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Supplementary material

Dead organic matter and the dynamics of carbon and greenhouse gas emissions in frequently burnt savannas

Garry D. Cook[A,D], *C. P.* (*Mick*) *Meyer*[B], *Maëlys Muepu*[C] *and Adam C. Liedloff*[A]

[A]CSIRO Land and Water, PMB 44, Winnellie, NT 0822, Australia.

[B]CSIRO Oceans and Atmosphere, PMB 1, Aspendale, Vic. 3195, Australia.

[C]Ecole Nationale Supérieure de Chimie, Biologie et Physique, Pessac, France (c/o PMB 44, Winnellie, NT 0822, Australia).

[D]Corresponding author. Email: garry.cook@csiro.au

Derivation of Eqn 15

The average value of the fuel load over the interval [0,r] is given by:

$\bar{\Phi} = \frac{1}{r}\int_0^r \Phi(t)\,dt$, therefore:

$$\bar{\Phi} = \frac{1}{r}\int_0^r \Phi_{max} - \left(\Phi_{max} - \Phi(0)\right)e^{-kt}\,dt$$

$$\bar{\Phi} = \frac{1}{r}\left[\int_0^r \Phi_{max}\,dt - \int_0^r \left(\Phi_{max} - \Phi(0)\right)e^{-kt}\,dt\right]$$

$$\bar{\Phi} = \frac{1}{r}\left[\Phi_{max}r - \left(\Phi_{max} - \Phi(0)\right)\int_0^r e^{-kt}\,dt\right]$$

$$\bar{\Phi} = \frac{1}{r}\left(\Phi_{max}r - \left(\Phi_{max} - \Phi(0)\right)\left[\frac{-1}{k}e^{-kt}\right]_0^r\right)$$

$$\bar{\Phi} = \frac{1}{r}\left(\Phi_{max}r + \left(\Phi_{max} - \Phi(0)\right)\left[\frac{1}{k}e^{-kt}\right]_0^r\right)$$

$$\bar{\Phi} = \frac{1}{r}\left(\Phi_{max}r + \left(\Phi_{max} - \Phi(0)\right)\frac{1}{k}\left(e^{-kr} - e^{0}\right)\right)$$

$$\bar{\Phi} = \frac{1}{r}\left(\Phi_{max} r + \left(\Phi_{max} - \Phi(0) \right) \frac{1}{k}\left(e^{-kr} - 1 \right) \right)$$

Since

$$\bar{\Phi} = \frac{\Phi_{max}\left(1 - e^{-kr} \right)}{\left(\frac{1}{\bar{\bar{R}}} - e^{-kr} \right)}$$

from Eqn 10, therefore:

$$\bar{\Phi} = \frac{1}{r}\left(\Phi_{max} r + \Phi_{max}\left(1 - \frac{\left(1 - e^{-kr} \right)}{\left(\frac{1}{\bar{\bar{R}}} - e^{-kr} \right)} \right) \frac{1}{k}\left(e^{-kr} - 1 \right) \right)$$

$$\bar{\Phi} = \frac{1}{r}(\Phi_{max} r + \Phi_{max} \frac{\left(\frac{1}{\bar{\bar{R}}} - e^{-kr} \right) - \left(1 - e^{-kr} \right)}{\left(\frac{1}{\bar{\bar{R}}} - e^{-kr} \right)} \frac{1}{k}\left(e^{-kr} - 1 \right))$$

$$\bar{\Phi} = \frac{1}{r}\left(\Phi_{max} r + \Phi_{max} \frac{\left(\frac{1}{\bar{\bar{R}}} - 1 \right)}{\left(\frac{1}{\bar{\bar{R}}} - e^{-kr} \right)} \frac{1}{k}\left(e^{-kr} - 1 \right) \right)$$

$$\bar{\Phi} = \frac{1}{r}(\Phi_{max} r + \Phi_{max} * \left(\frac{1}{\bar{\bar{R}}} - 1 \right) \frac{\left(e^{-kr} - 1 \right)}{\left(\frac{1}{\bar{\bar{R}}} - e^{-kr} \right)} * \frac{1}{k})$$

$$\bar{\Phi} = \frac{\Phi_{max}}{r}\left(r + \left(\frac{1}{\bar{\bar{R}}} - 1 \right) * \frac{\left(e^{-kr} - 1 \right)}{\left(\frac{1}{\bar{\bar{R}}} - e^{-kr} \right)} * \frac{1}{k} \right)$$

$$\bar{\Phi} = \frac{L}{kr}\left(r + \left(\frac{1}{\bar{\bar{R}}} - 1 \right) \frac{\left(e^{-kr} - 1 \right)}{\left(\frac{1}{\bar{\bar{R}}} - e^{-kr} \right)} \frac{1}{k} \right)$$

A simplified version is:

$$\bar{\Phi} = \left(\frac{L}{k} \right) \left[1 + \frac{\left(1 - \bar{R} \right)\left(e^{kr} - 1 \right)}{kr\left(\bar{R} - e^{kr} \right)} \right]$$