, ed., *A Handbook of Physical constants 1: Global Earth Physics.* Am. Geophys. Un., pp 346-355.

Newell, D., 2014, A more fundamental international system of units.: *Physics Today*, (July), 35–41. doi:10.1063/PT.3.2448

Stacey, F. D., and Banerjee, S. K. 1974, *The Physical Principles of Rock Magnetism*, Elsevier.

Tippler, P. A., and Mosca, G. 2008, *Physics for Scientists and Engineers, 6th Edition*, W.H Freeman and Company.

Whitworth, R. W., and Stopes-Roe, H. V., 1971, Experimental demonstration that the couple on a bar magnet depends on *H*, not *B*.: *Nature*, **234**, 31. doi:10.1038/234031a0