

# THE EXISTENCE AND STABILITY OF SELECTIVELY BALANCED POLYMORPHISM AT A SEX-LINKED LOCUS\*

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## Summary

Necessary and sufficient conditions are given for the existence and stability of a selectively balanced polymorphism at a sex-linked locus. It is shown that selective superiority of the heterozygote is neither necessary nor sufficient and also that the occurrence of heterozygotes with a frequency greater than that of homozygotes does not necessarily indicate that the heterozygote has a selective advantage over both homozygotes. These results are considered in relation to Wallace's published data for populations of *Drosophila pseudoobscura* in which selection is acting on the sex-linked condition "sex ratio".

## I. INTRODUCTION

In a large random mating population and in the absence of selective differences, the genotypes at an autosomal diploid locus with two allelomorphs  $A$  and  $a$  having population frequencies  $p$  and  $q$  respectively, where  $p+q=1$ , have the equilibrium population frequencies  $p^2 AA$ ,  $2pq Aa$ , and  $q^2 aa$ . In this case the frequency of heterozygotes ( $2pq$ ) cannot exceed the total frequency of homozygotes ( $p^2+q^2$ ). Heterozygotes and homozygotes are equally frequent only when  $p=q=0.5$ . When there are genotypic selective differences, the same in the two sexes, a stable balanced polymorphism exists if, and only if, the heterozygote is at a selective advantage over both homozygotes. In this case the equilibrium population frequency of the heterozygote may exceed the total frequency of homozygotes. When there are differences between the sexes in respect of the genotypic selective values, the situation may be much more complex. Owen (1953) has shown, for example, that in this case there may be two distinct stable states of balanced polymorphism. Owen's demonstration emphasizes the necessity of recording observations for males and females separately when studying polymorphic populations. The conditions for stability have been determined recently for polymorphic situations dependent on a number of other more complex genetic systems, such as those involving three or more allelomorphs (Owen 1954; Kimura 1956*a*; Penrose, Smith, and Sprott 1956), a pair of autosomal loci (Kimura 1956*b*), and a sex-linked locus (Bennett 1957). It is with an elaboration of the last case that we are particularly concerned here.

## II. POLYMORPHIC EQUILIBRIUM AND THE CONDITIONS FOR STABILITY

In a large random mating population and in the absence of selective differences, the genotypes at a completely sex-linked locus with two allelomorphs  $A$  and  $a$

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have the equilibrium frequencies  $p^2 AA$ ,  $2pq Aa$ , and  $q^2 aa$  in the homogametic sex (which we shall suppose to be female) and  $p A$ ,  $q a$  in the heterogametic (male) sex, where  $p$  and  $q$  are the population frequencies of the genes  $A$  and  $a$  respectively. It has commonly been assumed that when there are selective differences between the genotypes, the same stability conditions apply to the genotypic selective values for the homogametic sex as to those values for an autosomal locus with no sex difference, i.e. the heterozygote has been assumed to be at a selective advantage over both homozygotes. Da Cunha (1953) has gone on to argue that when the population frequency of heterozygotes is greater than 50 per cent., then the heterozygote may be presumed to be at a selective advantage over both homozygotes. Other authors also (e.g. Birch and Battaglia 1957) have based claims for the selective superiority of heterozygotes for genes or structural arrangements in the *X*-chromosome in *Drosophila* populations upon an examination of data for females only. However, it is not enough to consider population data for just one sex. The frequencies and relative selective values of the different genotypes in both sexes must be taken into account (Bennett 1957).

Suppose that the selective values of the two male genotypes  $A$  and  $a$  are in the ratio of  $t_A : 1$ , where  $t_A = 1 + h_A$ , say, will be taken to be greater than 1, and that the selective values of the female genotypes  $AA$ ,  $Aa$ , and  $aa$  are in the ratio  $S_{AA} : 1 : S_{aa}$ . If  $p_f$  and  $q_f$ , where  $p_f + q_f = 1$ , are the relative frequencies of the genes  $A$  and  $a$  in the gametic output of female members of a given generation, and  $p_m$ ,  $q_m$ , where  $p_m + q_m = 1$ , are the corresponding frequencies for males, then with random mating, the genotypes will appear in the next generation with the following frequencies:

Females	$p_f p_m AA$	$(p_f q_m + q_f p_m) Aa$	$q_f q_m aa$
Males	$p_f A$	$q_f a$	

The relative frequencies of the genotypes in the same generation *after* selection will be as follows:

Females	$S_{AA} p_f p_m AA$	$(p_f q_m + q_f p_m) Aa$	$S_{aa} q_f q_m aa$
Males	$t_A p_f A$	$q_f a$	

It is assumed that these are the frequencies with which the different genotypes contribute to the gametic output. When equilibrium is attained, the gametic output of females and males will show characteristic values for the gene ratio and these values will then be maintained throughout subsequent generations. The equilibrium values of the gene ratios in the gametic output of females and males are given by

$$u_f = \frac{p_f}{q_f} = \frac{2S_{AA}u_m u_f + (u_m + u_f)}{2S_{aa} + (u_m + u_f)},$$

and

$$u_m = \frac{p_m}{q_m} = t_A u_f.$$

It follows that

$$u_f = \frac{1 + t_A - 2S_{aa}}{1 + t_A - 2t_A S_{AA}}, \quad \dots \dots \dots (1)$$

and

$$u_m = t_A u_f. \quad \dots \dots \dots (2)$$

It can be shown that the gene ratios can have stable equilibrium values, other than zero or infinity, if, and only if, both numerator and denominator of the expression on the right hand side of equation (1) are positive, i.e.  $S_{aa}$  must be less than  $1 + \frac{1}{2}h_A$  and  $S_{AA}$  must be less than  $1 - [h_A/2(1 + h_A)]$ . In particular, when  $t_A = 1$ , i.e. when the gene substitution considered has no selective effect in males, the necessary and sufficient condition for the existence and stability of an equilibrium is that the heterozygote should be at a selective advantage over both homozygotes. Such a situation presumably exists only very rarely. Clearly, when  $t_A$  has any value other than unity, it is neither necessary nor sufficient for the heterozygote to be at a selective advantage with respect to both homozygotes in order that a stable balanced polymorphism may exist. Thus, for example, when  $t_A = 1.5$  and  $S_{AA} = 0.8$ ,  $S_{aa} = 1.1$ , there is a stable polymorphic equilibrium, but when  $t_A = 1.5$  and  $S_{AA} = S_{aa} = 0.9$ , only a trivial equilibrium exists, the gene  $a$  being eliminated from the population.

TABLE 1  
OVERALL SELECTIVE VALUES GIVEN BY WALLACE (1948)

Temp. (°C)	Selective Values	Males		Females		
		$SR$	$ST$	$SR/SR$	$SR/ST$	$ST/ST$
16.5	Maximum	1.452	1	0.343	1	0.849
16.5	Minimum	1.393	1	0.278	1	0.734
25	Maximum	0.724	1	0.021	1	0.314
25	Minimum	0.696	1	0.014	1	0.277

### III. AN APPLICATION TO WALLACE'S DATA ON "SEX RATIO"

Wallace (1948) has reported extensive observations on selection acting on sex ratio ( $SR$ ) and the normal  $X$ -chromosome ( $ST$ ) in experimental populations of *D. pseudoobscura*.  $SR$  males produce very few, if any,  $Y$ -bearing sperm (the actual proportion produced depending in part on the temperature), and have progenies consisting almost entirely of females. Since  $SR$  causes the elimination of the  $Y$ -chromosome and an additional division of the  $X$ -chromosome during spermatogenesis, an  $SR$  male contributes twice as many  $X$ -chromosomes to the population as does a normal male. However, selection acts to prevent  $X$ -chromosomes bearing  $SR$  from replacing normal  $X$ -chromosomes throughout the population.

Wallace studied four different populations, two of which he maintained at 16.5°C and two at 25°C, and he found that in all cases the frequency of  $SR$  declined from its initial high values.  $SR$  was soon eliminated from the populations kept at 25°C but in those kept at 16.5°C it was reduced only to about 6 per cent. when, it seemed, equilibrium existed. Adaptive values for various phases of the life cycle and maximum and minimum overall selective values were published by

Wallace for these populations. The overall selective values which he gave are shown in Table 1. When these selective values are tested in the manner indicated above, it is found that they all correspond with stable non-trivial equilibria. The expected equilibrium values of the genotypic frequencies corresponding to these selective values are given in Table 2. These expected frequencies differ considerably from Wallace's observed frequencies and we may conclude that the true selective values are not within the ranges given by that author.

This finding leads us to question Wallace's methods of estimating selective values. We shall not enter upon an examination of these methods here but we may note in passing that when a non-trivial equilibrium exists (as Wallace says is the case for the populations maintained at 16.5°C) and the equilibrium genotypic frequencies are known, it may be possible by using the equations given above to obtain estimates of the genotypic selective values in terms of the observed equilibrium frequencies.

TABLE 2  
EQUILIBRIUM VALUES FOR THE GENOTYPIC FREQUENCIES CORRESPONDING TO THE SELECTIVE  
VALUES OF TABLE 1

Temp. (°C)	Selective Values	Genotypic Frequencies	Males		Females		
			<i>SR</i>	<i>ST</i>	<i>SR/SR</i>	<i>SR/ST</i>	<i>ST/ST</i>
16.5	Maximum	Before selection	0.341	0.659	0.146	0.478	0.376
		After selection	0.429	0.571	0.059	0.564	0.377
	Minimum	Before selection	0.364	0.636	0.161	0.485	0.354
		After selection	0.443	0.557	0.057	0.614	0.329
25	Maximum	Before selection	0.393	0.607	0.125	0.461	0.413
		After selection	0.319	0.681	0.004	0.777	0.219
	Minimum	Before selection	0.405	0.595	0.130	0.466	0.404
		After selection	0.322	0.678	0.003	0.804	0.193

#### IV. EQUILIBRIA WITH AN EXCESS OF HETEROZYGOTES

Not only is a selective advantage of the heterozygote over both homozygotes in the homogametic sex neither necessary nor sufficient for there to exist a stable polymorphism, balanced by selection, at a sex-linked locus but also the occurrence of heterozygotes with a frequency greater than that of homozygotes does not necessarily indicate that the heterozygote has a selective advantage over both homozygotes. As an example of a population which, at equilibrium, has more heterozygotes than homozygotes but in which the heterozygote is not at a selective advantage over both homozygotes we may take the case where  $t_A = 6$ ,  $S_{AA} = \frac{1}{3}$ , and  $S_{aa} = 2$ ; 64 per cent. of adult females in such a population will have the genotype *Aa* when equilibrium is attained.

Populations with polymorphisms dependent on sex-linked loci are of great genetical interest because of the fact that such polymorphic situations can be maintained by selection with or without selective superiority of the heterozygote. Their further study, both in the field and the laboratory, may therefore be expected to give valuable information on the part played by heterosis in maintaining polymorphism.

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