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VOLUME 18, 2001

An international journal of astronomy and astrophysics

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Publ. Astron. Soc. Aust., 2001, 18, 374–383

Nonlinear Alfvén Waves in Weakly Ionised Dusty Plasmas

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Received 2001 March 14, accepted 2001 September 21

Abstract: The effects of charged dust on the steepening of the fields in nonlinear Alfvén waves in astrophysical weakly ionised plasmas are investigated. It is found that the formation of current singularities in the wave due to nonlinear ambipolar diffusion is strongly modified by the effects of the dust. The basic modes for propagation along the magnetic field in a dusty plasma are highly dispersive and split by the anisotropy of the magnetic field into two modes that are oppositely circularly polarised rather than linearly polarised. The right hand circularly polarised wave experiences a cutoff due to the presence of the dust. We derive nonlinear fluid equations describing the dusty plasma, and make approximations for strong coupling of the dust to the neutrals, and for stationary dust. Numerical solution of the equations shows that a nonlinear wave with sharp current features due to ambipolar diffusion involves a rotation of the wave magnetic field about the direction of propagation, and an oscillation of the field components, due to the mode splitting effects of the dust. This is in contrast to the dust-free case, where the sharp reversal of the transverse magnetic field component occurs in a single plane.

Keywords: plasmas — Sun: atmosphere — stars: atmospheres — ISM: clouds — MHD — waves

1 Introduction

Hydromagnetic waves are thought to play roles in processes in interstellar clouds. Alfvén and magnetoacoustic waves should be a mechanism for the transport and deposition of magnetic energy, and the transport of angular momentum in such regions. There is some possible direct evidence of hydromagnetic fluctuations and waves as the cause of the observed large widths of CO emission lines from molecular clouds (Arons & Max 1975). Balsara (1996) has investigated the damping of hydromagnetic waves in a partially ionised self-gravitating plasma under conditions typical of interstellar clouds. Nonlinear hydromagnetic waves in a partially ionised plasma have been postulated to provide support to a self-gravitating galactic molecular cloud (Pudritz 1990; Gammie & Ostriker 1996).

The weakly ionised plasmas that occur in protostellar disks and in the cores of interstellar molecular clouds generally also have a dust component. The ionisation fraction of molecular clouds is typically only $\approx 10^{-7}$, and the dust may contribute $\approx 1\%$ of the mass of the cloud (Pilipp et al. 1987). The process of ambipolar diffusion in weakly ionised astrophysical plasmas, the diffusion of the magnetic field attached to the ions through the neutral gas, contributes to the extraction of magnetic flux from molecular clouds, and to the damping of hydromagnetic fluctuations. The micron-sized dust particulates that are commonly present in molecular clouds are usually negatively charged due to the attachment of the background plasma electrons on the surface of the dust grains via collisions. The charge of the dust particle, $-Z_d e$, can vary significantly depending on the plasma parameters. For HII regions there can be several hundred electrons per grain, while for HI regions there are only a few per grain (Spitzer 1978). The presence of charged dust grains has been shown to affect the collective processes in so-called dusty plasmas, which are made up of ions, electrons, and charged grains. In the state of equilibrium the electron and ion densities are determined by the neutrality condition which is given by

$$-en_e + en_i - Z_d en_d = 0, \tag{1}$$

where $n_{e,i,d}$ is the number density of plasma electrons (with the charge $-e$), ions (for simplicity, we consider singly charged ions), and dust particles, respectively. The presence of neutral atoms and molecules in such a dusty plasma introduces extra collisional effects such as ambipolar diffusion. Some of the effects of dust in molecular clouds have been investigated, such as the effects on small amplitude magnetohydrodynamic waves (Pilipp et al. 1987; Cramer & Vladimirov 1997) and on shock waves (Pilipp & Hartquist 1994; Wardle 1998). Alfvén wave propagation parallel to the magnetic field in a dusty interstellar cloud was first investigated by Pilipp et al. (1987), and was applied to the problem of wave propagation in the dustless and dusty regions of stellar outflows such as α Ori (Havnes, Hartquist, & Pilipp 1989). It was argued that waves which propagate with no losses out to the zone of dust formation are dissipated rapidly in the dusty region.

It has been shown recently that the ion–neutral drift leading to ambipolar diffusion can lead to the steepening of the magnetic field profile and to the formation of singularities in the current density of hydromagnetic fluctuations and waves (Brandenburg & Zweibel 1994, 1995; Mac Low et al. 1995; Suzuki & Sakai 1996). The process

 10.1071/AS01049 1323-3580/01/04374$05.00

may contribute to the production of the large scale magnetic field in spiral galaxies, via dynamo generation of the fields and subsequent reconnection of the field at the small spatial scales produced by the ambipolar diffusion. The process may also aid the reconnection expected to occur during the star formation process, in neutral sheets formed during gravitational collapse (Chiueh 1998). It may also occur in clumpy molecular clouds as the field lines are dragged about by the turbulent gas, although ambipolar diffusion may reduce turbulent diffusion in a magnetised weakly ionised fluid (Kim 1997). The process of forming sharp magnetic field gradients is one of nonlinear diffusion, in contrast to the expected smoothing out of the field produced by linear ambipolar diffusion.

Suzuki & Sakai (1996) showed that such steepening occurs in a nonlinear Alfvén wave that is plane polarised, by finding a steady state solution for the wave, and also by using a one dimensional simulation to show that a pulse-like perturbation of an Alfvén wave evolves to stable current sheet structures. A two dimensional simulation showed that the current sheets which form from a large amplitude Alfvén wave are unstable to the formation of sharp current filaments and associated density filaments. The periodic density structure observed in the Orion A molecular cloud, oriented roughly perpendicular to the magnetic field, and quasi-periodic filament structures seen in soft X-ray images from the Yohkoh satellite of active regions on the Sun, may be due to this process.

In this paper we consider the effects of the presence of charged dust on the steepening of the field and the formation of current singularities due to nonlinear ambipolar diffusion. Even if the proportion of negative charge on the dust grains compared to that carried by free electrons is quite small (typically $\approx 10^{-4}$ in interstellar clouds), it can have a large effect on hydromagnetic Alfvén waves propagating at frequencies well below the ion–cyclotron frequency (Pilipp et al. 1987; Cramer & Vladimirov 1996; Vladimirov & Cramer 1996). This is because the ion Hall current is not compensated by the electron Hall current, with the result that the basic modes for propagation along the magnetic field are highly dispersive and split by the anisotropy of the magnetic field into two modes that are oppositely circularly polarised rather than linearly polarised. With a negligibly small charge on the dust grains, the waves have the usual shear and compressional Alfvén wave properties, while for a non-zero charge on the grains the left hand circularly polarised wave is better described as a whistler or helicon wave extending to low frequencies, while the right hand circularly polarised wave experiences a cutoff due to the presence of the dust. It is shown here that a nonlinear wave with sharp current features due to the ambipolar diffusion will involve a rotation of the wave magnetic field about the direction of propagation, and an oscillation of the field components, due to the mode splitting effects of the dust. This is in contrast to the dust-free case, where the sharp reversal of the transverse magnetic field component occurs in a single plane. The rotation of the magnetic field induced by charged dust in shock wave fronts has been discussed by Pilipp & Hartquist (1994) and Wardle (1998).

The paper is organised as follows. In Section 2 the basic equations are derived. Two limiting cases for the dynamics of the dust are derived. Section 3 discusses the case of dust strongly coupled to the neutral gas, including steady state wave solutions showing the effects of the dust. Section 4 discusses the case of stationary dust. Section 5 considers nonlinear periodic waves, and Sections 6 and 7 considers the time evolution of current sheets from velocity shear structures and pulses. In section 8 we make conclusions from the results found.

2 Model and Wave Equations

We consider waves in a uniform molecular cloud consisting of neutral atomic and molecular species, the ionised atomic and molecular species, the electrons, and negatively charged dust grains. For simplicity a single positively ionised ion species and a single neutral species are assumed, so that a 4-fluid model of the plasma can be used, which employs the fluid momentum equations for plasma ions (i) (singly charged), neutral molecules (n), charged dust grains (d) and electrons (e):

$$\rho_i\left(\frac{\partial \mathbf{v}_i}{\partial t}+\mathbf{v}_i\cdot\nabla\mathbf{v}_i\right)=-\nabla p_i+n_i e\left(\mathbf{E}+\mathbf{v}_i\times\mathbf{B}\right) - \rho_i\nu_{in}(\mathbf{v}_i-\mathbf{v}_n)-\rho_i\nu_{id}(\mathbf{v}_i-\mathbf{v}_d), \tag{2}$$

$$\rho_n\left(\frac{\partial \mathbf{v}_n}{\partial t}+\mathbf{v}_n\cdot\nabla\mathbf{v}_n\right)=-\nabla p_n-\rho_n\nu_{ni}(\mathbf{v}_n-\mathbf{v}_i) - \rho_n\nu_{nd}(\mathbf{v}_n-\mathbf{v}_d), \tag{3}$$

$$\rho_d\left(\frac{\partial \mathbf{v}_d}{\partial t}+\mathbf{v}_d\cdot\nabla\mathbf{v}_d\right)=-Z_d n_d e\left(\mathbf{E}+\mathbf{v}_d\times\mathbf{B}\right) - \rho_d\nu_{dn}(\mathbf{v}_d-\mathbf{v}_n)-\rho_d\nu_{di}(\mathbf{v}_d-\mathbf{v}_i), \tag{4}$$

$$0=-\,n_e e(\mathbf{E}+\mathbf{v}_e\times\mathbf{B})-\rho_e\nu_{en}(\mathbf{v}_e-\mathbf{v}_n) - \rho_e\nu_{ed}(\mathbf{v}_e-\mathbf{v}_d)-\rho_e\nu_{ei}(\mathbf{v}_e-\mathbf{v}_i), \tag{5}$$

where $\mathbf{E}$ is the wave electric field, $\rho_s = n_s m_s$ is the species mass density, where m_s is the species mass, ρ_s is the species mass density, $\mathbf{v}_s$ is the species velocity in the wave, p_i and p_n are the ion thermal and neutral thermal pressures, and ν_{st} is the collision frequency of a particle of species s with the particles of species t. Here $s, t = i, n, d, e$. We have neglected electron inertia and the thermal pressure gradient in (5), momentum transfer to ions from electrons in (2) and to dust grains from electrons in (4), and the dust thermal pressure gradient in (4).

The degree of ionisation, as measured by n_i/n_n, is assumed fixed, where n_n is the number density of neutrals. The parameter $\delta = n_e/n_i < 1$ measures the charge imbalance of the electrons and ions in the plasma, with the remainder of the negative charge residing on the dust particles, so that the total system is charge neutral according to (1). The analysis is valid for any size of δ, but a typical value of δ for molecular clouds is $\delta = 1-10^{-4}$. The charge on each dust grain is assumed constant, and for simplicity we also assume that δ is constant, even though n_n and thus n_i are variable. The neutral mass density obeys the continuity equation

$$\frac{\partial \rho_n}{\partial t} + \nabla \cdot (\rho_n \mathbf{v}_n) = 0. \tag{6}$$

To complete the system of equations, Maxwell's equations ignoring the displacement current are used, with the conduction current density given by

$$\mathbf{j} = e(n_i \mathbf{v}_i - n_e \mathbf{v}_e - n_d Z_d \mathbf{v}_d). \tag{7}$$

where equilibrium charge neutrality is expressed by (1).

Equations (5) and (7) lead to the following generalised Ohm's law:

$$\begin{aligned}\mathbf{E} &+ \left(\frac{\mathbf{v}_i}{\delta} - \frac{(1-\delta)}{\delta}\mathbf{v}_d\right) \times \mathbf{B} \\ &= \frac{\mathbf{j} \times \mathbf{B}}{n_e e} - \frac{m_e}{e}\nu_{en}(\mathbf{v}_e - \mathbf{v}_n) - \frac{m_e}{e}\nu_{ed}(\mathbf{v}_e - \mathbf{v}_d) \\ &\quad - \frac{m_e}{e}\nu_{ei}(\mathbf{v}_e - \mathbf{v}_i). \end{aligned} \tag{8}$$

The expression for $\mathbf{E}$ obtained from (8) can now be substituted into the ion equation (2). We neglect the contribution of the electron collisional momentum transfer terms in (8), compared to the ion momentum transfer terms (i.e. we are neglecting resistivity), so that we may write

$$\mathbf{E} + \mathbf{v}_i \times \mathbf{B} = -\frac{(1-\delta)}{\delta}(\mathbf{v}_i - \mathbf{v}_d) \times \mathbf{B} + \frac{\mathbf{j} \times \mathbf{B}}{n_e e}. \tag{9}$$

We now use the strong ion coupling approximation (Suzuki & Sakai 1996), whereby the ion inertia term (the left hand side) and the ion thermal pressure term are neglected in (2), leaving a balance between the remaining terms. This is equivalent to assuming that $\omega \ll \nu_{in}$. At this point it is useful to normalise the magnetic field by a reference field B_0, and define the Alfvén speed based on the field B_0 and the ion density as $v_A = B_0/(\mu_0 \rho_i)^{1/2}$. Equation (2) may then be written, using (9) and Faraday's law neglecting the displacement current,

$$\begin{aligned}\frac{v_A^2}{\delta}(\nabla \times \mathbf{B}) \times \mathbf{B} &= \Omega_m(\mathbf{v}_i - \mathbf{v}_d) \times \mathbf{B} + \nu_{in}(\mathbf{v}_i - \mathbf{v}_n) \\ &\quad + \nu_{id}(\mathbf{v}_i - \mathbf{v}_d),\end{aligned} \tag{10}$$

where $\Omega_m = \Omega_i(1-\delta)/\delta$ and Ω_i is the ion cyclotron frequency, $\Omega_i = B_0 e/m_i$. The frequency Ω_m is a cutoff frequency for right hand polarised waves in a collision-free dusty plasma (Vladimirov & Cramer 1996). The presence of dust introduces the first and third terms on the RHS of (10). In the absence of dust, with $\delta = 1$, $\Omega_m = 0$, and $\nu_{id} = 0$, (10) may be solved to give the relative drift velocity of ions and neutrals,

$$\mathbf{V}_i = \mathbf{v}_i - \mathbf{v}_n = \frac{v_A^2}{\nu_{in}}(\nabla \times \mathbf{B}) \times \mathbf{B}. \tag{11}$$

This is the strong coupling expression for the relative ion drift velocity used by Suzuki & Sakai (1996).

The dust equation of motion (4) becomes

$$\begin{aligned}\frac{\partial \mathbf{v}_d}{\partial t} + \mathbf{v}_d \cdot \nabla \mathbf{v}_d &= \frac{\Omega_d}{\Omega_i}\frac{v_A^2}{\delta}(\nabla \times \mathbf{B}) \times \mathbf{B} + \frac{\Omega_d}{\delta}(\mathbf{v}_i - \mathbf{v}_d) \\ &\quad \times \mathbf{B} - \nu_{dn}(\mathbf{v}_d - \mathbf{v}_n) + \nu_{di}(\mathbf{v}_i - \mathbf{v}_d),\end{aligned} \tag{12}$$

where $\Omega_d = Z_d B_0 e/m_d$ is the dust grain cyclotron frequency. For the dust grains typical of molecular clouds, $\Omega_d \approx 10^{-9}\Omega_i$.

Inspection of (12) shows that the acceleration of the dust due to the Lorentz force is proportional to Ω_d/Ω_i and is thus very small compared to that of the ions. To simplify the analysis we consider two limiting cases for the dust fluid velocity $\mathbf{v}_d$: (a) the dust grains and neutral gas are strongly coupled, such that the inertia of the dust is neglected, and (b) stationary dust grains, $\mathbf{v}_d = 0$. The first case holds approximately if the neutral gas density is high enough that $\nu_{dn} > \omega$, where ω is the characteristic frequency of the waves considered, which we assume here to be higher than the dust cyclotron frequency. For a cloud with $n_n = 10^4\ \mathrm{cm}^{-3}$, $\nu_{dn} \approx 0.4\Omega_d$ (Pilipp et al. 1987), so that strong dust–neutral coupling occurs for higher neutral densities. The second case occurs for high frequencies and lower neutral densities, such that $\omega > (\Omega_d, \nu_{dn})$, so that the high dust inertia and small dust collisional coupling implies that the dust is stationary on the time scale of interest. We proceed to discuss the two cases in detail.

3 Strong Dust–Neutral Coupling

For strong coupling of the dust with the neutral gas, we neglect the inertia of the dust, just as we neglect the ion inertia for strong ion coupling, so the LHS of (12) is zero. Defining the dust fluid velocity relative to the neutrals as

$$\mathbf{V}_d = \mathbf{v}_d - \mathbf{v}_n, \tag{13}$$

equations (10) and (12) can be written, neglecting the ion–dust momentum transfer terms, as

$$\frac{v_A^2}{\delta}(\nabla \times \mathbf{B}) \times \mathbf{B} = \Omega_m(\mathbf{V}_i - \mathbf{V}_d) \times \mathbf{B} + \nu_{in}\mathbf{V}_i, \tag{14}$$

$$\frac{\Omega_d}{\Omega_i}\frac{v_A^2}{\delta}(\nabla \times \mathbf{B}) \times \mathbf{B} = -\frac{\Omega_d}{\delta}(\mathbf{V}_i - \mathbf{V}_d) \times \mathbf{B} + \nu_{dn}\mathbf{V}_d. \tag{15}$$

These equations can be solved for $\mathbf{V}_i$ and $\mathbf{V}_d$, yielding

$$\begin{aligned}\mathbf{V}_i &= F\frac{v_A^2}{\nu_{in}\delta}[H(\nabla \times \mathbf{B}) \times \mathbf{B} \\ &\quad - (R/\rho)((\nabla \times \mathbf{B}) \times \mathbf{B}) \times \mathbf{B}],\end{aligned} \tag{16}$$

$$\mathbf{V}_d = \frac{\beta_d}{\delta\Omega_m}\left[v_A^2(\nabla \times \mathbf{B}) \times \mathbf{B} - \nu_{in}\mathbf{V}_i\right], \tag{17}$$

where $R = \Omega_m \rho_n/\nu_{in}\rho_0$, $B^2 = B_x^2 + B_y^2 + B_z^2$, $\beta_d = \Omega_d/\nu_{dn}$ is the dust Hall parameter (Wardle & Ng 1999),

$$F = \left[1 + \frac{R^2 B^2}{\rho^2}\left(1 + \frac{\beta_d \rho}{R}\right)^2\right]^{-1}, \tag{18}$$

and

$$H = \left[1 + \beta_d \frac{\nu_{dn}}{\nu_{in}} B^2 \left(1 + \frac{\beta_d \rho}{R}\right)\right]. \tag{19}$$

We have normalised the density by ρ_0, defining $\rho = \rho_n/\rho_0$. Note that the limit $\beta_d \to 0$ corresponds to very strong coupling of the dust to the neutrals, with $\mathbf{v}_d = \mathbf{v}_n$.

The inertia of the wave motion is provided predominantly by the neutrals. Summing the momentum conservation equations (2), (3), and (4) of the ions, neutrals, and dust, the equation of motion for the neutral velocity is obtained:

$$\rho_n \left(\frac{\partial \mathbf{v}_n}{\partial t} + \mathbf{v}_n \cdot \nabla \mathbf{v}_n\right) = -\nabla p_n + \frac{1}{\mu_0}(\nabla \times \mathbf{B}) \times \mathbf{B}. \tag{20}$$

The velocity is normalised by the Alfvén velocity based on ρ_0, $V_A = B_0/(\mu_0\rho_0)^{1/2}$, $\mathbf{v} = \mathbf{v}_n/V_A$, space and time are normalised by L_0 and $\tau_A = L_0/V_A$, and pressure is normalised by p_0. The result, in the normalised variables, is

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = -\beta \nabla p + (\nabla \times \mathbf{B}) \times \mathbf{B}, \tag{21}$$

where the plasma $\beta = 2\mu_0 p_0/B_0^2$.

The magnetic induction equation, using (8) with the collisional electron momentum transfer terms neglected, and neglecting the Hall term, gives

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[\frac{1}{\delta}(\mathbf{v}_i - (1-\delta)\mathbf{v}_d) \times \mathbf{B}\right]. \tag{22}$$

In the strong dust coupling limit, (22) becomes

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}_n \times \mathbf{B}) + \nabla \times \left[\frac{1}{\delta}(\mathbf{V}_i - (1-\delta)\mathbf{V}_d) \times \mathbf{B}\right]. \tag{23}$$

Substituting (16), and using the normalisations defined above, we obtain

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + A_D \nabla \times \left[\frac{F}{\rho}(H((\nabla \times \mathbf{B}) \times \mathbf{B}) \times \mathbf{B} + (R/\rho) B^2 (\nabla \times \mathbf{B}) \times \mathbf{B})\right], \tag{24}$$

where $B = |\mathbf{B}|$ and $A_D = \rho_n/\nu_{in}\tau_A\rho_i$, and we have neglected terms of order (Ω_d/Ω_i).

We consider the dispersion relation of linear waves for very strong dust coupling ($\beta_d \to 0$), with the waves propagating parallel to a steady magnetic field B_z in the z direction, with frequency ω and wavenumber k. This can be derived directly from equations (21) and (24), and also follows from the general dispersion relation for linear waves in a dusty partially ionised gas in a magnetic field derived by Cramer & Vladimirov (1997), in the limit $\nu_{dn} \gg \omega \gg \Omega_d$, $\omega > \nu_{ni}$ and $\Omega_m < \nu_{in}$. The result is

$$\omega^2 = k^2 v_{An}^2 \left(1 - i\frac{\omega}{\nu_{ni}} \pm \frac{\omega\Omega_m}{\nu_{ni}\nu_{in}}\right) = k^2 v_{An}^2 \left(1 - i A_D \omega \pm \frac{R}{\rho} A_D \omega\right), \tag{25}$$

where v_{An} is the Alfvén speed based on the magnetic field B_z and density ρ_n. The ($\pm$) sign corresponds to right (left) hand circular polarisation of the wave magnetic field, which is transverse to the steady magnetic field.

The dispersion relation (25) is a generalisation of the dispersion relation derived by Suzuki & Sakai (1996) to include dust, and shows the anisotropic effects of the dust in the different frequencies of the left and right hand circularly polarised modes for a given wavenumber. The dispersion and anisotropy of the modes are shown in Figure 1, where the frequency and damping rate obtained from (25) are plotted against the wavenumber, for three values of the dust parameter R ($R = 0$, 0.1, and 1). The ambipolar diffusion parameter A_D is held fixed at 0.5 in each case. It is seen that a single frequency exists for the two modes of polarisation in the absence of dust, and a linearly polarised Alfvén wave exists which is dispersive at higher frequencies where the damping becomes appreciable. However, in the presence of dust, the splitting of the two oppositely polarised modes, due to the effect of the dust on the ion and electron Hall currents, increases as R increases, and a linearly polarised wave can no longer be constructed. This splitting and dispersion is not, however, as strong as results when all collisions are neglected and the plasma becomes uncoupled from the neutral gas. In that case collisionless plasma theory for a two-ion plasma can be used (Melrose 1986); the right hand mode experiences a cutoff and the left hand mode has a whistler-type

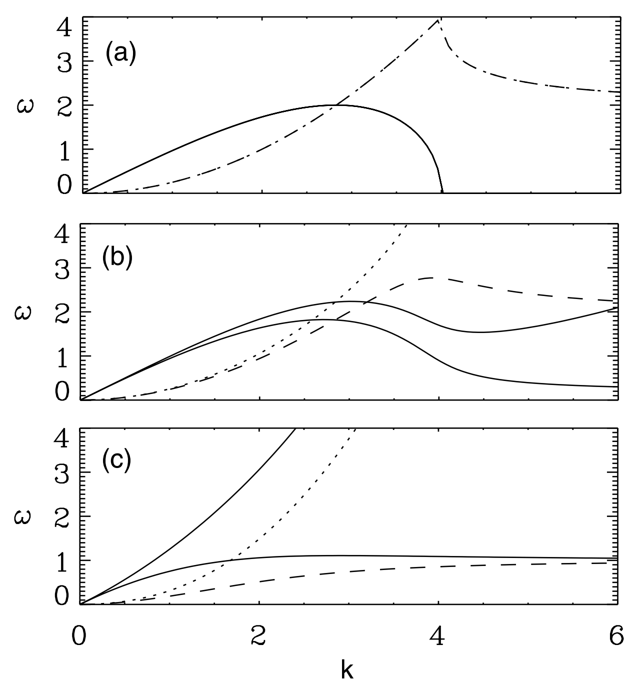

Figure 1 The dispersion relation for linear parallel propagating waves, and very strong coupling of dust. Frequencies and damping rates are plotted against wavenumber k (in normalised units). Here $A_D = 0.5$, and (a) $R = 0$, (b) $R = 0.1$, and (c) $R = 1$. The upper (lower) solid lines are the frequencies of the left (right) hand circularly polarised mode. The dotted (dashed) lines are the damping rates of the left (right) hand polarised mode.

dispersion relation (Vladimirov & Cramer 1996). The dispersion relation for parallel propagating waves, for strong dust coupling but arbitrary β_d, has been considered by Wardle & Ng (1999), who also take into account a spectrum of dust grain sizes. We find in the next sections that the anisotropic effect of the dust on the ion and electron Hall currents leads to a rotation of the magnetic field in nonlinear waves.

4 Stationary Dust

In the other limit of stationary dust ($\mathbf{v}_d = 0$) the strong ion coupling equation (10) gives

$$\mathbf{v}_i = \frac{F}{\nu_i}\left[\frac{v_A^2}{\delta}\left((\nabla\times\mathbf{B})\times\mathbf{B} - (R/\rho)((\nabla\times\mathbf{B})\times\mathbf{B})\times\mathbf{B}\right) - \Omega_m\mathbf{v}_n\times\mathbf{B} + \nu_i\mathbf{v}_n + (\Omega_m^2/\nu_i)(\mathbf{B}\cdot\mathbf{v}_n)\mathbf{B}\right], \tag{26}$$

which gives, on substitution into (3), the following normalised equation of motion for the neutrals (with $\rho = \rho_n/\rho_0$):

$$\begin{aligned}\rho\left(\frac{\partial\mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = {} & -\beta\nabla\rho + F[(\nabla\times\mathbf{B})\times\mathbf{B} \\ & - (R/\rho)((\nabla\times\mathbf{B})\times\mathbf{B})\times\mathbf{B}] \\ & - (RF/A_D)\mathbf{v}\times\mathbf{B} + (F\rho/A_D) \\ & \times[G\mathbf{v} + (R^2/\rho^2)(\mathbf{B}\cdot\mathbf{v})\mathbf{B}],\end{aligned} \tag{27}$$

where $G = 1 - (1 + \nu_{nd}/\nu_{ni})/F$.

The magnetic induction equation becomes, using (8),

$$\begin{aligned}\frac{\partial\mathbf{B}}{\partial t} = {} & \nabla\times\left[\frac{F}{\delta}\left(\mathbf{v}\times\mathbf{B} - (R/\rho)((\mathbf{B}\cdot\mathbf{v})\mathbf{B} - B^2\mathbf{v})\right)\right] \\ & + \frac{A_D}{\delta^2}\nabla\times\left[\frac{F}{\rho}\left(((\nabla\times\mathbf{B})\times\mathbf{B})\times\mathbf{B}\right.\right. \\ & \left.\left. + (R/\rho)B^2(\nabla\times\mathbf{B})\times\mathbf{B}\right)\right],\end{aligned} \tag{28}$$

The greater complexity of equations (27) and (28) compared to (21) and (24) is found to lead to greater anisotropic and dispersive effects on the waves. However, it is interesting to note that the linear dispersion relation, derived from equations (27) and (28), or from the general dispersion relation of Cramer & Vladimirov (1997), still with strong ion coupling but in the limit $\omega \gg \nu_{dn}$, is the same weakly dispersive relation (25) as in the very strongly coupled dust case. Thus strong linear anisotropic splitting and dispersion depends on the removal of the strong ion coupling assumption: the resulting dispersion relations in the absence of collisions and for stationary dust have been discussed by Vladimirov & Cramer (1996).

5 Steady State Solutions

5.1 Strong Dust Coupling

Very strong dust coupling ($\beta_d = 0$) is assumed for simplicity. We seek steady state solutions of (21) and (24), propagating in the z direction with the phase velocity V_0, and steady in the frame of reference with spatial coordinate $\xi = z - V_0 t$. The proportion of negative charge on the dust is typically $\approx 10^{-4}$, so we can set $\delta = 1$ where it appears as a factor in the equations. However, the cut-off frequency Ω_m can be of the order of the ion–neutral collision frequency ν_{in}, so we retain a non-zero value of R.

Equations (6), (21), and (24) become

$$\rho(v_z - V_0) = D, \tag{29}$$

$$\rho(v_z - V_0)\frac{dv_x}{d\xi} = B_z\frac{dB_x}{d\xi}, \tag{30}$$

$$\rho(v_z - V_0)\frac{dv_y}{d\xi} = B_z\frac{dB_y}{d\xi}, \tag{31}$$

$$\rho(v_z - V_0)\frac{dv_z}{d\xi} = -\beta\frac{d\rho}{d\xi} - \frac{1}{2}\frac{d(B_x^2 + B_y^2)}{d\xi}, \tag{32}$$

$$\begin{aligned}-V_0B_x = {} & v_xB_z - v_zB_x + (A_DF/\rho) \\ & \times\left[B_z^2\frac{dB_x}{d\xi} + \frac{B_x}{2}\frac{d(B_x^2 + B_y^2)}{d\xi}\right. \\ & \left. - (R/\rho)B^2B_z\frac{dB_y}{d\xi}\right] + C_x,\end{aligned} \tag{33}$$

$$\begin{aligned}-V_0B_y = {} & v_yB_z - v_zB_y + (A_DF/\rho) \\ & \times\left[B_z^2\frac{dB_y}{d\xi} + \frac{B_y}{2}\frac{d(B_x^2 + B_y^2)}{d\xi}\right. \\ & \left. + (R/\rho)B^2B_z\frac{dB_x}{d\xi}\right] + C_y,\end{aligned} \tag{34}$$

where D, C_x, and C_y are constants.

In the absence of dust ($R = 0$) the field variables v_x and B_x become uncoupled from v_y and B_y. If we choose $v_y = B_y = 0$, equation (33) simplifies to

$$(v_z - V_0)B_x = (A_D/\rho)(B_z^2 + B_x^2)\frac{dB_x}{d\xi} + C_x, \tag{35}$$

which, together with equations (29), (30), and (32) (with $B_y = 0$) were solved numerically by Suzuki & Sakai (1996).

If we set $B_z = 0$ in (35), the solution around $\xi = 0$ is then

$$B_x \sim \xi^{1/3}, \quad v_z \sim \xi^{2/3}. \tag{36}$$

This stationary solution was discussed by Brandenburg & Zweibel (1994). It leads to a singular current density, $J \sim \xi^{-2/3}$.

If $B_z \neq 0$, the solution of (35) in the vicinity of $\xi = 0$ is

$$B_x \sim \xi, \quad v_z \sim \xi^2, \tag{37}$$

so the current density is not singular. However, the length scale for variation of B_x can be small compared to the characteristic length, such as the wavelength of an Alfvén wave considered later, with a relatively large associated current density. A steady state solution of this type is shown by the dotted curves in Figure 2.

In the presence of dust, i.e. $R \neq 0$, if $B_z = 0$ there is no coupling between B_x and B_y and there is no effect of the dust apart from a variation of the factor F in equation (33). The fields again have the variation (36) around $\xi = 0$. If $B_z \neq 0$, there is coupling between B_x and B_y, i.e. a rotation of the magnetic field out of the x–z plane occurs. We have solved equations (29)–(34) for the parameter values $A_D = 0.5$, $V_0 = 0.07$, $\beta = 1$, $B_z = 0.1$, $C_x = 0.04$, and $C_y = 0$. The parameter D was varied, and a shock transition was obtained for $D = -0.066$, with uniform fields upstream and downstream of the transition. The boundary conditions chosen are $B_x = B_y = v_x = v_y = v_z = 0$ at $\xi = 0$. Figure 2 shows the B_x, B_y, v_z, and ρ profiles as functions of ξ for $R = 0$ (dotted curves) and $R = 1$ (solid curves). Small oscillations occur in B_x, v_z, and ρ, but an appreciable B_y is produced close to $\xi = 0$, with $B_y = 0$ upstream and downstream of the transition, i.e. around $\xi = 0$ the magnetic field rotates out of the x–z plane. Otherwise, it is apparent that anisotropic effects are fairly weak in this case.

5.2 Stationary Dust

In the case of stationary dust, the steady state equations derived from (6), (27), and (28) are

$$\rho(v_z - V_0) = D, \tag{38}$$

$$\begin{aligned}\rho(v_z - V_0)\frac{dv_x}{d\xi} &= F\left[B_z\frac{dB_x}{d\xi} - \frac{R}{\rho}\left(B_z^2\frac{dB_y}{d\xi} + \frac{B_y}{2}\frac{d(B_x^2+B_y^2)}{d\xi}\right)\right] \\ &\quad - (RF/A_D)(v_y B_z - v_z B_y) \\ &\quad + (F\rho/A_D)\left[Gv_x + (R^2/\rho^2)(\mathbf{B}\cdot\mathbf{v})B_x\right],\end{aligned} \tag{39}$$

$$\begin{aligned}\rho(v_z - V_0)\frac{dv_y}{d\xi} &= F\left[B_z\frac{dB_y}{d\xi} + \frac{R}{\rho}\left(B_z^2\frac{dB_x}{d\xi} + \frac{B_x}{2}\frac{d(B_x^2+B_y^2)}{d\xi}\right)\right] \\ &\quad - (RF/A_D)(v_z B_x - v_x B_z) \\ &\quad + (F\rho/A_D)\left[Gv_y + (R^2/\rho^2)(\mathbf{B}\cdot\mathbf{v})B_y\right],\end{aligned} \tag{40}$$

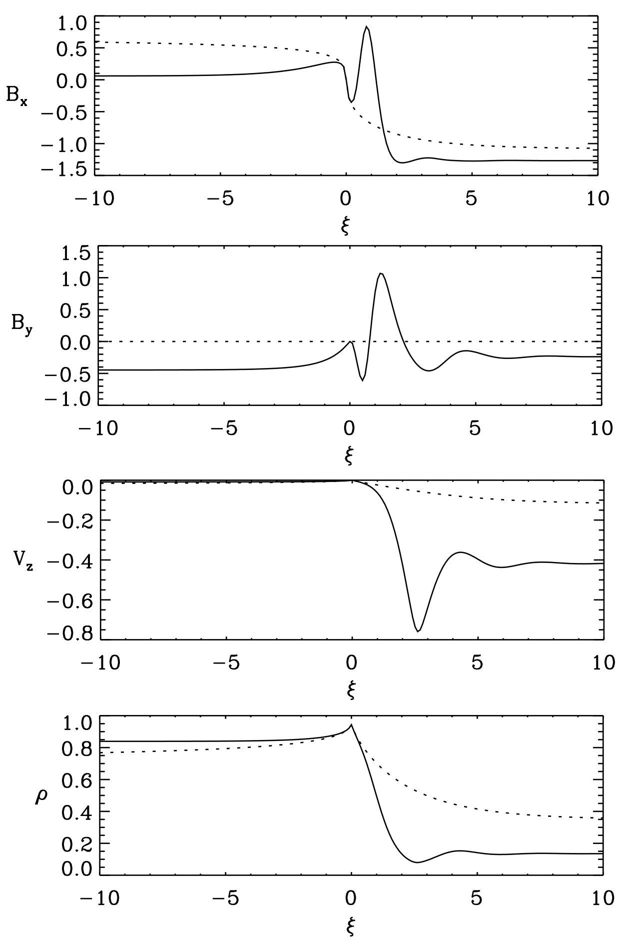

Figure 2 The two transverse components of the magnetic field, B_x and B_y, the longitudinal velocity v_z and the density ρ as a function of position, for a steady structure propagating with a normalised velocity of 0.07. The dotted curves correspond to no dust ($R = 0$) and the solid curves correspond to the presence of dust ($R = 1$). The very strongly coupled dust model is used.

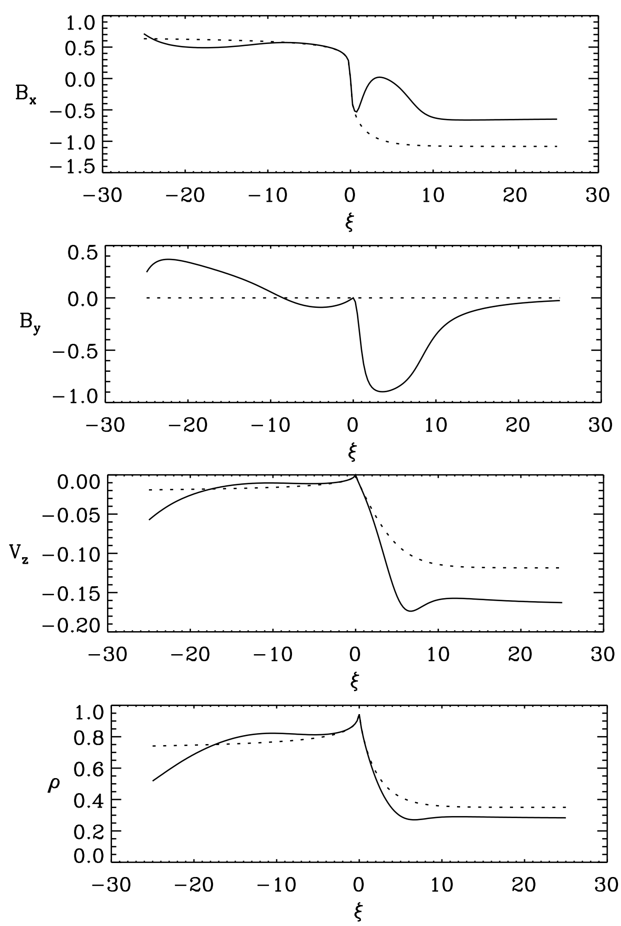

Figure 3 The two transverse components of the magnetic field, B_x and B_y, the longitudinal velocity v_z and the density ρ as a function of position, for a steady structure propagating with a normalised velocity of 0.07. The dotted curves correspond to no dust ($R = 0$) and the solid curves correspond to the presence of dust ($R = 0.1$). The stationary dust model is used.

$$\rho(v_z - V_0)\frac{dv_z}{d\xi} = -\beta\frac{d\rho}{d\xi} - F\left[\frac{1}{2}\frac{d(B_x^2+B_y^2)}{d\xi}\right.$$
$$\left. + (R/\rho)B_z^2\left(B_y\frac{dB_x}{d\xi} - B_x\frac{dB_y}{d\xi}\right)\right]$$
$$- (RF/A_D)(v_xB_y - v_yB_x)$$
$$+ (F\rho/A_D)\left[Gv_z + (R^2/\rho^2)(\mathbf{B}\cdot\mathbf{v})B_z\right], \tag{41}$$

$$-V_0B_x = F(v_xB_z - v_zB_x + (R/\rho)(\mathbf{B}\cdot\mathbf{v})B_y - (R/\rho)B^2v_y)$$
$$+ (A_DF/\rho)\left[B_z^2\frac{dB_x}{d\xi} + \frac{B_x}{2}\frac{d(B_x^2+B_y^2)}{d\xi}\right.$$
$$\left. - (R/\rho)B^2B_z\frac{dB_y}{d\xi}\right] + C_x, \tag{42}$$

$$-V_0B_y = F(v_yB_z - v_zB_y - (R/\rho)(\mathbf{B}\cdot\mathbf{v})B_x + (R/\rho)B^2v_x)$$
$$+ (A_DF/\rho)\left[B_z^2\frac{dB_y}{d\xi} + \frac{B_y}{2}\frac{d(B_x^2+B_y^2)}{d\xi}\right.$$
$$\left. + (R/\rho)B^2B_z\frac{dB_x}{d\xi}\right] + C_y. \tag{43}$$

If $B_z = 0$, in the presence of dust, B_x and B_y are coupled together, i.e. rotation of the magnetic field occurs, and B_x and B_y both have the variation $\sim\xi^{1/3}$, producing singular current densities in the x and y directions. This is in contrast to the strongly coupled dust case, where the magnetic field remains in the x direction. If $B_z \neq 0$, again the current density is non-singular. Equations (38)–(43) have been solved for the same parameters and boundary conditions as in the strong dust coupling case, except for the values of R. Also we take $\nu_{nd} = 0$. Figure 3 shows the profiles for $R = 0$ (dotted curves) and $R = 0.1$ (solid curves). We note that the $R = 0.1$ solution is uniform asymptotically for $\xi > 0$, but is nonuniform for $\xi < 0$. In this case the anisotropic rotational effects of the dust are very strong.

6 Periodic Nonlinear Waves

It was shown by Suzuki & Sakai (1996) that, in the absence of dust, sharp current sheets can be generated by the ambipolar diffusion effect in periodic nonlinear Alfvén waves propagating in one dimension. The waves were assumed to be initially polarised in a single plane. The planar polarisation was preserved in the time evolution of

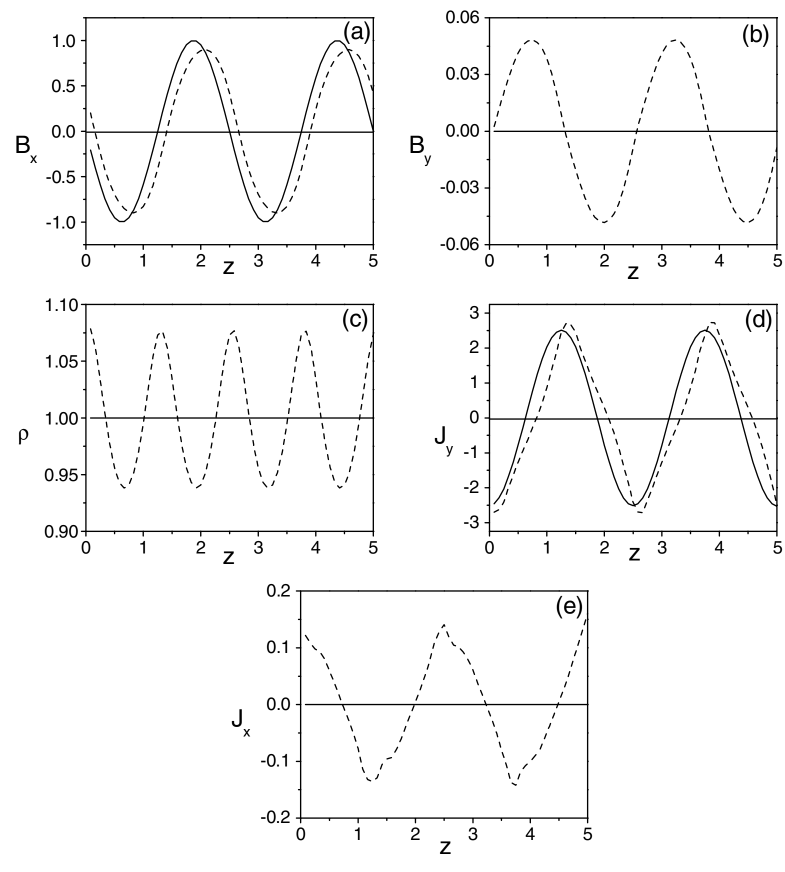

Figure 4 Field profiles for an Alfvén wave that is initially linearly polarised at time $t = 0$, with the transverse magnetic field initially in the x direction (part (a)). Here $B_z = 1$. The solid curves are the initial profiles, and the dotted curves are the profiles after the time $\tau_A/2\pi$. The very strongly coupled dust model is used, with $R = 1$.

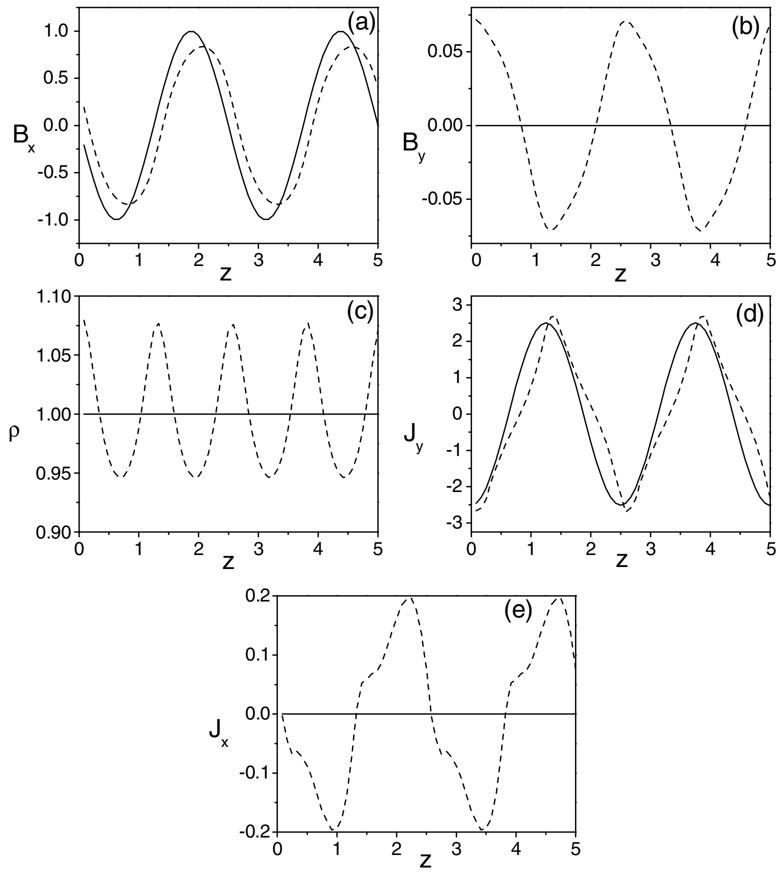

Figure 5 Field profiles for an Alfvén wave that is initially linearly polarised at time $t = 0$, with the transverse magnetic field initially in the x direction (part (a)). Here $B_z = 1$. The solid curves are the initial profiles, and the dotted curves are the profiles after the time $\tau_A/2\pi$. The stationary dust model is used, with $R = 0.1$.

the sharp current sheets in the waves. However, with the inclusion of the charged dust, we expect that the magnetic field will rotate out of the initial plane of polarisation. We investigate here the effect of that rotation on the formation of the current sheets. Figures 4 and 5 (corresponding to Figures 4–7 of Suzuki & Sakai (1996)) show the time evolution of the fields in a large amplitude periodic wave that is initially plane polarised, with magnetic field in the x–z plane. Figure 4 is for the very strongly coupled dust case, while Figure 5 is for the stationary dust case. In each case the magnetic field rotates out of the initial plane of polarisation, i.e. a B_y component is generated as shown in Figures 4(b) and 5(b), due to the presence of the dust. The nonlinear y component of the magnetic field so generated tends to have sharper gradients than the x component in each case, and so produces an appreciable current density in the x direction, as shown in Figures 4(e) and 5(e), even after only a very short time interval. The subsequent evolution of the wave appears to be a breakup of the wave into an irregular field with sharp gradients. This is an example of the evolution of nonlinear field structures with sharp gradients and strong resistive dissipation, induced by the presence of nonlinear ambipolar diffusion. The dust modifies the process by inducing a rotation of the fields.

7 Current Sheet Formation by Transient Localised Flows

In the formation of current sheets by transient gas flows with an initial reverse shear flow, and pulse-like structure, studied by Suzuki & Sakai (1996), the magnetic field preserves a planar polarisation. Again we expect, in the presence of charged dust, that the magnetic field will rotate out of the plane. This is shown in Figure 6 (corresponding to Figures 8–10 of Suzuki & Sakai (1996)), where the initial structure corresponds to a localised shear reversal in the fluid flow direction, and a corresponding reversal in the magnetic field B_x. After a time $\tau_A/4\pi$, the magnetic field has rotated out of the x–z plane, giving rise to an oscillatory B_y component. As well as the strong current sheet that appears in the y direction, there is a component of current J_x associated with the sharp gradients of the B_y component. This is another example of the evolution of structures with sharp gradients due to nonlinear ambipolar diffusion.

8 Conclusions

We have shown that charged dust can strongly influence the steepening of the fields of Alfvén waves in dusty

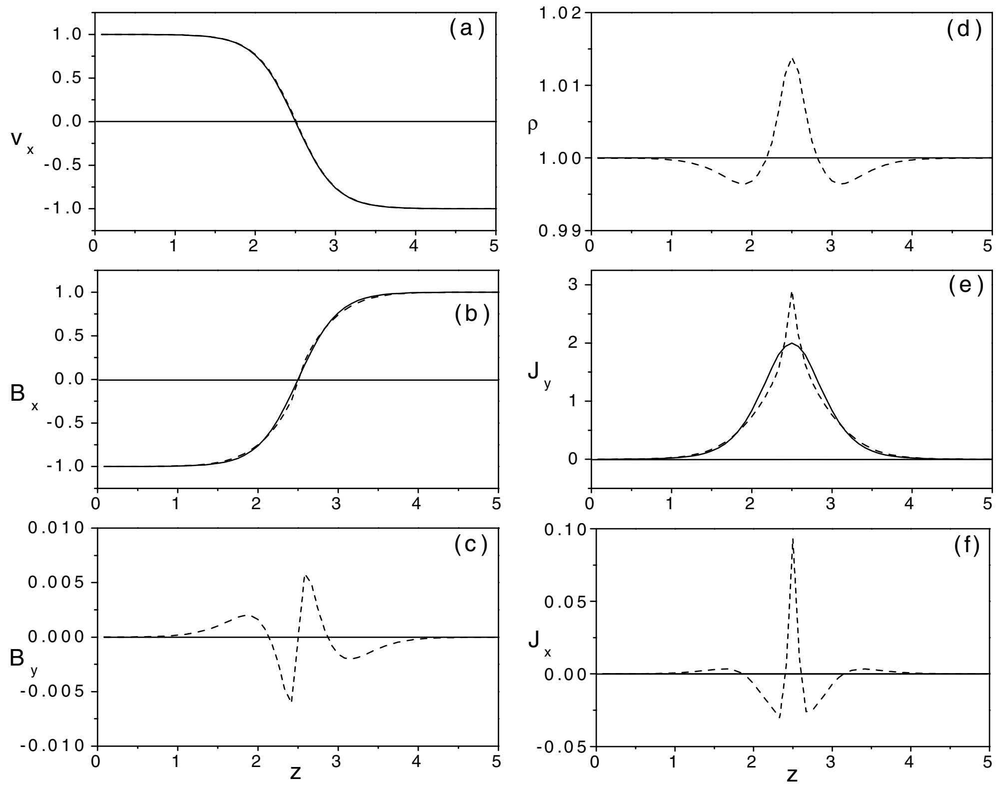

Figure 6 Field profiles for a shear perturbation with the velocity and the transverse magnetic field initially in the x direction (parts (a) and (b)). Here $B_z = 0.1$. The solid curves are the initial profiles, and the dotted curves are the profiles after the time $\tau_A/4\pi$. The very strongly coupled dust model is used, with $R = 1$.

weakly ionised astrophysical plasmas, and affects the formation of current singularities due to nonlinear ambipolar diffusion. The small proportion of negative charge carried on the dust grains can have a large effect on hydromagnetic Alfvén waves, because the basic modes for propagation along the magnetic field are highly dispersive and split by the anisotropy of the magnetic field into two modes that are oppositely circularly polarised rather than linearly polarised. We have related this behaviour to the usual shear and compressional Alfvén wave properties, and have derived nonlinear fluid equations describing the dusty plasma. Approximations for strong coupling of the dust to the neutrals, and for stationary dust, were made. Numerical solution of the equations shows that a nonlinear wave with sharp current features due to the ambipolar diffusion will involve a rotation of the wave magnetic field about the direction of propagation, and an oscillation of the field components, due to the mode splitting effects of the dust, in contrast to the dust-free case, where the sharp reversal of the transverse magnetic field component occurs in a single plane. Steady propagating structures were found, and the evolution of periodic waves was shown to lead to a rotation of the transverse magnetic field as well as sharp current features. Initially pulse-like structures were also shown to lead to a rotation of the magnetic field out of the initial plane of polarisation at the same time as evolving into structures with sharp gradients. These are examples of the evolution of initially smooth field profiles into nonlinear field structures with sharp gradients and potential strong resistive dissipation, induced by the presence of nonlinear ambipolar diffusion. The dust modifies the processes by inducing a rotation of the fields; if periodic current or density filaments can be observed, the signature of the presence of the dust could then be a spiral structure of the filaments, provided rotation of the source of the fields was eliminated as the cause of the spiral structure. An example is low frequency MHD waves emitted into a surrounding plasma by a rotating protostar. In the equatorial plane the waves would be linearly polarised in a dust-free plasma, but because of the dispersion introduced by the charged dust the waves would be circularly polarised.

Since charged dust is thought to be an important component of protostellar clouds and molecular interstellar clouds it is therefore evident that it should be taken into account in calculations of the transport and deposition of wave energy, and the transport of angular momentum, in such regions. The properties of nonlinear Alfvén waves in magnetic flux tubes (Sakai et al. 2000a, b) in the dusty atmospheres of cool stars will also be modified by the effects of the dust. The equations derived here provide a basis for the calculation of further important effects in magnetised dusty plasmas such as magnetic reconnection, filamentation instabilities, and the formation of current filaments (Suzuki & Sakai 1997). Further work could also allow for a spectrum of dust grain sizes and masses.

Acknowledgments

This work was supported by the Australian Research Council.

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