Appendices

**Appendix S1.** Derivation of time series of annual stage structure from the three population censuses by linear interpolation. Includes R code for extraction of projection matrix from time-series data, and Figures comparing projections using this matrix with mean and 95% CI of actual data.

Regression equations derived from 1980 and 2008 census data

- Sapling 1: \( y = 0.7228x + 14.377 \) (\( R^2 = 0.0698 \))
- Sapling 2: \( y = 0.2346x + 0.8242 \) (\( R^2 = 0.1134 \))
- Adult: \( y = 0.1428x + 7.326 \) (\( R^2 = 0.1176 \))

These equations were used to generate a time series of population structure spanning 29 years. This data was used as input to extract the pattern-derived projection matrix using Wood’s quadratic programming method, using the following script in R.

```r
## Acacia peuce - Pattern-derived matrix & analyses
## Script author: S. Raghu

## Analysis of Acacia peuce census data using Wood's quadratic programming
## method (see Caswell 2001: 144–149)

## Executing this script requires R packages 'popbio'
## (Stubben and Milligan 2007) and 'quadprog' to be installed
## on your computer.
require(popbio)
require(quadprog)

## Using density data (abundance/ha) recorded at 3 points in time
## (1980, 2001, 2008), a regression was constructed for each stage (S1,S2,A)
## Stage-specific regression was used to generate an annual time series
## spanning 1980-2008 resulting in the following data in the form of a
## stage-specific, time series vector
c(0.63,0.87,1.12,1.36,1.60,1.84,2.09,2.33,2.57,2.82,3.06,3.30,3.54,3.79,4.03,4.27,4.51,4.76,5.00,5.24,5.49,5.73,5.97,6.21,6.46,6.70,6.94,7.19,7.43),
c(7.21,7.39,7.57,7.75,7.93,8.11,8.29,8.47,8.65,8.83,9.01,9.19,9.37,9.55,9.73,9.91,10.09,10.27,10.45,10.64,10.82,11.00,11.18,11.36,11.54,11.72,11.90,12.08,12.26))

## List nonzero elements
nonzero<-c( 1, 2, 5, 6, 7, 9)

## Create C matrix
C<- rbind(diag(-1,6), c(1,1,0,0,0,0), c(0,0,1,1,0,0), c(0,0,0,0,0,1))

## Calculate b
b<-apply(C, 1, max)

## Calculate projection matrix
"
A <- QPmat(peuce,C,b,nonzero)

## Life cycle analysis and analysis of asymptotic dynamics
A
eigen.analysis(A)
Fig. S1. Comparison of observed (circles) and predicted (lines) for the three stages of *Acacia peuce*. Predictions were based on the matrix estimated by quadratic programming. The observed values are means over 14 and 30 plots in 1980 and 2008 respectively; the error bars are 95% confidence intervals for those means.
Appendix S2. Derivation of lower level demographic parameters from published and unpublished sources to enable the compilation the process-derived projection matrix.

Structure of *A. peuce* life-history (annual time-step)
**Seedbank**
Definition: Seeds in the soil. Short-lived (no more than 2 years from production)

*Parameters* (Source: Nano et al. in prep)
Viability of seeds in seedbank ($V_{SB}$) = 0.225
Viability of fresh seeds ($V_{Seeds}$) = 0.825
Germination rate ($g$) = 0.9725

*Annual Transition rates*
$SB \rightarrow S = V_{SB} \times g$
$A \rightarrow SB = f \times V_{Seeds} \times (1 - g)$

**Seedling**
Definition: Individuals in the establishment phase (first year) when they are still relying on their seed resources.

*Derivation of parameters*
Survival rate ($S_S$) is a function of soil moisture, which in turn is dependent on rainfall. These were determined from a lab-based germination trial (Nano et al. in prep)
If soil moisture is low (equivalent of rainfall ≤ 250mm),
$S_S = 0$
If soil moisture is high (equivalent of rainfall >250mm)
$S_S = 0.90$
Individuals cannot remain in the Seedling stage beyond 1 year, by definition. Therefore, by definition, survival rate = transition rate.

*Annual Transition rates*
$S \rightarrow S1 = S_S$
$S \rightarrow S = $ not possible by definition

The presence of a seedling stage in the model would invoke a false lag by one year for seeds produced in a given year but are capable of germinating in the subsequent year. Therefore, seedling survival to S1 stage has been incorporated into a direct transition from adults to S1.

Survival and transition parameters for Sapling 1, Sapling 2 and Adult stages were calculated as per Birt et al. (2009), where the probability of surviving and remaining in a stage = 1-(1/stage duration), and probability of surviving and transitioning into the next a stage = (overall stage survival/stage duration)

**Sapling 1**
Definition: Individuals that are ≤2m. The assumption is that plants in this stage are investing in belowground growth (i.e. roots foraging for moisture). The 2m cut-off was chosen as this appears to be an approximate cut-off in patterns of growth rate. Based on the limited data available, growth rate of individuals below 2m appear to be slower and more variable (NRETAS – unpublished data; Deveson 1980), than for individuals above this height.

*Derivation of parameters*
Duration of entire stage (=time taken for plant to grow 200cm)
If growth rate is 2.8 cm/year, stage duration = 200/2.8 = 71.43 years
If growth rate is 10.8 cm/year, stage duration = 200/10.8 = 18.52 years
If growth rate is 28.57 cm/year, stage duration = 200/28.57 = 7.00 years
If growth rate is 30 cm/year, stage duration = 6.66 years
Survival rate for Sapling 1 stage ($S_{S1}$) = 0.95 (⇒ stage mortality rate = 0.05)

**Annual Transition rates**
If growth rate is 2.8 cm/year,
$S1 \rightarrow S2 = 0.95/71.43 = 0.0133$
$S1 \rightarrow S1 = 1 - (1/71.43) = 0.9860$
S1 mortality rate = 1 - ($S1 \rightarrow S2$) - ($S1 \rightarrow S1$) = 0.0007
If growth rate is 10.8 cm/year,
$S1 \rightarrow S2 = 0.95/18.52 = 0.0513$
$S1 \rightarrow S1 = 1 - (1/18.52) = 0.9460$
S1 mortality rate = 1 - ($S1 \rightarrow S2$) - ($S1 \rightarrow S1$) = 0.0027
If growth rate is 28.57 cm/year,
$S1 \rightarrow S2 = 0.95/7 = 0.1357$
$S1 \rightarrow S1 = 1 - (1/7) = 0.8571$
S1 mortality rate = 1 - ($S1 \rightarrow S2$) - ($S1 \rightarrow S1$) = 0.0071
If growth rate is 30 cm/year,
$S1 \rightarrow S2 = 0.95/6.66 = 0.1426$
$S1 \rightarrow S1 = 1 - (1/6.66) = 0.8498$
S1 mortality rate = 1 - ($S1 \rightarrow S2$) - ($S1 \rightarrow S1$) = 0.0075

$A \rightarrow S1 = f^v V_{Seeds} g^* S_S$

**Sapling 2**
Definition: Individuals that are ≥2m and ≤4m; representative of a stage whose roots that has found the water table and returns to investment in aboveground growth towards reproductive maturity. Still with immature foliage and non-reproductive.

**Derivation of parameters**
Duration of entire stage (=time taken for plant to grow 200cm)
If growth rate is 24 cm/year, stage duration = 200/24 = 8.33 years
If growth rate is 30 cm/year, stage duration = 6.66 years
Survival rate for Sapling 2 stage ($S_{S2}$) = 0.95 (⇒ stage mortality rate = 0.05)

**Annual Transition rates**
If growth rate is 24/years,
$S1 \rightarrow S2 = 0.95/8.33 = 0.1140$
$S1 \rightarrow S1 = 1 - (1/8.33) = 0.8860$
S1 mortality rate = 1 - ($S1 \rightarrow S2$) - ($S1 \rightarrow S1$) = 0.0060
If growth rate is 30 cm/year,
$S1 \rightarrow S2 = 0.95/6.66 = 0.1426$
$S1 \rightarrow S1 = 1 - (1/6.66) = 0.8498$
S1 mortality rate = 1 - ($S1 \rightarrow S2$) - ($S1 \rightarrow S1$) = 0.0075

$S1 \rightarrow S1 = S_{S1}$
$S1 \rightarrow S2 = S_{S1S2}$
**Adult**
Definition: Individuals that are >4m; reproductive with adult foliage. The 4m cut-off was chosen because this appears to be the height at which individuals first appear to have pods (Deveson 1980, Chuk 1982).

Age estimates for individuals at ~height of 18m by Jon Luly (JCU; pers. comm) are at least 200 years. Some have argued that they can live up to 500 years. Taking these into account an average adult longevity of 200 years seems a reasonable guesstimate.

Annual survival rate of individuals in this stage is very high and was thought to be close to 95%

**Derivation of parameters**
Duration of entire stage = 200 years
Survival rate for Adult stage ($S_A$) = 0.95 ($\Rightarrow$ stage mortality rate = 0.05)

**Annual Transition rates**
$A \rightarrow A = 1-(1/200) = 0.9950$
A mortality rate = $1-(A \rightarrow A) = 0.0050$

$A \rightarrow A = S_A$
Appendix S3. R Code for deterministic perturbation analyses

```r
# Acacia peuce - Process-derived matrix & analyses
# Script author: S. Raghu
# Analysis exploring the elasticity of lambda to variation in lower
# level demographic parameters; formula from Caswell (2001: eq. 9.101);
# Adapted from MATLAB code of Davis et al. (2006) (Supplement: A016-076-S1)

# Executing this script requires R packages ‘popbio’
# (Stubben and Milligan 2007) to be installed and loaded for the session.
# An error message will be returned if these are not part of the R library
# on your computer.
require(popbio)

# Analysis 2.1. Parameter of interest = f = seeds/adult
require(popbio)

# Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
          dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

# Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss2<-0.0857
Ss2a<-0.1283
Sa<-0.9950

# Initializing lower-level parameter of interest for variable rate scenario
f<-200
max_f<-200
min_f<-2
step_f=(max_f-min_f)/100

# Initialization of counter variable to assist with iteration
# and storing of simulation output in array
i<-101

# Initialization of array for storing of simulation output
ElastOut_f<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_f[iis.na(ElastOut_f) <- 0

while (i > 0) {
  # Computation of projection matrix elements from lower level parameters
  # Proportion of seed rain that enters the seedbank
  A[1,4]<-f*Vseeds*(1-g)
  # Proportion of seedbank that germinates and survives to become Sapling 1
  A[2,1]<-Vsb*g*Ss
  # Proportion of seedrains that germinates and survives to become Sapling 1
  A[2,4]<-f*Vseeds*g*Ss
  # Survival of Sapling 1 remaining in stage
  A[2,2]<-Ss1
```
## Survival of Sapling 1 transitioning to Sapling 2 stage
\[ A[3,2] \rightarrow Ss1s2 \]
## Survival of Sapling 2 remaining in stage
\[ A[3,3] \rightarrow Ss2 \]
## Survival of Sapling 2 transitioning to Adult stage
\[ A[4,3] \rightarrow Ss2a \]
## Survival of Adults remaining in stage
\[ A[4,4] \rightarrow Sa \]
## Computation of lambda and matrices of sensitivities & elasticities
### for the projection matrix (\( A \))
\[ L \leftarrow \text{lambda}(A) \]
\[ \text{SensA} \leftarrow \text{sensitivity}(A) \]
\[ \text{ElastA} \leftarrow \text{elasticity}(A) \]
## Calculating elasticities to lower level parameters by
### solving the partial for each \( a_{ij} \) with respect to
### lower level parameter of interest
## Calculating elasticities of lambda to lower level parameters
### (solving partial for each \( a_{ij} \) with respect
### to lower level demographic parameter of interest
## Value of variable for simulation
\[ \text{ElastOut} \mid f[i,1] \leftarrow f \]
### Elasticity with respect to \( f \)
\[ \text{ElastOut} \mid f[i,2] \leftarrow ((f/L) \times ((\text{SensA}[1,4] \times \text{Vseeds} \times (1-g)) + (\text{SensA}[2,4] \times \text{Vseeds} \times g \times \text{Ss})) \]
### Elasticity with respect to \( \text{Vseeds} \)
\[ \text{ElastOut} \mid f[i,3] \leftarrow ((\text{Vseeds}/L) \times ((\text{SensA}[1,4] \times f \times (1-g)) + (\text{SensA}[2,4] \times f \times g \times \text{Ss})) \]
### Elasticity with respect to \( \text{Vsb} \)
\[ \text{ElastOut} \mid f[i,4] \leftarrow ((\text{Vsb}/L) \times ((\text{SensA}[1,4] \times \text{f} \times (1-g)) + (\text{SensA}[2,4] \times \text{f} \times \text{Vseeds} \times \text{Ss})) \]
### Elasticity with respect to \( \text{g} \)
\[ \text{ElastOut} \mid f[i,5] \leftarrow ((\text{g}/L) \times ((\text{SensA}[2,1] \times \text{Vsb} \times \text{Ss}) + (\text{SensA}[1,4] \times -f \times \text{Vseeds}) + (\text{SensA}[2,4] \times f \times \text{Vseeds} \times \text{Ss})) \]
### Elasticity with respect to \( \text{Ss} \)
\[ \text{ElastOut} \mid f[i,6] \leftarrow ((\text{Ss}/L) \times ((\text{SensA}[2,1] \times \text{Vsb} \times g) + (\text{SensA}[2,4] \times f \times \text{Vseeds} \times g)) \]
### Elasticity with respect to \( \text{Ss1} \)
\[ \text{ElastOut} \mid f[i,7] \leftarrow ((\text{Ss1}/L) \times ((\text{SensA}[2,2])) \]
### Elasticity with respect to \( \text{Ss1s2} \)
\[ \text{ElastOut} \mid f[i,8] \leftarrow ((\text{Ss1s2}/L) \times ((\text{SensA}[3,2])) \]
### Elasticity with respect to \( \text{Ss2} \)
\[ \text{ElastOut} \mid f[i,9] \leftarrow ((\text{Ss2}/L) \times ((\text{SensA}[3,3])) \]
### Elasticity with respect to \( \text{Ss2a} \)
\[ \text{ElastOut} \mid f[i,10] \leftarrow ((\text{Ss2a}/L) \times ((\text{SensA}[4,3])) \]
## Elasticity with respect to \( \text{Sa} \)
\[ \text{ElastOut} \mid f[i,11] \leftarrow ((\text{Sa}/L) \times ((\text{SensA}[4,4])) \]
### Storing lambda for each run
\[ \text{ElastOut} \mid f[i,12] \leftarrow L \]
## Changing value of lower-level parameter of interest for
### variable rate scenario
\[ f = f - \text{step}_f \]
## Changing counter
\[ i = i - 1 \]
}
colnames(ElastOut_f) <<- c("f", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1", "E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_f,digits=6))
require(popbio)

## Analysis 2.2. Parameter of interest = Vseeds = Viability of fresh seeds

## Initialization of projection matrix

A <- matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0), nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters

f <- 10
Vseeds <- 0.8250
Vsb <- 0.2250
g <- 0.9725
Ss <- 0.1000
Ss1 <- 0.9097
Ss1s2 <- 0.0857
Ss2 <- 0.8649
Ss2a <- 0.1283
Sa <- 0.9950

## Initializing lower-level parameter of interest for variable rate scenario
Vseeds <- 0.90
max_Vseeds <- 0.90
min_Vseeds <- 0.80
step_Vseeds = (max_Vseeds - min_Vseeds) / 100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i <- 101

## Initialization of array for storing of simulation output
ElastOut_Vseeds <- matrix(data=NA, nrow=i, ncol=12)
ElastOut_Vseeds[is.na(ElastOut_Vseeds)] <- 0

while (i > 0) {
    ## Computation of projection matrix elements from lower level parameters
    ## Proportion of seed rain that enters the seedbank
    A[1,4] <- f * Vseeds * (1 - g)
    ## Proportion of seedbank that germinates and survives to become Sapling 1
    A[2,1] <- Vsb * g * Ss
    ## Proportion of seedrain that germinates and survives to become Sapling 1
    A[2,4] <- f * Vseeds * g * Ss
    ## Survival of Sapling 1 remaining in stage
    A[2,2] <- Ss1
    ## Survival of Sapling 1 transitioning to Sapling 2 stage
    A[3,2] <- Ss1s2
    ## Survival of Sapling 2 remaining in stage
    A[3,3] <- Ss2
    ## Survival of Sapling 2 transitioning to Adult stage
    A[4,3] <- Ss2a
    ## Survival of Adults remaining in stage
    A[4,4] <- Sa
    ## Computation of lambda and matrices of sensitivities & elasticities
    ## for the projection matrix (A)
    L <- lambda(A)
    SensA <- sensitivity(A)
    ElastA <- elasticity(A)

    ## Calculating elasticities to lower level parameters by
    ## solving the partial for each aij with respect to
}
## lower level parameter of interest
## Calculating elasticities of lambda to lower level parameters
## (solving partial for each aij with respect to lower level demographic parameter of interest

## Value of variable for simulation
ElastOut_Vseeds[i,1]<-Vseeds

## Elasticity with respect to f
ElastOut_Vseeds[i,2]<-(f/L)*((SensA[1,4]*Vseeds*(1-g))+(SensA[2,4]*Vseeds*g*Ss))

## Elasticity with respect to Vseeds
ElastOut_Vseeds[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+(SensA[2,4]*f*g*Ss))

## Elasticity with respect to Vsb
ElastOut_Vseeds[i,4]<-(Vsb/L)*(SensA[2,1]*g*Ss)

## Elasticity with respect to g
ElastOut_Vseeds[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*f*Vseeds)+(SensA[2,4]*f*Vseeds*Ss))

## Elasticity with respect to Ss
ElastOut_Vseeds[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*Ss)+(SensA[2,4]*f*Vseeds*g))

## Elasticity with respect to Ss1
ElastOut_Vseeds[i,7]<-(Ss1/L)*(SensA[2,2])

## Elasticity with respect to Ss2
ElastOut_Vseeds[i,8]<-(Ss2/L)*(SensA[3,2])

## Elasticity with respect to Ss2a
ElastOut_Vseeds[i,9]<-(Ss2a/L)*(SensA[4,3])

## Elasticity with respect to Sa
ElastOut_Vseeds[i,10]<-(Sa/L)*(SensA[4,4])

## Elasticity with respect to Sa
ElastOut_Vseeds[i,11]<-(Sa/L)*(SensA[4,4])

## Storing lambda for each run
ElastOut_Vseeds[i,12]<-L

## Changing value of lower-level parameter of interest for
## variable rate scenario
Vseeds=Vseeds-step_Vseeds

## Changing counter
i=i-1

colnames(ElastOut_Vseeds)<- c("Vseeds", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1", "E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Vseeds,digits=6))

##########################################################################
## Analysis 2.3. Parameter of interest = Vsb = Viability of seeds in seedbank
## Initialization of projection matrix
require(popbio)
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A")), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss2a<-0.0857
## Initializing lower level parameter of interest for variable rate scenario

\[
\begin{align*}
S_s^2 &< 0.8649 \\
S_s^{2a} &< 0.1283 \\
S_s &< 0.9950
\end{align*}
\]

## Initialization of counter variable to assist with iteration

## and storing of simulation output in array

\[
i < 101
\]

## Initialization of array for storing of simulation output

\[
\text{ElastOut}_{Vsb} \leftarrow \begin{pmatrix}
\text{data}=\text{NA}, \text{nrow}=i, \text{ncol}=12
\end{pmatrix}
\]

while (i > 0) {
  ## Computation of projection matrix elements from lower level parameters
  ## Proportion of seed rain that enters the seedbank
  \[
  A[1,4] \leftarrow f \cdot Vseeds \cdot (1 - g)
  \]
  ## Proportion of seedbank that germinates and survives to become Sapling 1
  \[
  A[2,1] \leftarrow Vsb \cdot g \cdot S_s
  \]
  ## Proportion of seedrain that germinates and survives to become Sapling 1
  \[
  A[2,4] \leftarrow f \cdot Vseeds \cdot g \cdot S_s
  \]
  ## Survival of Sapling 1 remaining in stage
  \[
  A[2,2] \leftarrow S_s 1
  \]
  ## Survival of Sapling 1 transitioning to Sapling 2 stage
  \[
  A[3,2] \leftarrow S_s 1 s2
  \]
  ## Survival of Sapling 2 remaining in stage
  \[
  A[3,3] \leftarrow S_s 2
  \]
  ## Survival of Sapling 2 transitioning to Adult stage
  \[
  A[4,3] \leftarrow S_s 2 a
  \]
  ## Survival of Adults remaining in stage
  \[
  A[4,4] \leftarrow S_a
  \]
  ## Computation of lambda and matrices of sensitivities & elasticities
  ## for the projection matrix (A)
  \[
  L \leftarrow \text{lambda}(A)
  \]
  \[
  \text{SensA} \leftarrow \text{sensitivity}(A)
  \]
  \[
  \text{ElastA} \leftarrow \text{elasticity}(A)
  \]
  ## Calculating elasticities to lower level parameters by
  ## solving the partial for each aij with respect to
  ## lower level parameter of interest
  ## Calculating elasticities of lambda to lower level parameters
  ## (solving partial for each aij with respect
  ## to lower level demographic parameter of interest

  ## Value of variable for simulation
  \[
  \text{ElastOut}_{Vsb}[i,1] \leftarrow Vsb
  \]
  ## Elasticity with respect to f
  \[
  \text{ElastOut}_{Vsb}[i,2] \leftarrow \frac{f}{L} \cdot ((\text{SensA}[1,4] \cdot Vseeds \cdot (1 - g)) + (\text{SensA}[2,4] \cdot Vseeds \cdot g \cdot S_s))
  \]
  ## Elasticity with respect to Vseeds
  \[
  \text{ElastOut}_{Vsb}[i,3] \leftarrow \frac{Vseeds}{L} \cdot ((\text{SensA}[1,4] \cdot f \cdot (1 - g)) + (\text{SensA}[2,4] \cdot f \cdot g \cdot S_s))
  \]
  ## Elasticity with respect to Vsb
  \[
  \text{ElastOut}_{Vsb}[i,4] \leftarrow \frac{Vsb}{L} \cdot ((\text{SensA}[2,1] \cdot g \cdot S_s)
  \]
  ## Elasticity with respect to g
}
ElastOut_Vsb[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[2,4]*f*Vseeds*g)+(SensA[1,4]*f*Vseeds*Ss))
## Elasticity with respect to Ss
ElastOut_Vsb[i,6]<-((Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))
## Elasticity with respect to Ss1
ElastOut_Vsb[i,7]<-((Ss1/L)*(SensA[2,2]))
## Elasticity with respect to Ss1s2
ElastOut_Vsb[i,8]<-((Ss1s2/L)*(SensA[3,2]))
## Elasticity with respect to Ss2
ElastOut_Vsb[i,9]<-((Ss2/L)*(SensA[3,3]))
## Elasticity with respect to Ss2a
ElastOut_Vsb[i,10]<-((Ss2a/L)*(SensA[4,3]))
## Elasticity with respect to Sa
ElastOut_Vsb[i,11]<-((Sa/L)*(SensA[4,4]))
## Storing lambda for each run
ElastOut_Vsb[i,12]<-L

## Changing value of lower-level parameter of interest for
## variable rate scenario
Vsb=Vsb-step_Vsb

## Changing counter
i=i-1

colnames(ElastOut_Vsb)<-c("Vsb", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1", "E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Vsb,digits=6))

##########################################################################
## Analysis 2.4. Parameter of interest = g = germination rate
require(popbio)
## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
t<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
g<-1.00
max_g<-1.00
min_g<-0.95
step_g=(max_g-min_g)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101
## Initialization of array for storing of simulation output
ElastOut_g<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_g[is.na(ElastOut_g)] <- 0
while (i > 0) {

    # Computation of projection matrix elements from lower level parameters
    # Proportion of seed rain that enters the seedbank
    A[1,4] <- f*Vseeds*(1-g)
    # Proportion of seedbank that germinates and survives to become Sapling 1
    A[2,1] <- Vsb*g*Ss
    # Proportion of seed rain that germinates and survives to become Sapling 1
    A[2,4] <- f*Vseeds*g*Ss
    # Survival of Sapling 1 remaining in stage
    A[2,2] <- Ss
    # Survival of Sapling 1 transitioning to Sapling 2 stage
    A[3,2] <- Ss1
    # Survival of Sapling 2 remaining in stage
    A[3,3] <- Ss2
    # Survival of Sapling 2 transitioning to Adult stage
    A[4,3] <- Ss2a
    # Survival of Adults remaining in stage
    A[4,4] <- Sa

    # Computation of lambda and matrices of sensitivities & elasticities
    # for the projection matrix (A)
    L <- lambda(A)
    SensA <- sensitivity(A)
    ElastA <- elasticity(A)

    # Calculating elasticities to lower level parameters by
    # solving the partial for each aij with respect to
    # lower level parameter of interest
    # Calculating elasticities of lambda to lower level parameters
    # (solving partial for each aij with respect
    # to lower level demographic parameter of interest

    # Value of variable for simulation
    ElastOut_g[i,1] <- g

    # Elasticity with respect to f
    ElastOut_g[i,2] <- (f/L)*(SensA[1,4]*Vseeds*(1-g)) +
                     (SensA[2,4]*Vseeds*g*Ss)
    # Elasticity with respect to Vseeds
    ElastOut_g[i,3] <- (Vseeds/L)*(SensA[1,4]*f*(1-g)) +
                     (SensA[2,4]*f*g*Ss)
    # Elasticity with respect to Vsb
    ElastOut_g[i,4] <- (Vsb/L)*(SensA[2,1]*g*Ss)
    # Elasticity with respect to g
    ElastOut_g[i,5] <- (g/L)*(SensA[2,1]*Vsb*Ss)+(SensA[1,4]*f*Vseeds)+
                     (SensA[2,4]*f*Vseeds*Ss)
    # Elasticity with respect to Ss
    ElastOut_g[i,6] <- (Ss/L)*(SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))
    # Elasticity with respect to Ss1
    ElastOut_g[i,7] <- (Ss1/L)*(SensA[2,2])
    # Elasticity with respect to Ss1s2
    ElastOut_g[i,8] <- (Ss1s2/L)*(SensA[3,2])
    # Elasticity with respect to Ss2
    ElastOut_g[i,9] <- (Ss2/L)*(SensA[3,3])
    # Elasticity with respect to Ss2a
    ElastOut_g[i,10] <- (Ss2a/L)*(SensA[4,3])
    # Elasticity with respect to Sa
    ElastOut_g[i,11] <- (Sa/L)*(SensA[4,4])
    # Storing lambda for each run
    ElastOut_g[i,12] <- L
## Changing value of lower-level parameter of interest for variable rate scenario

g=g-step_g

## Changing counter

i=i-1

```r
colnames(ElastOut_g)<- c("g", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1", "E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_g,digits=6))
```

##########################################################################
## Analysis 2.5. Parameter of interest = Ss = seedling survival

```r
require(popbio)
## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4, dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))
## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950
```

## Initializing lower-level parameter of interest for variable rate scenario

```r
Ss<-0.90
max_Ss<-0.90
min_Ss<-0.0
step_Ss=(max_Ss-min_Ss)/100
```

## Initialization of counter variable to assist with iteration

```r
## and storing of simulation output in array
i<-101
## Initialization of array for storing of simulation output
ElastOut_Ss<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_Ss[is.na(ElastOut_Ss)] <- 0
```

while (i > 0) {

```r
## Computation of projection matrix elements from lower level parameters
## Proportion of seed rain that enters the seedbank
A[1,4]<-f*Vseeds*(1-g)
## Proportion of seedbank that germinates and survives to become Sapling 1
A[2,1]<-Vsb*g*Ss
## Proportion of seedrain that germinates and survives to become Sapling 1
A[2,4]<-f*Vseeds*g*Ss
## Survival of Sapling 1 remaining in stage
A[2,2]<-Ss1
## Survival of Sapling 1 transitioning to Sapling 2 stage
A[3,2]<-Ss1s2
## Survival of Sapling 2 remaining in stage
A[3,3]<-Ss2
## Survival of Sapling 2 transitioning to Adult stage
A[4,3]<-Ss2a
## Survival of Adults remaining in stage
```

## Computation of lambda and matrices of sensitivities & elasticities
## for the projection matrix (A)

L <- lambda(A)
SensA <- sensitivity(A)
ElastA <- elasticity(A)

## Calculating elasticities to lower level parameters by
## solving the partial for each aij with respect to
## lower level parameter of interest
## Calculating elasticities of lambda to lower level parameters
## (solving partial for each aij with respect
## to lower level demographic parameter of interest)

## Value of variable for simulation
ElastOut_Ss[,1] <- Ss

## Elasticity with respect to f
ElastOut_Ss[,2] <- -(f/L)*((SensA[1,4]*Vseeds*(1-g)) +
(SensA[2,4]*Vseeds*g*Ss))

## Elasticity with respect to Vseeds
ElastOut_Ss[,3] <- -(Vseeds/L)*((SensA[1,4]*f*(1-g)) +
(SensA[2,4]*f*g*Ss))

## Elasticity with respect to Vsb
ElastOut_Ss[,4] <- -(Vsb/L)*((SensA[2,4]*Vseeds*g*Ss))

## Elasticity with respect to g
ElastOut_Ss[,5] <- -(g/L)*((SensA[2,4]*Vsb*Ss) +
(SensA[2,1]*g*Ss))

## Elasticity with respect to Ss
ElastOut_Ss[,6] <- -(Ss/L)*((SensA[2,4]*Vseeds*Ss) +
(SensA[2,1]*Vsb))

## Elasticity with respect to Ss1
ElastOut_Ss[,7] <- -(Ss1/L)*((SensA[2,4]*Vsb*Ss) +
(SensA[2,1]*g*Ss))

## Elasticity with respect to Ss1s2
ElastOut_Ss[,8] <- -(Ss1s2/L)*(SensA[2,1]*Vsb*Ss) +
(SensA[2,4]*Vseeds*Ss)

## Elasticity with respect to Ss2
ElastOut_Ss[,9] <- -(Ss2/L)*(SensA[2,1]*Vsb*Ss) +
(SensA[2,4]*Vseeds*Ss)

## Elasticity with respect to Ss2a
ElastOut_Ss[,10] <- -(Ss2a/L)*(SensA[2,1]*Vsb*Ss) +
(SensA[2,4]*Vseeds*Ss)

## Elasticity with respect to Sa
ElastOut_Ss[,11] <- -(Sa/L)*(SensA[2,1]*Vsb*Ss) +
(SensA[2,4]*Vseeds*Ss)

## Storing lambda for each run
ElastOut_Ss[,12] <- L

## Changing value of lower-level parameter of interest for
## variable rate scenario
Ss = Ss - step_Ss

## Changing counter
i = i - 1
}
colnames(ElastOut_Ss) <- c("Ss", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1",
"E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Ss,digits=6))

# Analysis 2.6. Parameter of interest = Ss1 = Sapling 1 survival & remaining
# in stage
require(popbio)
## Initialization of projection matrix
A <- matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A"))

## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
Ss1<-0.9097
max_Ss1<-0.9097
min_Ss1<-0.8498
step_Ss1=(max_Ss1-min_Ss1)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101

## Initialization of array for storing of simulation output
ElastOut_Ss1<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_Ss1[is.na(ElastOut_Ss1)] <- 0

while (i > 0) {

## Computation of projection matrix elements from lower level parameters
## Proportion of seed rain that enters the seedbank
A[1,4]<-f*Vseeds*(1-g)
## Proportion of seedbank that germinates and survives to become Sapling 1
A[2,1]<-Vsb*g*Ss
## Proportion of seedrain that germinates and survives to become Sapling 1
A[2,4]<-f*Vseeds*g*Ss
## Survival of Sapling 1 remaining in stage
A[2,2]<-Ss1
## Survival of Sapling 1 transitioning to Sapling 2 stage
A[3,2]<-Ss1s2
## Survival of Sapling 2 remaining in stage
A[3,3]<-Ss2
## Survival of Sapling 2 transitioning to Adult stage
A[4,3]<-Ss2a
## Survival of Adults remaining in stage
A[4,4]<-Sa
## Computation of lambda and matrices of sensitivities & elasticities
## for the projection matrix (A)
L<-lambda(A)
SensA<-sensitivity(A)
ElastA<-elasticity(A)

## Calculating elasticities to lower level parameters by
## solving the partial for each aij with respect to
## lower level parameter of interest
## Calculating elasticities of lambda to lower level parameters
## (solving partial for each aij with respect
## to lower level demographic parameter of interest

## Value of variable for simulation
ElastOut_Ss1[i,1]<-Ss1
## Elasticity with respect to f
ElastOut_Ss1[i,2]<-((f/L)*((SensA[1,4]*Vseeds*(1-g))+(SensA[2,4]*Vseeds*g*Ss))

## Elasticity with respect to Vseeds
ElastOut_Ss1[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+(SensA[2,4]*f*g*Ss))

## Elasticity with respect to Vsb
ElastOut_Ss1[i,4]<-(Vsb/L)*((SensA[2,1]*g*Ss))

## Elasticity with respect to g
ElastOut_Ss1[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*f*Vseeds)+(SensA[2,4]*f*Vseeds*Ss))

## Elasticity with respect to Ss
ElastOut_Ss1[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))

## Elasticity with respect to Ss1
ElastOut_Ss1[i,7]<-(Ss1/L)*(SensA[2,2])

## Elasticity with respect to Ss1s2
ElastOut_Ss1[i,8]<-(Ss1s2/L)*(SensA[3,2])

## Elasticity with respect to Ss2
ElastOut_Ss1[i,9]<-(Ss2/L)*(SensA[3,3])

## Elasticity with respect to Ss2a
ElastOut_Ss1[i,10]<-(Ss2a/L)*(SensA[4,3])

## Elasticity with respect to Sa
ElastOut_Ss1[i,11]<-(Sa/L)*(SensA[4,4])

## Storing lambda for each run
ElastOut_Ss1[i,12]<-L

colnames(ElastOut_Ss1)<- c("Ss1", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1", 
"E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Ss1,digits=6))

##########################################################################
## Analysis 2.7. Parameter of interest = Ss1s2 = Sapling 1 survival &
## transitioning to Sapling 2 in stage
require(popbio)

## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
Ss1s2<-0.0857
max_Ss1s2<-0.0857
min_Ss1s2<-0.0133
step_Ss1s2=(max_Ss1s2-min_Ss1s2)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101
## Initialization of array for storing of simulation output
ElastOut_Ss1s2<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_Ss1s2[is.na(ElastOut_Ss1s2)] <- 0

while (i > 0)
{
## Computation of projection matrix elements from lower level parameters
## Proportion of seed rain that enters the seedbank
A[1,4]<-f*Vseeds*(1-g)
## Proportion of seedbank that germinates and survives to become Sapling 1
A[2,1]<-Vsb*g*Ss
## Proportion of seedrain that germinates and survives to become Sapling 1
A[2,4]<-f*Vseeds*g*Ss
## Survival of Sapling 1 remaining in stage
A[2,2]<-Ss1
## Survival of Sapling 1 transitioning to Sapling 2 stage
A[3,2]<-Ss1s2
## Survival of Sapling 2 remaining in stage
A[3,3]<-Ss2
## Survival of Sapling 2 transitioning to Adult stage
A[4,3]<-Ss2a
## Survival of Adults remaining in stage
A[4,4]<-Sa
## Computation of lambda and matrices of sensitivities & elasticities
## for the projection matrix (A)
L<-lambda(A)
SensA<-sensitivity(A)
ElastA<-elasticity(A)

## Calculating elasticities to lower level parameters by
## solving the partial for each aij with respect to
## lower level parameter of interest
## Calculating elasticities of lambda to lower level parameters
## (solving partial for each aij with respect
## to lower level demographic parameter of interest

## Value of variable for simulation
ElastOut_Ss1s2[i,1]<-Ss1s2
## Elasticity with respect to f
ElastOut_Ss1s2[i,2]<-(f/L)*((SensA[1,4]*Vseeds*(1-g))+(SensA[2,4]*Vseeds*g*Ss))
## Elasticity with respect to Vseeds
ElastOut_Ss1s2[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+(SensA[2,4]*f*g*Ss))
## Elasticity with respect to Vsb
ElastOut_Ss1s2[i,4]<-(Vsb/L)*(SensA[2,1]*Vsb*Ss)
## Elasticity with respect to g
ElastOut_Ss1s2[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*f*Vseeds)+(SensA[2,4]*f*Vseeds*Ss))
## Elasticity with respect to Ss
ElastOut_Ss1s2[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))
## Elasticity with respect to Ss1
ElastOut_Ss1s2[i,7]<-(Ss1/L)*(SensA[2,2])}
## Elasticity with respect to Ss1s2
ElastOut_Ss1s2[i,8]<-(Ss1s2/L)*(SensA[3,2])

## Elasticity with respect to Ss2
ElastOut_Ss1s2[i,9]<-(Ss2/L)*(SensA[3,3])

## Elasticity with respect to Ss2a
ElastOut_Ss1s2[i,10]<-(Ss2a/L)*(SensA[4,3])

## Elasticity with respect to Sa
ElastOut_Ss1s2[i,11]<-(Sa/L)*(SensA[4,4])

## Storing lambda for each run
ElastOut_Ss1s2[i,12]<-L

## Changing value of lower-level parameter of interest for variable rate scenario
Ss1s2=Ss1s2-step_Ss1s2

## Changing counter
i=i-1

} 

colnames(ElastOut_Ss1s2)<- c("Ss1s2", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Ss1s2,digits=6))
A[2,1]<-Vsb*g*Ss
## Proportion of seedrain that germinates and survives to become Sapling 1
A[2,4]<-f*Vseeds*g*Ss
## Survival of Sapling 1 remaining in stage
A[2,2]<-Ss1
## Survival of Sapling 1 transitioning to Sapling 2 stage
A[3,2]<-Ss1s2
## Survival of Sapling 2 remaining in stage
A[3,3]<-Ss2
## Survival of Sapling 2 transitioning to Adult stage
A[4,3]<-Ss2a
## Survival of Adults remaining in stage
A[4,4]<-Sa
## Computation of lambda and matrices of sensitivities & elasticities
## for the projection matrix (A)
L<-lambda(A)
SensA<-sensitivity(A)
ElastA<-elasticity(A)

## Calculating elasticities to lower level parameters by
## solving the partial for each aij with respect to
## lower level parameter of interest
## Calculating elasticities of lambda to lower level parameters
## (solving partial for each aij with respect
## to lower level demographic parameter of interest)

## Value of variable for simulation
ElastOut_Ss2[i,1]<-Ss2

## Elasticity with respect to f
ElastOut_Ss2[i,2]<-((f/L)*((SensA[1,4]*Vseeds*(1-g))+(SensA[2,4]*Vseeds*g*Ss)))
## Elasticity with respect to Vseeds
ElastOut_Ss2[i,3]<-((Vseeds/L)*((SensA[1,4]*f*(1-g))+(SensA[2,4]*f*g*Ss)))
## Elasticity with respect to Vsb
ElastOut_Ss2[i,4]<-((Vsb/L)*((SensA[2,1]*Vsb*Ss)+(SensA[2,4]*f*Vseeds*Ss)))
## Elasticity with respect to Ss
ElastOut_Ss2[i,5]<-((Ss/L)*((SensA[2,1]*Vsb*Ss)+(SensA[2,4]*f*Vseeds*Ss)))
## Elasticity with respect to Ss1
ElastOut_Ss2[i,6]<-((Ss1/L)*((SensA[2,1]*Vsb*Ss)+(SensA[2,4]*f*Vseeds*Ss)))
## Elasticity with respect to Ss1s2
ElastOut_Ss2[i,7]<-((Ss1s2/L)*((SensA[3,2])))
## Elasticity with respect to Ss2
ElastOut_Ss2[i,8]<-((Ss2/L)*((SensA[3,2])))
## Elasticity with respect to Ss2a
ElastOut_Ss2[i,9]<-((Ss2a/L)*((SensA[4,3]))
## Elasticity with respect to Sa
ElastOut_Ss2[i,10]<-((Sa/L)*((SensA[4,4])))
## Storing lambda for each run
ElastOut_Ss2[i,12]<-L

## Changing value of lower-level parameter of interest for
## variable rate scenario
Ss2=Ss2-step_Ss2
## Changing counter
i=i-1

colnames(ElastOut_Ss2)<- c("Ss2", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1", "E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Ss2,digits=6))

##########################################################################
## Analysis 2.9. Parameter of interest = Ss2a = Sapling 2 survival &
## transitioning to Adult stage
require(popbio)
## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),nrow=4, ncol=4,
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))
## Initialization of lower level parameters
f<-10
Vseeds<-0.8250
Vsb<-0.2250
g<-0.9725
Ss<-0.1000
Ss1<-0.9097
Ss1s2<-0.0857
Ss2<-0.8649
Ss2a<-0.1283
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
Ss2a<-0.1283
max_Ss2a<-0.1283
min_Ss2a<-0.1140
step_Ss2a=(max_Ss2a-min_Ss2a)/100

## Initialization of counter variable to assist with iteration
## and storing of simulation output in array
i<-101
## Initialization of array for storing of simulation output
ElastOut_Ss2a<-matrix(data=NA,nrow=i, ncol=12)
ElastOut_Ss2a[is.na(ElastOut_Ss2a)] <- 0

while (i > 0) {

## Computation of projection matrix elements from lower level parameters
## Proportion of seed rain that enters the seedbank
A[1,4]<-f*Vseeds*(1-g)
## Proportion of seedbank that germinates and survives to become Sapling 1
A[2,1]<-Vsb*g*Ss
## Proportion of seedrain that germinates and survives to become Sapling 1
A[2,4]<-f*Vseeds*g*Ss
## Survival of Sapling 1 remaining in stage
A[2,2]<-Ss1
## Survival of Sapling 1 transitioning to Sapling 2 stage
A[3,2]<-Ss1s2
## Survival of Sapling 2 remaining in stage
A[3,3]<-Ss2
## Survival of Sapling 2 transitioning to Adult stage
A[4,3]<-Ss2a
## Survival of Adults remaining in stage
A[4,4]<-Sa
## Computation of lambda and matrices of sensitivities & elasticities
## for the projection matrix (A)
L<-lambda(A)
SensA<-sensitivity(A)
ElastA<-elasticity(A)

## Calculating elasticities to lower level parameters by
## solving the partial for each aij with respect to
## lower level parameter of interest
## Calculating elasticities of lambda to lower level parameters
## (solving partial for each aij with respect
## to lower level demographic parameter of interest)

## Value of variable for simulation
ElastOut_Ss2a[i,1]<-Ss2a

## Elasticity with respect to f
ElastOut_Ss2a[i,2]<-(b/L)*((SensA[1,4]*Vseeds*(1-g))+(SensA[2,4]*Vseeds*g*Ss))

## Elasticity with respect to Vseeds
ElastOut_Ss2a[i,3]<-(Vseeds/L)*((SensA[1,4]*f*(1-g))+(SensA[2,4]*f*g*Ss))

## Elasticity with respect to Vsb
ElastOut_Ss2a[i,4]<-(Vsb/L)*((SensA[2,1]*g*Ss))

## Elasticity with respect to g
ElastOut_Ss2a[i,5]<-(g/L)*((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*f*Vseeds)+(SensA[2,4]*f*Vseeds*Ss))

## Elasticity with respect to Ss
ElastOut_Ss2a[i,6]<-(Ss/L)*((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))

## Elasticity with respect to Ss1
ElastOut_Ss2a[i,7]<-(Ss1/L)*((SensA[2,2]))

## Elasticity with respect to Ss1s2
ElastOut_Ss2a[i,8]<-(Ss1s2/L)*((SensA[3,2]))

## Elasticity with respect to Ss2
ElastOut_Ss2a[i,9]<-(Ss2/L)*((SensA[3,3]))

## Elasticity with respect to Sa
ElastOut_Ss2a[i,10]<-(Ss2a/L)*((SensA[4,3]))

## Elasticity with respect to Ss2a
ElastOut_Ss2a[i,11]<-(Ss2a/L)*(SensA[4,4])

## Storing lambda for each run
ElastOut_Ss2a[i,12]<-L

## Changing value of lower-level parameter of interest for
## variable rate scenario
Ss2a=Ss2a-step_Ss2a

## Changing counter
i=i-1

colnames(ElastOut_Ss2a)<- c("Ss2a", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1", 
"E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Ss2a,digits=6))

##########################################################################
## Analysis 2.10. Parameter of interest = Sa = Sapling 2 survival &
## transitioning to Adult stage
require(popbio)

## Initialization of projection matrix
A<-matrix(c(0,0,0,0, 0,0,0,0, 0,0,0,0),ncol=4, row=4, 
dimnames = list(c("SB", "S1", "S2", "A"), c("SB", "S1", "S2", "A")))

## Initialization of lower level parameters
f<-10
Vseeds<-0.8250  
Vsb<-0.2250   
g<-0.9725   
Ss<-0.1000  
Ss1<-0.9097  
Ss1s2<-0.0857  
Ss2<-0.8649  
Ss2a<-0.1283  
Sa<-0.9950

## Initializing lower-level parameter of interest for variable rate scenario
Sa<-1.00  
max_Sa<-1.00  
min_Sa<-0.95  
step_Sa=(max_Sa-min_Sa)/100

## Initialization of counter variable to assist with iteration  
## and storing of simulation output in array
i<-101  
## Initialization of array for storing of simulation output
ElastOut_Sa<-matrix(data=NA,nrow=i, ncol=12)  
ElastOut_Sa[is.na(ElastOut_Sa)] <- 0

while (i > 0) {
    ## Computation of projection matrix elements from lower level parameters
    ## Proportion of seed rain that enters the seedbank
    A[1,4]<-f*Vseeds*(1-g)  
    ## Proportion of seedbank that germinates and survives to become Sapling 1
    A[2,1]<-Vsb*g*Ss  
    ## Proportion of seedrain that germinates and survives to become Sapling 1
    A[2,4]<-f*Vseeds*g*Ss  
    ## Survival of Sapling 1 remaining in stage
    A[2,2]<-Ss1  
    ## Survival of Sapling 1 transitioning to Sapling 2 stage
    A[3,2]<-Ss1s2  
    ## Survival of Sapling 2 remaining in stage
    A[3,3]<-Ss2  
    ## Survival of Sapling 2 transitioning to Adult stage
    A[4,3]<-Ss2a  
    ## Survival of Adults remaining in stage
    A[4,4]<-Sa  
    ## Computation of lambda and matrices of sensitivities & elasticities
    ## for the projection matrix (A)
    L<-lambda(A)  
    SensA<-sensitivity(A)  
    ElastA<-elasticity(A)

    ## Calculating elasticities to lower level parameters by
    ## solving the partial for each aij with respect to
    ## lower level parameter of interest
    ## Calculating elasticities of lambda to lower level parameters
    ## (solving partial for each aij with respect
    ## to lower level demographic parameter of interest

    ## Value of variable for simulation
    ElastOut_Sa[i,1]<-Sa

    ## Elasticity with respect to f
    ElastOut_Sa[i,2]<-(f/L)*((SensA[1,4]*Vseeds*(1-g))+
                       (SensA[2,4]*Vseeds*g*Ss))
## Elasticity with respect to Vseeds
\[
Elasticity_{Vseeds} = \frac{(Vseeds/L) \times (SensA[1,4]*f*(1-g)) + (SensA[2,4]*f*g*Ss)}{ElastOut_{Sa}[i,3]}
\]
## Elasticity with respect to Vsb
\[
Elasticity_{Vsb} = \frac{(Vsb/L) \times (SensA[2,1]*g*Ss)}{ElastOut_{Sa}[i,4]}
\]
## Elasticity with respect to g
\[
Elasticity_{g} = \frac{(g/L) \times ((SensA[2,1]*Vsb*Ss)+(SensA[1,4]*f*Vseeds)+(SensA[2,4]*f*Vseeds*Ss))}{ElastOut_{Sa}[i,5]}
\]
## Elasticity with respect to Ss
\[
Elasticity_{Ss} = \frac{(Ss/L) \times ((SensA[2,1]*Vsb*g)+(SensA[2,4]*f*Vseeds*g))}{ElastOut_{Sa}[i,6]}
\]
## Elasticity with respect to Ss1
\[
Elasticity_{Ss1} = \frac{(Ss1/L) \times SensA[2,2]}{ElastOut_{Sa}[i,7]}
\]
## Elasticity with respect to Ss1s2
\[
Elasticity_{Ss1s2} = \frac{(Ss1s2/L) \times SensA[3,2]}{ElastOut_{Sa}[i,8]}
\]
## Elasticity with respect to Ss2
\[
Elasticity_{Ss2} = \frac{(Ss2/L) \times SensA[3,3]}{ElastOut_{Sa}[i,9]}
\]
## Elasticity with respect to Ss2a
\[
Elasticity_{Ss2a} = \frac{(Ss2a/L) \times SensA[4,3]}{ElastOut_{Sa}[i,10]}
\]
## Elasticity with respect to Sa
\[
Elasticity_{Sa} = \frac{(Sa/L) \times SensA[4,4]}{ElastOut_{Sa}[i,11]}
\]
## Storing lambda for each run
\[
Elasticity_{L} = L
\]

### Changing value of lower-level parameter of interest for
### variable rate scenario
\[
Sa = Sa-step_{Sa}
\]

### Changing counter
\[
i = i - 1
\]

```r
colnames(ElastOut_Sa) <- c("Sa", "E_f", "E_Vseeds", "E_Vsb", "E_g", "E_Ss", "E_Ss1", "E_Ss1s2", "E_Ss2", "E_Ss2a", "E_Sa", "L")
write.table(round(ElastOut_Sa,digits=6))

##########################################################################
Appendix S4. R Code for stochastic simulations

## Acacia peuce – Stochastic simulation and analyses
## Script author: S. Raghu

### ANALYSIS 4
### Incorporating environmental stochasticity by generating projection matrix from normal, moderate and good lower level demographic parameters to represent transition matrix in normal, moderate and good years respectively

## Executing this script requires R packages 'popbio' and 'demogR' (Jones 2007, Stubben and Milligan 2007) to be installed and loaded for the session.

```r
require(popbio)
require(demogR)
```

### Normal, Moderate and Good value for fecundity
```r
fnormal <- 2
fgood <- 200
fmoderate <- 10
```

### Normal, Moderate and Good value for Vseeds
```r
Vseedsnormal <- 0.80
Vseedsgood <- 0.90
Vseedsmoderate <- 0.8250
```

### Normal, Moderate and Good value for Vsb
```r
Vsbnormal <- 0.20
Vsbgood <- 0.30
Vsbmoderate <- 0.2250
```

### Normal, Moderate and Good value for g
```r
gnormal <- 0.95
ggood <- 1.00
gmoderate <- 0.9725
```

### Normal, Moderate and Good value for Ss
```r
Ssnormal <- 0.00
Ssgood <- 0.90
Ssmmoderate <- 0.10
```

### Normal, Moderate and Good value for Ss1
```r
Ss1normal <- 0.8498
Ss1good <- 0.9097
Ss1moderate <- 0.9097
```

### Normal, Moderate and Good value for Ss1s2
```r
Ss1s2normal <- 0.0133
Ss1s2good <- 0.0857
Ss1s2moderate <- 0.0857
```

### Normal, Moderate and Good value for Ss2
```r
Ss2normal <- 0.8498
Ss2good <- 0.8649
Ss2moderate <- 0.8649
```
## Normal, Moderate and Good value for Ss2a

Ss2anormal <- 0.1140
Ss2agood <- 0.1283
Ss2amode <- 0.1283

## Normal, Moderate and Good value for Sa

Sanormal <- 0.95
Sagood <- 1.00
Samoderate <- 0.9950

## Initialization of "master matrix" to hold "normal", "moderate" and "good" annual projection matrices in a column, with elements filled by columns

matrices <- matrix(0, 16, 3)

## Computation of A matrix elements of "normal", "moderate" and "good" annual matrices and storing them by column in "master matrix"

## Proportion of seed rain that enters the seedbank

matrices[13, 1] <- fnormal * Vseedsnormal * (1 - gnormal)
matrices[13, 2] <- fmoderate * Vseedsmoderate * (1 - gmoderate)
matrices[13, 3] <- fgood * Vseedsgood * (1 - ggood)

## Proportion of seedbank that germinates and survives to become Sapling 1

matrices[2, 1] <- Vsbnormal * gnormal * Ssnormal
matrices[2, 2] <- Vsbmoderate * gmoderate * Ssmoderate
matrices[2, 3] <- Vsbgood * ggood * Ssgood

## Proportion of seedrain that germinates and survives to become Sapling 1

matrices[14, 1] <- fnormal * Vseedsnormal * gnormal * Ssnormal
matrices[14, 2] <- fmoderate * Vseedsmoderate * gmoderate * Ssmoderate
matrices[14, 3] <- fgood * Vseedsgood * ggood * Ssgood

## Survival of Sapling 1 remaining in stage

matrices[6, 1] <- Ss1normal
matrices[6, 2] <- Ss1moderate
matrices[6, 3] <- Ss1good

## Survival of Sapling 1 transitioning to Sapling 2 stage

matrices[7, 1] <- Ss1s2normal
matrices[7, 2] <- Ss1s2moderate
matrices[7, 3] <- Ss1s2good

## Survival of Sapling 2 remaining in stage

matrices[11, 1] <- Ss2normal
matrices[11, 2] <- Ss2moderate
matrices[11, 3] <- Ss2good

## Survival of Sapling 2 transitioning to Adult stage

matrices[12, 1] <- Ss2anormal
matrices[12, 2] <- Ss2amode
matrices[12, 3] <- Ss2agood

## Survival of Adults remaining in stage

matrices[16, 1] <- Sanormal
matrices[16, 2] <- Samoderate
matrices[16, 3] <- Sagood

## Relative probability of normal vs. moderate vs. good years

## These can be adjusted based on current climate vs. projected

## If climate change is going to increase variability maybe

## vital rates will become bimodal, i.e. moderate

## weather conditions for A. peuce may no longer occur under

## climate change scenarios
## Must sum to 1

### Relative probabilities under
### Current climate
\[ p_{1\text{\_normal}} < 0.75 \]
\[ p_{1\text{\_moderate}} < 0.20 \]
\[ p_{1\text{\_good}} < 0.05 \]

### Moderate climate change
\[ p_{2\text{\_normal}} < 0.85 \]
\[ p_{2\text{\_moderate}} < 0.10 \]
\[ p_{2\text{\_good}} < 0.05 \]

### Major climate change
\[ p_{3\text{\_normal}} < 0.90 \]
\[ p_{3\text{\_moderate}} < 0.05 \]
\[ p_{3\text{\_good}} < 0.05 \]

### Constructing a vector of probabilities
\[ p_1 \leftarrow c(p_{1\text{\_normal}}, p_{1\text{\_moderate}}, p_{1\text{\_good}}) \]
\[ p_2 \leftarrow c(p_{2\text{\_normal}}, p_{2\text{\_moderate}}, p_{2\text{\_good}}) \]
\[ p_3 \leftarrow c(p_{3\text{\_normal}}, p_{3\text{\_moderate}}, p_{3\text{\_good}}) \]

### Computation of stochastic growth rate using Tuljapurkar's approximation
### and by simulation
\[ p_{1\text{\_st.gr}} \leftarrow \text{stoch.growth.rate(matrices, prob=2)} \]
\[ p_{2\text{\_st.gr}} \leftarrow \text{stoch.growth.rate(matrices, prob=2)} \]
\[ p_{3\text{\_st.gr}} \leftarrow \text{stoch.growth.rate(matrices, prob=2)} \]

### Display log stochastic growth rate (both by Tuljapurkar's approximation
### and simulation) and confidence intervals for simulated log lambda
### Display stochastic lambda from Tuljapurkar's approximation
### and from simulation, and CI for simulation
\[ \exp(p_{1\text{\_st.gr}}) \]
\[ \exp(p_{1\text{\_st.gr}} \_sim) \]
\[ \exp(p_{1\text{\_st.gr}} \_sim\_CI) \]
\[ \exp(p_{2\text{\_st.gr}}) \]
\[ \exp(p_{2\text{\_st.gr}} \_sim) \]
\[ \exp(p_{2\text{\_st.gr}} \_sim\_CI) \]
\[ \exp(p_{3\text{\_st.gr}}) \]
\[ \exp(p_{3\text{\_st.gr}} \_sim) \]
\[ \exp(p_{3\text{\_st.gr}} \_sim\_CI) \]

### Computing a vector representing independently and randomly
### distributed runs of normal, moderate and good years over a 1000 year
### period under current climate, moderate climate change and major
### climate change
\[ \text{current.env} \leftarrow \text{sample(c(rep(1,750), rep(2,200), rep(3,50))}) \]
\[ \text{moderate.env} \leftarrow \text{sample(c(rep(1,850), rep(2,100), rep(3,50))}) \]
\[ \text{major.env} \leftarrow \text{sample(c(rep(1,900), rep(2,50), rep(3,50))}) \]

### Calculation of stochastic sensitivity & elasticity of stochastic growth
### rate
\[ \text{current.st.sens} \leftarrow \text{stoch.sens(current.env, matrices, k=4) \}
\text{current.st.sens} \]
moderate.st.sens<-stoch.sens(moderate.env,matrices,k=4)
moderate.st.sens

major.st.sens<-stoch.sens(major.env,matrices,k=4)
major.st.sens

## Computation of quasi-extinction probability of a stage structured
## population in an an independently and identically
distributed stochastic environment

## Initial population structure is 1980 density
n<-c(0,1273,91,636)
names(n)<-c("SB", "S1", "S2", "A")

## Risk of decline of population (SB not included) below 1980 density
## Current Climate (including graphs)
par(mfrow=c(3,1))
x1<-stoch.quasi.ext(matrices, n, Nx=2000, tmax=50, maxruns=10, nreps=10000, prob=p1,sumweight=c(0,1,1,1))
matplot(x1, xlab="Years", ylab="Quasi-extinction probability", type='l', lty=1, col=rainbow(1), las=1, main="Time to reach a quasi-extinction threshold of 2000 above-ground individuals (=1980 population")

## Moderate Climate Change
x2<-stoch.quasi.ext(matrices, n, Nx=2000, tmax=50, maxruns=10, nreps=10000, prob=p2,sumweight=c(0,1,1,1))
matplot(x2, xlab="Years", ylab="Quasi-extinction probability", type='l', lty=1, col=rainbow(1), las=1, main="Time to reach a quasi-extinction threshold of 2000 above-ground individuals (=1980 population")

## Major Climate Change
x3<-stoch.quasi.ext(matrices, n, Nx=2000, tmax=50, maxruns=10, nreps=10000, prob=p3,sumweight=c(0,1,1,1))
matplot(x3, xlab="Years", ylab="Quasi-extinction probability", type='l', lty=1, col=rainbow(1), las=1, main="Time to reach a quasi-extinction threshold of 2000 above-ground individuals (=1980 population")

**Appendix S5.** Transition matrices and summary output for deterministic and stochastic perturbation analyses

**A. Deterministic Perturbation Analysis**

PaDM
A Matrix

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<th>A</th>
</tr>
</thead>
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Sensitivity

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Elasticity

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PrDM

A Matrix

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Sensitivity

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Elasticity

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### B. Stochastic Perturbation Analysis (PrDM)

A Matrix for ‘normal’ rainfall year

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A Matrix for ‘moderate’ rainfall year

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A Matrix for ‘good’ rainfall year

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**Current Climate**

(Relative probability of normal:moderate:good rainfall years = 0.75:0.20:0.05)

**Sensitivity**

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**Elasticity**

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</tr>
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**Moderate Climate Change**

(Relative probability of normal:moderate:good rainfall years = 0.85:0.10:0.05)

**Sensitivity**

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<th>SB</th>
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<th>S2</th>
<th>A</th>
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### Elasticity

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### Major Climate Change

(Relative probability of normal:moderate:good rainfall years = 0.90:0.05:0.05)

### Sensitivity

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### Elasticity

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References: