10.1071/BT12221_AC
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Appendices

Appendix S1. Derivation of time series of annual stage structure from the three population censuses by linear interpolation. Includes R code for extraction of projection matrix from time-series data, and Figures comparing projections using this matrix with mean and 95% CI of actual data.

Regression equations derived from 1980 and 2008 census data
Sapling 1: y = 0.7228x + 14.377 (R^2 = 0.0698)
Sapling 2: y = 0.2346x + 0.8242 (R^2 = 0.1134)
Adult: y = 0.1428x + 7.326 (R^2 = 0.1176)

These equations were used to generate a time series of population structure spanning 29 years. This data was used as input to extract the patter-derived projection matrix using Wood's quadratic programming method, using the following script in R.

```
## Acacia peuce - Pattern-derived matrix & analyses
## Script author: S. Raghu

## Analysis of Acacia peuce census data using Wood's quadratic programming
## method (see Caswell 2001: 144–149)

## Executing this script requires R packages 'popbio'
## (Stubben and Milligan 2007) and 'quadprog' to be installed
## and loaded for the session.
## An error message will be returned if these are not part of the R library
## on your computer.
require(popbio)
require(quadprog)

## Using density data (abundance/ha) recorded at 3 points in time
## (1980, 2001, 2008), a regression was constructed for each stage (S1,S2,A)
## Stage-specific regression was used to generate an annual time series
## spanning 1980-2008 resulting in the following data in the form of a
## stage-specific, time series vector

peuce<-
rbind(c(15.18,15.93,16.67,17.42,18.17,18.91,19.66,20.40,21.15,21.90,22.64,23.39,24.13,24.88,25.63,2
6.37,27.12,27.86,28.61,29.36,30.10,30.85,31.59,32.34,33.09,33.83,34.58,35.32,36.07),
c(0.63,0.87,1.12,1.36,1.60,1.84,2.09,2.33,2.57,2.82,3.06,3.30,3.54,3.79,4.03,4.27,4.51,4.76,5.00,5.24,5.
49,5.73,5.97,6.21,6.46,6.70,6.94,7.19,7.43),
c(7.21,7.39,7.57,7.75,7.93,8.11,8.29,8.47,8.65,8.83,9.01,9.19,9.37,9.55,9.73,9.91,10.09,10.27,10.45,10.
64,10.82,11.00,11.18,11.36,11.54,11.72,11.90,12.08,12.26))
## List nonzero elements
nonzero<- c( 1, 2, 5, 6, 7, 9)

## Create C matrix
C<- rbind(diag(-1,6), c(1,1,0,0,0,0), c(0,0,1,1,0,0), c(0,0,0,0,0,1))

## Calculate b
b<-apply(C, 1, max)

## Calculate projection matrix
```

```
A<-QPmat(peuce,C,b,nonzero)

## Life cycle analysis and analysis of asymptotic dynamics
A
eigen.analysis(A)
```

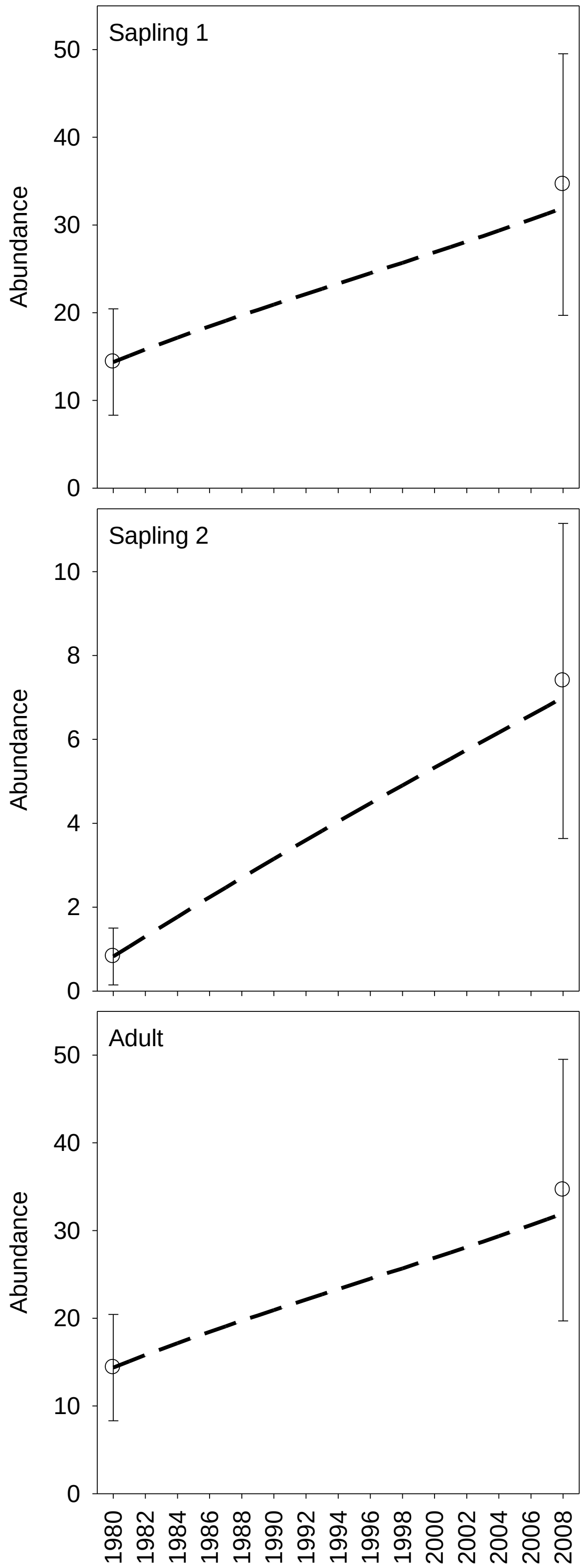
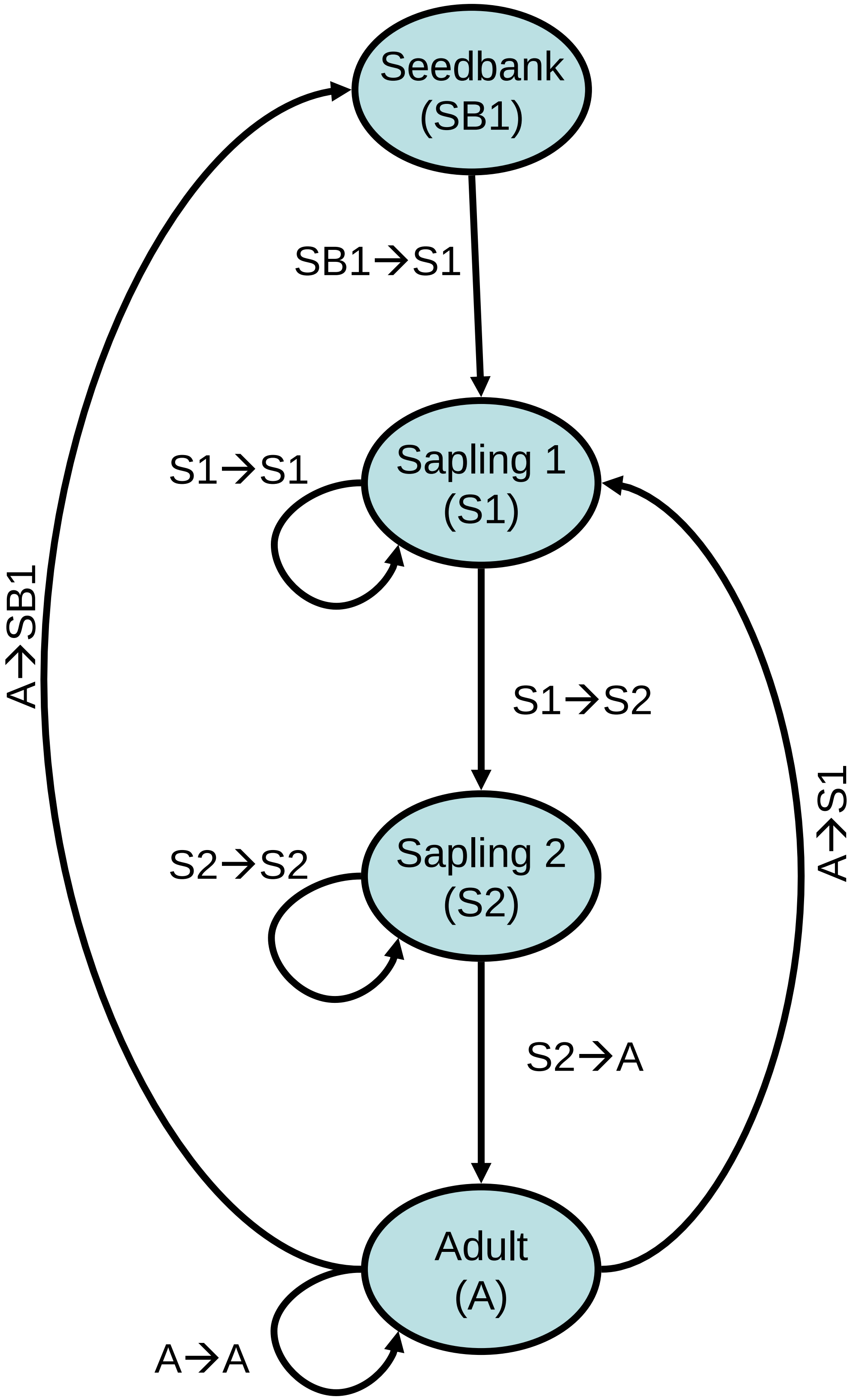


Fig. S1. Comparison of observed (circles) and predicted (lines) for the three stages of *Acacia peuce*. Predictions were based on the matrix estimated by quadratic programming. The observed values are means over 14 and 30 plots in 1980 and 2008 respectively; the error bars are 95% confidence intervals for those means.

Appendix S2. Derivation of lower level demographic parameters from published and unpublished sources to enable the compilation the process-derived projection matrix.

Structure of A. peuce life-history (annual time-step)



Seedbank
Definition: Seeds in the soil. Short-lived (no more than 2 years from production)

Parameters (Source: Nano et al. in prep)
Viability of seeds in seedbank (V_{SB}) = 0.225
Viability of fresh seeds (V_{Seeds}) = 0.825
Germination rate (g) = 0.9725

Annual Transition rates
SB→S = $V_{SB}*g$
A→SB = $f*V_{Seeds}*(1-g)$

Seedling
Definition: Individuals in the establishment phase (first year) when they are still relying on their seed resources.

Derivation of parameters
Survival rate (S_S) is a function of soil moisture, which in turn is dependent on rainfall. These were determined from a lab-based germination trial (Nano et al. in prep)
If soil moisture is low (equivalent of rainfall ≤ 250mm),
$S_S = 0$
If soil moisture is high (equivalent of rainfall >250mm)
$S_S = 0.90$
Individuals cannot remain in the Seedling stage beyond 1 year, by definition. Therefore, by definition, survival rate = transition rate.

Annual Transition rates
S→S1 = S_S
S→S = not possible by definition

The presence of a seedling stage in the model would invoke a false lag by one year for seeds produced in a given year but are capable of germinating in the subsequent year. Therefore, seedling survival to S1 stage has been incorporated into a direct transition from adults to S1.

Survival and transition parameters for Sapling 1, Sapling 2 and Adult stages were calculated as per Birt et al. (2009), where the probability of surviving and remaining in a stage = 1-(1/stage duration), and probability of surviving and transitioning into the next a stage = (overall stage survival/stage duration)

Sapling 1
Definition: Individuals that are ≤2m. The assumption is that plants in this stage are investing in belowground growth (i.e. roots foraging for moisture). The 2m cut-off was chosen as this appears to be an approximate cut-off in patterns of growth rate. Based on the limited data available, growth rate of individuals below 2m appear to be slower and more variable (NRETAS – unpublished data; Deveson 1980), than for individuals above this height.

Derivation of parameters
Duration of entire stage (=time taken for plant to grow 200cm)

If growth rate is 2.8cm/year, stage duration = 200/2.8 = 71.43 years
If growth rate is 10.8cm/year, stage duration = 200/10.8 = 18.52 years
If growth rate is 28.57/year, stage duration = 200/28.57 = 7.00 years
If growth rate is 30cm/year, stage duration = 6.66 years

Survival rate for Sapling 1 stage (S_{S1}) = 0.95 (⇒ stage mortality rate = 0.05)

Annual Transition rates

If growth rate is 2.8cm/year,

S1→S2 = 0.95/71.43 = 0.0133
S1→S1 = 1-(1/71.43) = 0.9860
S1 mortality rate = 1-(S1→S2)-(S1→S1) = 0.0007

If growth rate is 10.8cm/year,

S1→S2 = 0.95/18.52 = 0.0513
S1→S1 = 1-(1/18.52) = 0.9460
S1 mortality rate = 1-(S1→S2)-(S1→S1) = 0.0027

If growth rate is 28.57/year,

S1→S2 = 0.95/7 = 0.1357
S1→S1 = 1-(1/7) = 0.8571
S1 mortality rate = 1-(S1→S2)-(S1→S1) = 0.0071

If growth rate is 30cm/year,

S1→S2 = 0.95/6.66 = 0.1426
S1→S1 = 1-(1/6.66) = 0.8498
S1 mortality rate = 1-(S1→S2)-(S1→S1) = 0.0075

A→S1 = $f*V_{Seeds}*g*S_S$

Sapling 2

Definition: Individuals that are ≥2m and ≤4m; representative of a stage whose roots that has found the water table and returns to investment in aboveground growth towards reproductive maturity. Still with immature foliage and non-reproductive.

Derivation of parameters

Duration of entire stage (=time taken for plant to grow 200cm)

If growth rate is 24cm/year, stage duration = 200/24 = 8.33 years
If growth rate is 30cm/year, stage duration = 6.66 years

Survival rate for Sapling 2 stage (S_{S2}) = 0.95 (⇒ stage mortality rate = 0.05)

Annual Transition rates

If growth rate is 24/year,

S1→S2 = 0.95/8.33 = 0.1140
S1→S1 = 1-(1/8.33) = 0.8800
S1 mortality rate = 1-(S1→S2)-(S1→S1) = 0.0060

If growth rate is 30cm/year,

S1→S2 = 0.95/6.66 = 0.1426
S1→S1 = 1-(1/6.66) = 0.8498
S1 mortality rate = 1-(S1→S2)-(S1→S1) = 0.0075

S1→S1 = S_{S1}
S1→S2 = S_{S1S2}

Adult
Definition: Individuals that are >4m; reproductive with adult foliage. The 4m cut-off was chosen because this appears to be the height at which individuals first appear to have pods (Deveson 1980, Chuk 1982).

Age estimates for individuals at ~height of 18m by Jon Luly (JCU; pers. comm) are at least 200 years. Some have argued that they can live up to 500 years. Taking these into account an average adult longevity of 200 years seems a reasonable guesstimate Annual survival rate of individuals in this stage is very high and was thought to be close to 95%

Derivation of parameters
Duration of entire stage = 200 years
Survival rate for Adult stage (S_A) = 0.95 ($\Rightarrow$ stage mortality rate = 0.05)

Annual Transition rates
A→A = 1-(1/200) = 0.9950
A mortality rate = 1-(A→A) = 0.0050

A→A = S_A

Appendix S3. R Code for deterministic perturbation analyses

```
###########################################################################
## Acacia peuce - Process-derived matrix & analyses
## Script author: S. Raghu
###########################################################################
###########################################################################
## ANALYSIS 2
## Analysis exploring the elasticity of lambda to variation in lower
## level demographic parameters; formula from Caswell (2001: eq. 9.101);
## Adapted from MATLAB code of Davis et al. (2006) (Supplement: A016-076-S1)

## Executing this script requires R packages 'popbio'
## (Stubben and Milligan 2007) to be installed and loaded for the session.
## An error message will be returned if these are not part of the R library
## on your computer.
require(popbio)

###########################################################################
## Analysis 2.1. Parameter of interest = f = seeds/adult
require(popbio)
## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
f<-200
max_f<-200
min_f<-2
step_f=(max_f-min_f)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101

## Initialization of array for storing of simulation output
ElastOut_f<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_f[is.na(ElastOut_f)] <- 0

while (i > 0) {
        ## Computation of projection matrix elements from lower level parameters
        ## Proportion of seed rain that enters the seedbank
        A[1,4]<-f*Vseeds*(1-g)
        ## Proportion of seedbank that germinates and survives to become Sapling 1
        A[2,1]<-Vsb*g*Ss
        ## Proportion of seedrain that germinates and survives to become Sapling 1
        A[2,4]<-f*Vseeds*g*Ss
        ## Survival of Sapling 1 remaining in stage
        A[2,2]<-Ss1
```

```
        ## Survival of Sapling 1 transitioning to Sapling 2 stage
        A[3,2]<-Ss1s2
        ## Survival of Sapling 2 remaining in stage
        A[3,3]<-Ss2
        ## Survival of Sapling 2 transitioning to Adult stage
        A[4,3]<-Ss2a
        ## Survival of Adults remaining in stage
        A[4,4]<-Sa
        ## Computation of lambda and matrices of sensitivities & elasticities
        ## for the projection matrix (A)
        L<-lambda(A)
        SensA<-sensitivity(A)
        ElastA<-elasticity(A)

        ## Calculating elasticities to lower level parameters by
        ## solving the partial for each aij with respect to
        ## lower level parameter of interest
        ## Calculating elasticities of lambda to lower level parameters
        ## (solving partial for each aij with respect
        ## to lower level demographic parameter of interest

        ## Value of variable for simulation
        ElastOut_f[i,1]<-f

        ## Elasticity with respect to f
        ElastOut_f[i,2]<-(f/L)*((SensA[1,4]*Vseeds*(1-g))+
        (SensA[2,4]*Vseeds*g*Ss))
        ##Elasticity with respect to Vseeds
        ElastOut_f[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+
        (SensA[2,4]*f*g*Ss))
        ## Elasticity with respect to Vsb
        ElastOut_f[i,4]<-(Vsb/L)*(SensA[2,1]*g*Ss)
        ## Elasticity with respect to g
        ElastOut_f[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*-f*Vseeds)+
        (SensA[2,4]*f*Vseeds*Ss))
        ## Elasticity with respect to Ss
        ElastOut_f[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))
        ## Elasticity with respect to Ss1
        ElastOut_f[i,7]<-(Ss1/L)*(SensA[2,2])
        ## Elasticity with respect to Ss1s2
        ElastOut_f[i,8]<-(Ss1s2/L)*(SensA[3,2])
        ## Elasticity with respect to Ss2
        ElastOut_f[i,9]<-(Ss2/L)*(SensA[3,3])
        ##Elasticity with respect to Ss2a
        ElastOut_f[i,10]<-(Ss2a/L)*(SensA[4,3])
        ## Elasticity with respect to Sa
        ElastOut_f[i,11]<-(Sa/L)*(SensA[4,4])
        ## Storing lambda for each run
        ElastOut_f[i,12]<-L

        ## Changing value of lower-level parameter of interest for
        ## variable rate scenario
        f=f-step_f

        ## Changing counter
        i=i-1
}
colnames(ElastOut_f)<- c("f", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1",
"E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_f,digits=6))
```

```
##########################################################################
## Analysis 2.2. Parameter of interest = Vseeds = Viability of fresh seeds
require(popbio)
## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
Vseeds<-0.90
max_Vseeds<-0.90
min_Vseeds<-0.80
step_Vseeds=(max_Vseeds-min_Vseeds)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101

## Initialization of array for storing of simulation output
ElastOut_Vseeds<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_Vseeds[is.na(ElastOut_Vseeds)] <- 0

while (i > 0) {
	## Computation of projection matrix elements from lower level parameters
	## Proportion of seed rain that enters the seedbank
	A[1,4]<-f*Vseeds*(1-g)
	## Proportion of seedbank that germinates and survives to become Sapling 1
	A[2,1]<-Vsb*g*Ss
	## Proportion of seedrain that germinates and survives to become Sapling 1
	A[2,4]<-f*Vseeds*g*Ss
	## Survival of Sapling 1 remaining in stage
	A[2,2]<-Ss1
	## Survival of Sapling 1 transitioning to Sapling 2 stage
	A[3,2]<-Ss1s2
	## Survival of Sapling 2 remaining in stage
	A[3,3]<-Ss2
	## Survival of Sapling 2 transitioning to Adult stage
	A[4,3]<-Ss2a
	## Survival of Adults remaining in stage
	A[4,4]<-Sa
	## Computation of lambda and matrices of sensitivities & elasticities
	## for the projection matrix (A)
	L<-lambda(A)
	SensA<-sensitivity(A)
	ElastA<-elasticity(A)

	## Calculating elasticities to lower level parameters by
	## solving the partial for each aij with respect to
```

```
            ## lower level parameter of interest
            ## Calculating elasticities of lambda to lower level parameters
            ## (solving partial for each aij with respect
            ## to lower level demographic parameter of interest

            ## Value of variable for simulation
            ElastOut_Vseeds[i,1]<-Vseeds

            ## Elasticity with respect to f
            ElastOut_Vseeds[i,2]<-(f/L)*((SensA[1,4]*Vseeds*(1-g))+
            (SensA[2,4]*Vseeds*g*Ss))
            ##Elasticity with respect to Vseeds
            ElastOut_Vseeds[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+
            (SensA[2,4]*f*g*Ss))
            ## Elasticity with respect to Vsb
            ElastOut_Vseeds[i,4]<-(Vsb/L)*(SensA[2,1]*g*Ss)
            ## Elasticity with respect to g
            ElastOut_Vseeds[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*-f*Vseeds)+
            (SensA[2,4]*f*Vseeds*Ss))
            ## Elasticity with respect to Ss
            ElastOut_Vseeds[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))
            ## Elasticity with respect to Ss1
            ElastOut_Vseeds[i,7]<-(Ss1/L)*(SensA[2,2])
            ## Elasticity with respect to Ss1s2
            ElastOut_Vseeds[i,8]<-(Ss1s2/L)*(SensA[3,2])
            ## Elasticity with respect to Ss2
            ElastOut_Vseeds[i,9]<-(Ss2/L)*(SensA[3,3])
            ##Elasticity with respect to Ss2a
            ElastOut_Vseeds[i,10]<-(Ss2a/L)*(SensA[4,3])
            ## Elasticity with respect to Sa
            ElastOut_Vseeds[i,11]<-(Sa/L)*(SensA[4,4])
            ## Storing lambda for each run
            ElastOut_Vseeds[i,12]<-L

            ## Changing value of lower-level parameter of interest for
            ## variable rate scenario
            Vseeds=Vseeds-step_Vseeds

            ## Changing counter
            i=i-1
}
colnames(ElastOut_Vseeds)<- c("Vseeds", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1",
"E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Vseeds,digits=6))

###########################################################################
## Analysis 2.3. Parameter of interest = Vsb = Viability of seeds in seedbank
## Initialization of projection matrix
require(popbio)
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
```

```
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
Vsb<-0.30
max_Vsb<-0.30
min_Vsb<-0.20
step_Vsb=(max_Vsb-min_Vsb)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101
## Initialization of array for storing of simulation output
ElastOut_Vsb<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_Vsb[is.na(ElastOut_Vsb)] <- 0

while (i > 0) {
        ## Computation of projection matrix elements from lower level parameters
        ## Proportion of seed rain that enters the seedbank
        A[1,4]<-f*Vseeds*(1-g)
        ## Proportion of seedbank that germinates and survives to become Sapling 1
        A[2,1]<-Vsb*g*Ss
        ## Proportion of seedrain that germinates and survives to become Sapling 1
        A[2,4]<-f*Vseeds*g*Ss
        ## Survival of Sapling 1 remaining in stage
        A[2,2]<-Ss1
        ## Survival of Sapling 1 transitioning to Sapling 2 stage
        A[3,2]<-Ss1s2
        ## Survival of Sapling 2 remaining in stage
        A[3,3]<-Ss2
        ## Survival of Sapling 2 transitioning to Adult stage
        A[4,3]<-Ss2a
        ## Survival of Adults remaining in stage
        A[4,4]<-Sa
        ## Computation of lambda and matrices of sensitivities & elasticities
        ## for the projection matrix (A)
        L<-lambda(A)
        SensA<-sensitivity(A)
        ElastA<-elasticity(A)

        ## Calculating elasticities to lower level parameters by
        ## solving the partial for each aij with respect to
        ## lower level parameter of interest
        ## Calculating elasticities of lambda to lower level parameters
        ## (solving partial for each aij with respect
        ## to lower level demographic parameter of interest

        ## Value of variable for simulation
        ElastOut_Vsb[i,1]<-Vsb

        ## Elasticity with respect to f
        ElastOut_Vsb[i,2]<-(f/L)*((SensA[1,4]*Vseeds*(1-g))+
        (SensA[2,4]*Vseeds*g*Ss))
        ##Elasticity with respect to Vseeds
        ElastOut_Vsb[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+
        (SensA[2,4]*f*g*Ss))
        ## Elasticity with respect to Vsb
        ElastOut_Vsb[i,4]<-(Vsb/L)*(SensA[2,1]*g*Ss)
        ## Elasticity with respect to g
```

```
		ElastOut_Vsb[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*-f*Vseeds)+
		(SensA[2,4]*f*Vseeds*Ss))
		## Elasticity with respect to Ss
		ElastOut_Vsb[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))
		## Elasticity with respect to Ss1
		ElastOut_Vsb[i,7]<-(Ss1/L)*(SensA[2,2])
		## Elasticity with respect to Ss1s2
		ElastOut_Vsb[i,8]<-(Ss1s2/L)*(SensA[3,2])
		## Elasticity with respect to Ss2
		ElastOut_Vsb[i,9]<-(Ss2/L)*(SensA[3,3])
		##Elasticity with respect to Ss2a
		ElastOut_Vsb[i,10]<-(Ss2a/L)*(SensA[4,3])
		## Elasticity with respect to Sa
		ElastOut_Vsb[i,11]<-(Sa/L)*(SensA[4,4])
		## Storing lambda for each run
		ElastOut_Vsb[i,12]<-L

		## Changing value of lower-level parameter of interest for
		## variable rate scenario
		Vsb=Vsb-step_Vsb

		## Changing counter
		i=i-1
}
colnames(ElastOut_Vsb)<- c("Vsb", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1",
"E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Vsb,digits=6))

##########################################################################
## Analysis 2.4. Parameter of interest = g = germination rate
require(popbio)
## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
g<-1.00
max_g<-1.00
min_g<-0.95
step_g=(max_g-min_g)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101
## Initialization of array for storing of simulation output
ElastOut_g<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_g[is.na(ElastOut_g)] <- 0
```

```
while (i > 0) {
        ## Computation of projection matrix elements from lower level parameters
        ## Proportion of seed rain that enters the seedbank
        A[1,4]<-f*Vseeds*(1-g)
        ## Proportion of seedbank that germinates and survives to become Sapling 1
        A[2,1]<-Vsb*g*Ss
        ## Proportion of seedrain that germinates and survives to become Sapling 1
        A[2,4]<-f*Vseeds*g*Ss
        ## Survival of Sapling 1 remaining in stage
        A[2,2]<-Ss1
        ## Survival of Sapling 1 transitioning to Sapling 2 stage
        A[3,2]<-Ss1s2
        ## Survival of Sapling 2 remaining in stage
        A[3,3]<-Ss2
        ## Survival of Sapling 2 transitioning to Adult stage
        A[4,3]<-Ss2a
        ## Survival of Adults remaining in stage
        A[4,4]<-Sa
        ## Computation of lambda and matrices of sensitivities & elasticities
        ## for the projection matrix (A)
        L<-lambda(A)
        SensA<-sensitivity(A)
        ElastA<-elasticity(A)

        ## Calculating elasticities to lower level parameters by
        ## solving the partial for each aij with respect to
        ## lower level parameter of interest
        ## Calculating elasticities of lambda to lower level parameters
        ## (solving partial for each aij with respect
        ## to lower level demographic parameter of interest

        ## Value of variable for simulation
        ElastOut_g[i,1]<-g

        ## Elasticity with respect to f
        ElastOut_g[i,2]<-(f/L)*((SensA[1,4]*Vseeds*(1-g))+
        (SensA[2,4]*Vseeds*g*Ss))
        ##Elasticity with respect to Vseeds
        ElastOut_g[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+
        (SensA[2,4]*f*g*Ss))
        ## Elasticity with respect to Vsb
        ElastOut_g[i,4]<-(Vsb/L)*(SensA[2,1]*g*Ss)
        ## Elasticity with respect to g
        ElastOut_g[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*-f*Vseeds)+
        (SensA[2,4]*f*Vseeds*Ss))
        ## Elasticity with respect to Ss
        ElastOut_g[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))
        ## Elasticity with respect to Ss1
        ElastOut_g[i,7]<-(Ss1/L)*(SensA[2,2])
        ## Elasticity with respect to Ss1s2
        ElastOut_g[i,8]<-(Ss1s2/L)*(SensA[3,2])
        ## Elasticity with respect to Ss2
        ElastOut_g[i,9]<-(Ss2/L)*(SensA[3,3])
        ##Elasticity with respect to Ss2a
        ElastOut_g[i,10]<-(Ss2a/L)*(SensA[4,3])
        ## Elasticity with respect to Sa
        ElastOut_g[i,11]<-(Sa/L)*(SensA[4,4])
        ## Storing lambda for each run
        ElastOut_g[i,12]<-L
```

```
	## Changing value of lower-level parameter of interest for
	## variable rate scenario
	g=g-step_g

	## Changing counter
	i=i-1
}
colnames(ElastOut_g)<- c("g", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1",
"E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_g,digits=6))

##########################################################################
## Analysis 2.5. Parameter of interest = Ss = seedling survival
require(popbio)
## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
Ss<-0.90
max_Ss<-0.90
min_Ss<-0.0
step_Ss=(max_Ss-min_Ss)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101
## Initialization of array for storing of simulation output
ElastOut_Ss<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_Ss[is.na(ElastOut_Ss)] <- 0

while (i > 0) {
	## Computation of projection matrix elements from lower level parameters
	## Proportion of seed rain that enters the seedbank
	A[1,4]<-f*Vseeds*(1-g)
	## Proportion of seedbank that germinates and survives to become Sapling 1
	A[2,1]<-Vsb*g*Ss
	## Proportion of seedrain that germinates and survives to become Sapling 1
	A[2,4]<-f*Vseeds*g*Ss
	## Survival of Sapling 1 remaining in stage
	A[2,2]<-Ss1
	## Survival of Sapling 1 transitioning to Sapling 2 stage
	A[3,2]<-Ss1s2
	## Survival of Sapling 2 remaining in stage
	A[3,3]<-Ss2
	## Survival of Sapling 2 transitioning to Adult stage
	A[4,3]<-Ss2a
	## Survival of Adults remaining in stage
```

```
        A[4,4]<-Sa
        ## Computation of lambda and matrices of sensitivities & elasticities
        ## for the projection matrix (A)
        L<-lambda(A)
        SensA<-sensitivity(A)
        ElastA<-elasticity(A)

        ## Calculating elasticities to lower level parameters by
        ## solving the partial for each aij with respect to
        ## lower level parameter of interest
        ## Calculating elasticities of lambda to lower level parameters
        ## (solving partial for each aij with respect
        ## to lower level demographic parameter of interest

        ## Value of variable for simulation
        ElastOut_Ss[i,1]<-Ss

        ## Elasticity with respect to f
        ElastOut_Ss[i,2]<-(f/L)*((SensA[1,4]*Vseeds*(1-g))+
        (SensA[2,4]*Vseeds*g*Ss))
        ##Elasticity with respect to Vseeds
        ElastOut_Ss[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+
        (SensA[2,4]*f*g*Ss))
        ## Elasticity with respect to Vsb
        ElastOut_Ss[i,4]<-(Vsb/L)*(SensA[2,1]*g*Ss)
        ## Elasticity with respect to g
        ElastOut_Ss[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*-f*Vseeds)+
        (SensA[2,4]*f*Vseeds*Ss))
        ## Elasticity with respect to Ss
        ElastOut_Ss[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))
        ## Elasticity with respect to Ss1
        ElastOut_Ss[i,7]<-(Ss1/L)*(SensA[2,2])
        ## Elasticity with respect to Ss1s2
        ElastOut_Ss[i,8]<-(Ss1s2/L)*(SensA[3,2])
        ## Elasticity with respect to Ss2
        ElastOut_Ss[i,9]<-(Ss2/L)*(SensA[3,3])
        ##Elasticity with respect to Ss2a
        ElastOut_Ss[i,10]<-(Ss2a/L)*(SensA[4,3])
        ## Elasticity with respect to Sa
        ElastOut_Ss[i,11]<-(Sa/L)*(SensA[4,4])
        ## Storing lambda for each run
        ElastOut_Ss[i,12]<-L

        ## Changing value of lower-level parameter of interest for
        ## variable rate scenario
        Ss=Ss-step_Ss

        ## Changing counter
        i=i-1
}
colnames(ElastOut_Ss)<- c("Ss", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1",
"E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Ss,digits=6))

##########################################################################
## Analysis 2.6. Parameter of interest = Ss1 = Sapling 1 survival & remaining
## in stage
require(popbio)
## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
```

```
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
Ss1<-0.9097
max_Ss1<-0.9097
min_Ss1<-0.8498
step_Ss1=(max_Ss1-min_Ss1)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101
## Initialization of array for storing of simulation output
ElastOut_Ss1<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_Ss1[is.na(ElastOut_Ss1)] <- 0

while (i > 0) {
        ## Computation of projection matrix elements from lower level parameters
        ## Proportion of seed rain that enters the seedbank
        A[1,4]<-f*Vseeds*(1-g)
        ## Proportion of seedbank that germinates and survives to become Sapling 1
        A[2,1]<-Vsb*g*Ss
        ## Proportion of seedrain that germinates and survives to become Sapling 1
        A[2,4]<-f*Vseeds*g*Ss
        ## Survival of Sapling 1 remaining in stage
        A[2,2]<-Ss1
        ## Survival of Sapling 1 transitioning to Sapling 2 stage
        A[3,2]<-Ss1s2
        ## Survival of Sapling 2 remaining in stage
        A[3,3]<-Ss2
        ## Survival of Sapling 2 transitioning to Adult stage
        A[4,3]<-Ss2a
        ## Survival of Adults remaining in stage
        A[4,4]<-Sa
        ## Computation of lambda and matrices of sensitivities & elasticities
        ## for the projection matrix (A)
        L<-lambda(A)
        SensA<-sensitivity(A)
        ElastA<-elasticity(A)

        ## Calculating elasticities to lower level parameters by
        ## solving the partial for each aij with respect to
        ## lower level parameter of interest
        ## Calculating elasticities of lambda to lower level parameters
        ## (solving partial for each aij with respect
        ## to lower level demographic parameter of interest

        ## Value of variable for simulation
        ElastOut_Ss1[i,1]<-Ss1
```

```
        ## Elasticity with respect to f
        ElastOut_Ss1[i,2]<-(f/L)*((SensA[1,4]*Vseeds*(1-g))+
        (SensA[2,4]*Vseeds*g*Ss))
        ##Elasticity with respect to Vseeds
        ElastOut_Ss1[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+
        (SensA[2,4]*f*g*Ss))
        ## Elasticity with respect to Vsb
        ElastOut_Ss1[i,4]<-(Vsb/L)*(SensA[2,1]*g*Ss)
        ## Elasticity with respect to g
        ElastOut_Ss1[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*-f*Vseeds)+
        (SensA[2,4]*f*Vseeds*Ss))
        ## Elasticity with respect to Ss
        ElastOut_Ss1[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))
        ## Elasticity with respect to Ss1
        ElastOut_Ss1[i,7]<-(Ss1/L)*(SensA[2,2])
        ## Elasticity with respect to Ss1s2
        ElastOut_Ss1[i,8]<-(Ss1s2/L)*(SensA[3,2])
        ## Elasticity with respect to Ss2
        ElastOut_Ss1[i,9]<-(Ss2/L)*(SensA[3,3])
        ##Elasticity with respect to Ss2a
        ElastOut_Ss1[i,10]<-(Ss2a/L)*(SensA[4,3])
        ## Elasticity with respect to Sa
        ElastOut_Ss1[i,11]<-(Sa/L)*(SensA[4,4])
        ## Storing lambda for each run
        ElastOut_Ss1[i,12]<-L

        ## Changing value of lower-level parameter of interest for
        ## variable rate scenario
        Ss1=Ss1-step_Ss1

        ## Changing counter
        i=i-1
}
colnames(ElastOut_Ss1)<- c("Ss1", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1",
"E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Ss1,digits=6))

##########################################################################
## Analysis 2.7. Parameter of interest = Ss1s2 = Sapling 1 survival &
## transitioning to Sapling 2 in stage
require(popbio)
## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
Ss1s2<-0.0857
```

```
max_Ss1s2<-0.0857
min_Ss1s2<-0.0133
step_Ss1s2=(max_Ss1s2-min_Ss1s2)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101
## Initialization of array for storing of simulation output
ElastOut_Ss1s2<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_Ss1s2[is.na(ElastOut_Ss1s2)] <- 0

while (i > 0) {
	## Computation of projection matrix elements from lower level parameters
	## Proportion of seed rain that enters the seedbank
	A[1,4]<-f*Vseeds*(1-g)
	## Proportion of seedbank that germinates and survives to become Sapling 1
	A[2,1]<-Vsb*g*Ss
	## Proportion of seedrain that germinates and survives to become Sapling 1
	A[2,4]<-f*Vseeds*g*Ss
	## Survival of Sapling 1 remaining in stage
	A[2,2]<-Ss1
	## Survival of Sapling 1 transitioning to Sapling 2 stage
	A[3,2]<-Ss1s2
	## Survival of Sapling 2 remaining in stage
	A[3,3]<-Ss2
	## Survival of Sapling 2 transitioning to Adult stage
	A[4,3]<-Ss2a
	## Survival of Adults remaining in stage
	A[4,4]<-Sa
	## Computation of lambda and matrices of sensitivities & elasticities
	## for the projection matrix (A)
	L<-lambda(A)
	SensA<-sensitivity(A)
	ElastA<-elasticity(A)

	## Calculating elasticities to lower level parameters by
	## solving the partial for each aij with respect to
	## lower level parameter of interest
	## Calculating elasticities of lambda to lower level parameters
	## (solving partial for each aij with respect
	## to lower level demographic parameter of interest

	## Value of variable for simulation
	ElastOut_Ss1s2[i,1]<-Ss1s2

	## Elasticity with respect to f
	ElastOut_Ss1s2[i,2]<-(f/L)*((SensA[1,4]*Vseeds*(1-g))+
	(SensA[2,4]*Vseeds*g*Ss))
	##Elasticity with respect to Vseeds
	ElastOut_Ss1s2[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+
	(SensA[2,4]*f*g*Ss))
	## Elasticity with respect to Vsb
	ElastOut_Ss1s2[i,4]<-(Vsb/L)*(SensA[2,1]*g*Ss)
	## Elasticity with respect to g
	ElastOut_Ss1s2[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*-f*Vseeds)+
	(SensA[2,4]*f*Vseeds*Ss))
	## Elasticity with respect to Ss
	ElastOut_Ss1s2[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))
	## Elasticity with respect to Ss1
	ElastOut_Ss1s2[i,7]<-(Ss1/L)*(SensA[2,2])
```

```
		## Elasticity with respect to Ss1s2
		ElastOut_Ss1s2[i,8]<-(Ss1s2/L)*(SensA[3,2])
		## Elasticity with respect to Ss2
		ElastOut_Ss1s2[i,9]<-(Ss2/L)*(SensA[3,3])
		##Elasticity with respect to Ss2a
		ElastOut_Ss1s2[i,10]<-(Ss2a/L)*(SensA[4,3])
		## Elasticity with respect to Sa
		ElastOut_Ss1s2[i,11]<-(Sa/L)*(SensA[4,4])
		## Storing lambda for each run
		ElastOut_Ss1s2[i,12]<-L

		## Changing value of lower-level parameter of interest for
		## variable rate scenario
		Ss1s2=Ss1s2-step_Ss1s2

		## Changing counter
		i=i-1
}
colnames(ElastOut_Ss1s2)<- c("Ss1s2", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1",
"E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Ss1s2,digits=6))

##########################################################################
## Analysis 2.8. Parameter of interest = Ss2 = Sapling 2 survival & remaining
## in stage
require(popbio)
## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
Ss2<-0.8649
max_Ss2<-0.8649
min_Ss2<-0.8498
step_Ss2=(max_Ss2-min_Ss2)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101
## Initialization of array for storing of simulation output
ElastOut_Ss2<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_Ss2[is.na(ElastOut_Ss2)] <- 0

while (i > 0) {
		## Computation of projection matrix elements from lower level parameters
		## Proportion of seed rain that enters the seedbank
		A[1,4]<-f*Vseeds*(1-g)
		## Proportion of seedbank that germinates and survives to become Sapling 1
```

```
A[2,1]<-Vsb*g*Ss
## Proportion of seedrain that germinates and survives to become Sapling 1
A[2,4]<-f*Vseeds*g*Ss
## Survival of Sapling 1 remaining in stage
A[2,2]<-Ss1
## Survival of Sapling 1 transitioning to Sapling 2 stage
A[3,2]<-Ss1s2
## Survival of Sapling 2 remaining in stage
A[3,3]<-Ss2
## Survival of Sapling 2 transitioning to Adult stage
A[4,3]<-Ss2a
## Survival of Adults remaining in stage
A[4,4]<-Sa
## Computation of lambda and matrices of sensitivities & elasticities
## for the projection matrix (A)
L<-lambda(A)
SensA<-sensitivity(A)
ElastA<-elasticity(A)

## Calculating elasticities to lower level parameters by
## solving the partial for each aij with respect to
## lower level parameter of interest
## Calculating elasticities of lambda to lower level parameters
## (solving partial for each aij with respect
## to lower level demographic parameter of interest

## Value of variable for simulation
ElastOut_Ss2[i,1]<-Ss2

## Elasticity with respect to f
ElastOut_Ss2[i,2]<-(f/L)*((SensA[1,4]*Vseeds*(1-g))+
(SensA[2,4]*Vseeds*g*Ss))
##Elasticity with respect to Vseeds
ElastOut_Ss2[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+
(SensA[2,4]*f*g*Ss))
## Elasticity with respect to Vsb
ElastOut_Ss2[i,4]<-(Vsb/L)*(SensA[2,1]*g*Ss)
## Elasticity with respect to g
ElastOut_Ss2[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*-f*Vseeds)+
(SensA[2,4]*f*Vseeds*Ss))
## Elasticity with respect to Ss
ElastOut_Ss2[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))
## Elasticity with respect to Ss1
ElastOut_Ss2[i,7]<-(Ss1/L)*(SensA[2,2])
## Elasticity with respect to Ss1s2
ElastOut_Ss2[i,8]<-(Ss1s2/L)*(SensA[3,2])
## Elasticity with respect to Ss2
ElastOut_Ss2[i,9]<-(Ss2/L)*(SensA[3,3])
##Elasticity with respect to Ss2a
ElastOut_Ss2[i,10]<-(Ss2a/L)*(SensA[4,3])
## Elasticity with respect to Sa
ElastOut_Ss2[i,11]<-(Sa/L)*(SensA[4,4])
## Storing lambda for each run
ElastOut_Ss2[i,12]<-L

## Changing value of lower-level parameter of interest for
## variable rate scenario
Ss2=Ss2-step_Ss2

## Changing counter
```

```
        i=i-1
}
colnames(ElastOut_Ss2)<- c("Ss2", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1",
"E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Ss2,digits=6))

############################################################################
## Analysis 2.9. Parameter of interest = Ss2a = Sapling 2 survival &
## transitioning to Adult stage
require(popbio)
## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
Ss2a<-0.1283
max_Ss2a<-0.1283
min_Ss2a<-0.1140
step_Ss2a=(max_Ss2a-min_Ss2a)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101
## Initialization of array for storing of simulation output
ElastOut_Ss2a<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_Ss2a[is.na(ElastOut_Ss2a)] <- 0

while (i > 0) {
        ## Computation of projection matrix elements from lower level parameters
        ## Proportion of seed rain that enters the seedbank
        A[1,4]<-f*Vseeds*(1-g)
        ## Proportion of seedbank that germinates and survives to become Sapling 1
        A[2,1]<-Vsb*g*Ss
        ## Proportion of seedrain that germinates and survives to become Sapling 1
        A[2,4]<-f*Vseeds*g*Ss
        ## Survival of Sapling 1 remaining in stage
        A[2,2]<-Ss1
        ## Survival of Sapling 1 transitioning to Sapling 2 stage
        A[3,2]<-Ss1s2
        ## Survival of Sapling 2 remaining in stage
        A[3,3]<-Ss2
        ## Survival of Sapling 2 transitioning to Adult stage
        A[4,3]<-Ss2a
        ## Survival of Adults remaining in stage
        A[4,4]<-Sa
        ## Computation of lambda and matrices of sensitivities & elasticities
        ## for the projection matrix (A)
        L<-lambda(A)
```

```
	SensA<-sensitivity(A)
	ElastA<-elasticity(A)

	## Calculating elasticities to lower level parameters by
	## solving the partial for each aij with respect to
	## lower level parameter of interest
	## Calculating elasticities of lambda to lower level parameters
	## (solving partial for each aij with respect
	## to lower level demographic parameter of interest

	## Value of variable for simulation
	ElastOut_Ss2a[i,1]<-Ss2a

	## Elasticity with respect to f
	ElastOut_Ss2a[i,2]<-(f/L)*((SensA[1,4]*Vseeds*(1-g))+
	(SensA[2,4]*Vseeds*g*Ss))
	##Elasticity with respect to Vseeds
	ElastOut_Ss2a[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+
	(SensA[2,4]*f*g*Ss))
	## Elasticity with respect to Vsb
	ElastOut_Ss2a[i,4]<-(Vsb/L)*(SensA[2,1]*g*Ss)
	## Elasticity with respect to g
	ElastOut_Ss2a[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*-f*Vseeds)+
	(SensA[2,4]*f*Vseeds*Ss))
	## Elasticity with respect to Ss
	ElastOut_Ss2a[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))
	## Elasticity with respect to Ss1
	ElastOut_Ss2a[i,7]<-(Ss1/L)*(SensA[2,2])
	## Elasticity with respect to Ss1s2
	ElastOut_Ss2a[i,8]<-(Ss1s2/L)*(SensA[3,2])
	## Elasticity with respect to Ss2
	ElastOut_Ss2a[i,9]<-(Ss2/L)*(SensA[3,3])
	##Elasticity with respect to Ss2a
	ElastOut_Ss2a[i,10]<-(Ss2a/L)*(SensA[4,3])
	## Elasticity with respect to Sa
	ElastOut_Ss2a[i,11]<-(Sa/L)*(SensA[4,4])
	## Storing lambda for each run
	ElastOut_Ss2a[i,12]<-L

	## Changing value of lower-level parameter of interest for
	## variable rate scenario
	Ss2a=Ss2a-step_Ss2a

	## Changing counter
	i=i-1
}
colnames(ElastOut_Ss2a)<- c("Ss2a", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1",
"E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Ss2a,digits=6))

############################################################################
## Analysis 2.10. Parameter of interest = Sa = Sapling 2 survival &
## transitioning to Adult stage
require(popbio)
## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
f<-10
```

```
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
Sa<-1.00
max_Sa<-1.00
min_Sa<-0.95
step_Sa=(max_Sa-min_Sa)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101
## Initialization of array for storing of simulation output
ElastOut_Sa<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_Sa[is.na(ElastOut_Sa)] <- 0

while (i > 0) {
        ## Computation of projection matrix elements from lower level parameters
        ## Proportion of seed rain that enters the seedbank
        A[1,4]<-f*Vseeds*(1-g)
        ## Proportion of seedbank that germinates and survives to become Sapling 1
        A[2,1]<-Vsb*g*Ss
        ## Proportion of seedrain that germinates and survives to become Sapling 1
        A[2,4]<-f*Vseeds*g*Ss
        ## Survival of Sapling 1 remaining in stage
        A[2,2]<-Ss1
        ## Survival of Sapling 1 transitioning to Sapling 2 stage
        A[3,2]<-Ss1s2
        ## Survival of Sapling 2 remaining in stage
        A[3,3]<-Ss2
        ## Survival of Sapling 2 transitioning to Adult stage
        A[4,3]<-Ss2a
        ## Survival of Adults remaining in stage
        A[4,4]<-Sa
        ## Computation of lambda and matrices of sensitivities & elasticities
        ## for the projection matrix (A)
        L<-lambda(A)
        SensA<-sensitivity(A)
        ElastA<-elasticity(A)

        ## Calculating elasticities to lower level parameters by
        ## solving the partial for each aij with respect to
        ## lower level parameter of interest
        ## Calculating elasticities of lambda to lower level parameters
        ## (solving partial for each aij with respect
        ## to lower level demographic parameter of interest

        ## Value of variable for simulation
        ElastOut_Sa[i,1]<-Sa

        ## Elasticity with respect to f
        ElastOut_Sa[i,2]<-(f/L)*((SensA[1,4]*Vseeds*(1-g))+
        (SensA[2,4]*Vseeds*g*Ss))
```

```
        ##Elasticity with respect to Vseeds
        ElastOut_Sa[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+
        (SensA[2,4]*f*g*Ss))
        ## Elasticity with respect to Vsb
        ElastOut_Sa[i,4]<-(Vsb/L)*(SensA[2,1]*g*Ss)
        ## Elasticity with respect to g
        ElastOut_Sa[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*-f*Vseeds)+
        (SensA[2,4]*f*Vseeds*Ss))
        ## Elasticity with respect to Ss
        ElastOut_Sa[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))
        ## Elasticity with respect to Ss1
        ElastOut_Sa[i,7]<-(Ss1/L)*(SensA[2,2])
        ## Elasticity with respect to Ss1s2
        ElastOut_Sa[i,8]<-(Ss1s2/L)*(SensA[3,2])
        ## Elasticity with respect to Ss2
        ElastOut_Sa[i,9]<-(Ss2/L)*(SensA[3,3])
        ##Elasticity with respect to Ss2a
        ElastOut_Sa[i,10]<-(Ss2a/L)*(SensA[4,3])
        ## Elasticity with respect to Sa
        ElastOut_Sa[i,11]<-(Sa/L)*(SensA[4,4])
        ## Storing lambda for each run
        ElastOut_Sa[i,12]<-L

        ## Changing value of lower-level parameter of interest for
        ## variable rate scenario
        Sa=Sa-step_Sa

        ## Changing counter
        i=i-1
}
colnames(ElastOut_Sa)<- c("Sa", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1",
"E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Sa,digits=6))

########################################################################
```

Appendix S4. R Code for stochastic simulations

```
##########################################################################
## Acacia peuce – Stochastic simulation and analyses
## Script author: S. Raghu
##########################################################################
##########################################################################
## ANALYSIS 4
## Incorporating environmental stochasticity by generating projection
## matrix from normal, moderate and good lower level demographic parameters
## to represent transition matrix in normal, moderate and good years
## respectively

## Executing this script requires R packages 'popbio'  and 'demogR'
## (Jones 2007, Stubben and Milligan 2007) to be installed and loaded for
## the session.
## An error message will be returned if these are not part of the R library
## on your computer.
require(popbio)
require(demogR)

########################################
## Normal, Moderate and Good value for fecundity
fnormal<-2
fgood<-200
fmoderate<-10

## Normal, Moderate and Good value for Vseeds
Vseedsnormal<-0.80
Vseedsgood<-0.90
Vseedsmoderate<-0.8250

## Normal, Moderate and Good value for Vsb
Vsbnormal<-0.20
Vsbgood<-0.30
Vsbmoderate<-0.2250

## Normal, Moderate and Good value for g
gnormal<-0.95
ggood<-1.00
gmoderate<-0.9725

## Normal, Moderate and Good value for Ss
Ssnormal<-0.00
Ssgood<-0.90
Ssmoderate<-0.10

## Normal, Moderate and Good value for Ss1
Ss1normal<-0.8498
Ss1good<-0.9097
Ss1moderate<-0.9097

## Normal, Moderate and Good value for Ss1s2
Ss1s2normal<-0.0133
Ss1s2good<-0.0857
Ss1s2moderate<-0.0857

## Normal, Moderate and Good value for Ss2
Ss2normal<-0.8498
Ss2good<-0.8649
Ss2moderate<-0.8649
```

```
## Normal, Moderate and Good value for Ss2a
Ss2anormal<-0.1140
Ss2agood<-0.1283
Ss2amode<-0.1283

## Normal, Moderate and Good value for Sa
Sanormal<-0.95
Sagood<-1.00
Samoderate<-0.9950

## Initialization of "master matrix" to hold "normal", "moderate" and
## "good" annual projection matrices in a column,
## with elements filled by columns
matrices<-matrix(0,16,3)

## Computation of A matrix elements        of "normal", "moderate" and "good"
## annual matrices and storing them by column in "master matrix"
## Proportion of seed rain that enters the seedbank
matrices[13,1]<-fnormal*Vseedsnormal*(1-gnormal)
matrices[13,2]<-fmoderate*Vseedsmoderate*(1-gmoderate)
matrices[13,3]<-fgood*Vseedsgood*(1-ggood)
## Proportion of seedbank that germinates and survives to become Sapling 1
matrices[2,1]<-Vsbnormal*gnormal*Ssnormal
matrices[2,2]<-Vsbmoderate*gmoderate*Ssmoderate
matrices[2,3]<-Vsbgood*ggood*Ssgood
## Proportion of seedrain that germinates and survives to become Sapling 1
matrices[14,1]<-fnormal*Vseedsnormal*gnormal*Ssnormal
matrices[14,2]<-fmoderate*Vseedsmoderate*gmoderate*Ssmoderate
matrices[14,3]<-fgood*Vseedsgood*ggood*Ssgood
## Survival of Sapling 1 remaining in stage
matrices[6,1]<-Ss1normal
matrices[6,2]<-Ss1moderate
matrices[6,3]<-Ss1good
## Survival of Sapling 1 transitioning to Sapling 2 stage
matrices[7,1]<-Ss1s2normal
matrices[7,2]<-Ss1s2moderate
matrices[7,3]<-Ss1s2good
## Survival of Sapling 2 remaining in stage
matrices[11,1]<-Ss2normal
matrices[11,2]<-Ss2moderate
matrices[11,3]<-Ss2good
## Survival of Sapling 2 transitioning to Adult stage
matrices[12,1]<-Ss2anormal
matrices[12,2]<-Ss2amode
matrices[12,3]<-Ss2agood
## Survival of Adults remaining in stage
matrices[16,1]<-Sanormal
matrices[16,2]<-Samoderate
matrices[16,3]<-Sagood

matrices


## Relative probability of normal vs. moderate vs. good years
## These can be adjusted based on current climate vs. projected
## If climate change is going to increase variability maybe
## vital rates will become bimodal, i.e. moderate
## weather conditions for A. peuce may no longer occur under
## climate change scenarios
```

```
## Must sum to 1

# Relative probabilities under
# Current climate
p1_normal<-0.75
p1_moderate<-0.20
p1_good<-0.05

# Moderate climate change
p2_normal<-0.85
p2_moderate<-0.10
p2_good<-0.05

# Major climate change
p3_normal<-0.90
p3_moderate<-0.05
p3_good<-0.05

## Constructing a vector of probabilities
p1<-c(p1_normal,p1_moderate,p1_good)
p2<-c(p2_normal,p2_moderate,p2_good)
p3<-c(p3_normal,p3_moderate,p3_good)

## Computation of stochastic growth rate using Tuljapurkar's approximation
## and by simulation
p1.sgr<-stoch.growth.rate(matrices,prob=p1)
p2.sgr<-stoch.growth.rate(matrices,prob=p2)
p3.sgr<-stoch.growth.rate(matrices,prob=p3)

## Display log stochastic growth rate (both by Tuljapurkar's approximation
## and simulation) and confidence intervals for simulated log lambda
## Display stochastic lambda from Tuljapurkar's approximation
## and from simulation, and CI for simulation
p1.sgr
exp(p1.sgr$approx)
exp(p1.sgr$sim)
exp(p1.sgr$sim.CI)

p2.sgr
exp(p2.sgr$approx)
exp(p2.sgr$sim)
exp(p2.sgr$sim.CI)

p3.sgr
exp(p3.sgr$approx)
exp(p3.sgr$sim)
exp(p3.sgr$sim.CI)

## Computing a vector representing independently and randomly
## distributed runs of normal, moderate and good years over a 1000 year
## period under current climate, moderate climate change and major
## climate change
current.env<-sample(c(rep(1,750),rep(2,200),rep(3,50)))
moderate.env<-sample(c(rep(1,850),rep(2,100),rep(3,50)))
major.env<-sample(c(rep(1,900),rep(2,50),rep(3,50)))

## Calculation of stochastic sensitivity & elasticity of stochastic growth
## rate
current.st.sens<-stoch.sens(current.env,matrices,k=4)
current.st.sens
```

```
moderate.st.sens<-stoch.sens(moderate.env,matrices,k=4)
moderate.st.sens

major.st.sens<-stoch.sens(major.env,matrices,k=4)
major.st.sens


## Computation of quasi-extinction probability of a stage structured
## population in an an independently and identically
## distributed stochastic environment

## Initial population structure is 1980 density
n<-c(0,1273,91,636)
names(n)<-c("SB",  "S1", "S2", "A")

## Risk of decline of population (SB not included) below 1980 density
## Current Climate (including graphs)
par(mfrow=c(3,1))

x1<-stoch.quasi.ext(matrices, n, Nx=2000, tmax=50, maxruns=10, nreps=10000,
prob=p1,sumweight=c(0,1,1,1))

matplot(x1, xlab="Years", ylab="Quasi-extinction probability",
 type='l', lty=1, col=rainbow(1), las=1,
 main="Time to reach a quasi-extinction threshold
of 2000 above-ground individuals (=1980 population)")

## Moderate Climate Change
x2<-stoch.quasi.ext(matrices, n, Nx=2000, tmax=50, maxruns=10, nreps=10000,
prob=p2,sumweight=c(0,1,1,1))

matplot(x2, xlab="Years", ylab="Quasi-extinction probability",
 type='l', lty=1, col=rainbow(1), las=1,
 main="Time to reach a quasi-extinction threshold
of 2000 above-ground individuals (=1980 population)")

## Major Climate Change
x3<-stoch.quasi.ext(matrices, n, Nx=2000, tmax=50, maxruns=10, nreps=10000,
prob=p3,sumweight=c(0,1,1,1))

matplot(x3, xlab="Years", ylab="Quasi-extinction probability",
 type='l', lty=1, col=rainbow(1), las=1,
 main="Time to reach a quasi-extinction threshold
of 2000 above-ground individuals (=1980 population)")
```

Appendix S5. Transition matrices and summary output for deterministic and stochastic perturbation analyses

A. Deterministic Perturbation Analysis

<u>PaDM</u>

A Matrix

	S1	S2	A
S1	0.968	0.000	0.161
S2	0.020	0.939	0.000
A	0.000	0.029	1.000

Sensitivity

	S1	S2	A
S1	0.197	0.000	0.070
S2	0.804	0.184	0.000
A	0.000	0.400	0.619

Elasticity

	S1	S2	A
S1	0.182	0.000	0.014
S2	0.014	0.170	0.000
A	0.000	0.014	0.605

<u>PrDM</u>

A Matrix

	SB	S1	S2	A
SB	0.000	0.000	0.000	0.227
S1	0.022	0.910	0.000	0.802
S2	0.000	0.086	0.865	0.000
A	0.000	0.000	0.128	0.995

Sensitivity

	SB	S1	S2	A
SB	0.000	0.000	0.000	0.002
S1	0.016	0.291	0.000	0.082
S2	0.000	0.774	0.243	0.000
A	0.000	0.000	0.517	0.465

Elasticity

	SB	S1	S2	A
SB	0.000	0.000	0.000	0.000
S1	0.000	0.233	0.000	0.058
S2	0.000	0.058	0.185	0.000
A	0.000	0.000	0.058	0.407

B. Stochastic Perturbation Analysis (PrDM)

A Matrix for ‘normal’ rainfall year

	SB	S1	S2	A
SB	0.000	0.000	0.000	0.080
S1	0.000	0.850	0.000	0.000
S2	0.000	0.013	0.850	0.000
A	0.000	0.000	0.114	0.950

A Matrix for ‘moderate’ rainfall year

	SB	S1	S2	A
SB	0.000	0.000	0.000	0.227
S1	0.022	0.910	0.000	0.802
S2	0.000	0.086	0.865	0.000
A	0.000	0.000	0.128	0.995

A Matrix for ‘good’ rainfall year

	SB	S1	S2	A
SB	0.000	0.000	0.000	0.000
S1	0.270	0.910	0.000	162.000
S2	0.000	0.086	0.865	0.000
A	0.000	0.000	0.128	1.000

Current Climate
(Relative probability of normal:moderate:good rainfall years = 0.75:0.20:0.05)
Sensitivity

	SB	S1	S2	A
SB	0.000	0.002	0.000	0.000
S1	0.004	0.249	0.033	0.043
S2	0.029	1.744	0.221	0.285
A	0.050	3.181	0.390	0.481

Elasticity

	SB	S1	S2	A
SB	0.000	0.000	0.000	0.000
S1	0.000	0.214	0.000	0.046
S2	0.000	0.045	0.188	0.000
A	0.000	0.000	0.045	0.461

Moderate Climate Change
(Relative probability of normal:moderate:good rainfall years = 0.85:0.10:0.05)
Sensitivity

	SB	S1	S2	A
SB	0.000	0.002	0.000	0.000
S1	0.004	0.232	0.025	0.042
S2	0.031	2.033	0.216	0.339
A	0.048	3.391	0.347	0.521

Elasticity

	SB	S1	S2	A
SB	0.000	0.000	0.000	0.000
S1	0.000	0.198	0.000	0.041
S2	0.000	0.040	0.184	0.000
A	0.000	0.000	0.040	0.498

Major Climate Change
(Relative probability of normal:moderate:good rainfall years = 0.90:0.05:0.05)
Sensitivity

	SB	S1	S2	A
SB	0.000	0.002	0.000	0.000
S1	0.004	0.219	0.023	0.041
S2	0.033	2.110	0.211	0.382
A	0.047	3.649	0.314	0.551

Elasticity

	SB	S1	S2	A
SB	0.000	0.000	0.000	0.000
S1	0.000	0.187	0.000	0.036
S2	0.000	0.036	0.179	0.000
A	0.000	0.000	0.036	0.525

References:

Birt, A., R. M. Feldman, D. M. Cairns, R. N. Coulson, M. Tchakerian, W. Xi, and J. M. Guldin. 2009. Stage-structured matrix models for organisms with non-geometric development times. Ecology **90**:57–68.

Caswell, H. 2001. Matrix Population Models. Construction, Analysis, and Interpretation. Sinauer Associates Inc., Sunderland, MA.

Chuk, M. 1982. The Status and Ecology of *Acacia peuce* in the Northern Territory. Conservation Commission of the Northern Territory, Alice Springs.

Davis, A. S., D. A. Landis, V. Nuzzo, B. Blossey, E. Gerber, and H. L. Hinz. 2006. Demographic models inform selection of biocontrol agents for garlic mustard (*Alliaria petiolata*). Ecological Applications **16**:2399–2410.

Deveson. 1980. An Inventory of *Acacia peuce* (F. Muell.) Stands in central Australia: Biogeography and Ecology. Honours Thesis, Department of Geography, Australian National University, Canberra.

Jones, J. H. 2007. demogR: A package for the construction and analysis of age-structured demographic models. Journal of Statistical Software **22**:10: 11–28 http://www.jstatsoft.org/v22/i10.

Nano, C. E. M., A. E. Bowland, and C. R. Pavey. in prep. Factors limiting regeneration in a rare and long-lived desert tree *Acacia peuce*: A seed bank dynamics perspective.

Stubben, C., and B. Milligan. 2007. Estimating and analyzing demograhic models using the popbio package in R. Journal of Statistical Software **22**:11: 11–23 http://www.jstatsoft.org/v22/i11.