Seismic Lithologic Modelling of Amplitude-Versus-Offset Data

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Summary

For small angles of incidence, \( \theta \), the reflection amplitude of plane p-waves from a planar interface between two elastic media is nearly linear with \( \sin^2 \theta \). For an NMO-corrected common-midpoint (CMP) gather, the linear fit of amplitude versus \( \sin^2 \theta \) at each time sample yields two new kinds of seismic trace. The trace constructed from the zero-offset intercept, the so-called ‘p-wave stack’, represents the response to changes in acoustic impedance at reflecting boundaries. The trace constructed from the slope, the ‘gradient stack’, represents the response to changes in s-wave velocity as well as p-wave velocity and bulk density. Under certain assumptions, the difference between the p-wave trace and the gradient trace reflects changes in s-wave velocity alone, and the sum reflects changes solely in Poisson’s ratio. Here, we demonstrate how the method of seismic lithologic modelling, a parameter-estimation technique that refines thin-layer models of p-wave velocity and density by iterative matching of synthetic data to CMP stack sections, can be extended to include s-wave velocity in the model. In this extended approach, a new form of synthetic data trace is matched to the gradient stack (or to either the ‘s-wave stack’ or ‘Poisson’s ratio stack’) as well. As with any other approach that attempts to obtain s-wave velocity structure from the offset-dependence of amplitude, success in application to field data depends heavily on the extent to which data can be conditioned so that they satisfy the assumptions upon which amplitude-versus-offset analysis is based.

Introduction

Depending on changes in elastic parameters across a reflecting boundary, the reflection coefficient for incident plane p-waves can increase or decrease (and even change polarity) with increasing incidence angle. Consequently, reflection character as well as event amplitude and continuity on conventional common-midpoint (CMP) stacks can differ from their counterparts in zero-offset sections. Such departures of the CMP stack from the zero-offset section bias attempts to invert amplitudes of CMP-stacked data to obtain thin-layer models of acoustic impedance or velocity.

Shuey (1985) has shown that for angles of incidence \( \theta \) less than about 25 degrees, the amplitude of a p-wave reflected from a planar interface between two elastic media varies approximately linearly with \( \sin^2 \theta \). The regression coefficients are simple functions of \( \Delta V_p/V_p, \Delta V_s/V_s, \) and \( \Delta \rho/\rho \), where \( V_p, V_s \), and \( \rho \) are the average p-wave velocity, s-wave velocity, and bulk density, respectively. We estimate the media on either side of the reflecting interface, and \( \Delta V_p, \Delta V_s \), and \( \Delta \rho \) are the (assumed small) differences between those quantities across the interface.

Given this angle-dependent behaviour, one can readily compute and display seismic traces that are derived from NMO-corrected multifold data, but, unlike the conventional stack, give information about offset-dependence of reflection magnitude. In particular, from a linear fit to amplitude-versus-\( \sin^2 \theta \) at each time sample of an NMO-corrected gather, two traces can be inferred: an intercept trace, and a slope trace. The intercept trace is indicative of contrasts in p-wave impedance; the difference between the intercept and slope traces, under certain assumptions, shows contrasts in s-wave impedance; the sum of the intercept and slope traces, under the same set of assumptions, reveals contrasts in Poisson’s ratio. We call the trace obtained from the determined intercepts the ‘zero-offset stack’ or ‘p-wave stack’, and the trace determined from the slope the ‘gradient stack’. The value of these types of data would be enhanced if, from them, one could derive models of the elastic-parameter behaviour with depth.

Extended seismic lithologic modelling

The method of seismic lithologic modelling (Gelfand and Larner, 1984) has proven useful in helping interpreters refine thin-layer models of p-wave velocity or acoustic impedance. Starting with an initial model interpreted from borehole information, seismic data, and other geologic information, the seismic lithologic modelling (SLIM) approach uses a parameter-estimation procedure to perturb parameters of the model. The perturbations, done systematically and iteratively,
are aimed at minimizing the rms difference between migrated, CMP-stacked data and a vertical-path synthetic section computed for the model.

The p-wave stack section from the linear fit of amplitude versus sin²θ should be a better approximation to the zero-offset section than is the CMP stack and, hence, more appropriate for the matching process. Application of the SLIM process to p-wave-stack data yields a thin-layer, lithologic model of p-wave velocity, density, and layer thickness. Suppose, next, that this derived model is combined with an initial model for \( V_s \) (based on VSP, full-waveform sonic data, or on an assumed value for the ratio \( V_p/V_s \)) to construct a synthetic gradient stack. Then, in a second application of the SLIM process, the \( V_s \) model is perturbed so as to minimize the rms difference between the gradient stack obtained from the field data and the synthetic gradient stack computed for the model.

**Synthetic data example**

We can demonstrate the extended method for modelling amplitude-versus-offset data with a simplistic 1-D model (see Table 1) consisting of two separated sand layers embedded in shale. The upper sand layer is dry, and the lower one, distinguished by its low value of Poisson's ratio (0.10), is gas-saturated. This lower layer exhibits a higher value of \( V_s \) and a lower value of \( V_p \) than are evident in the background shale.

On the left of Fig. 1 is an NMO-corrected CMP gather of synthetic traces computed for the model in Table 1. P-wave reflections from the tops and bottoms of the two sand layers are evident. Note that reflection amplitude decreases with offset for reflections at the boundaries of the dry sand, but increases with offset at the boundaries of the gas-saturated sand. Therefore, the CMP stack (trace a in Fig. 1) exhibits stronger reflection events for the deep layer than for the shallow one.

Trace d in Fig. 1 is the difference between the p-wave stack (trace b) and the gradient stack (trace c), and trace e is the sum of the two. Note that the wavelets are identical in these various derived traces, and the polarity of the p-wave reflections at the gas sand is opposite to that of the s-wave reflections.

In conventional applications of the SLIM method, we estimate the thin-layer acoustic structure by matching synthetic data from our developing model with the observed CMP stack.

### Table 1. Sand-Shale Model.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Depth (m)</th>
<th>( V_p ) (m/s)</th>
<th>( V_s ) (m/s)</th>
<th>( \rho ) (g/cm³)</th>
<th>( \sigma )</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>shale</td>
<td>2030</td>
<td>830</td>
<td>2.07</td>
<td>.40</td>
<td>.087</td>
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<tr>
<td>dry sand</td>
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<td>2325</td>
<td>1165</td>
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<td>.087</td>
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<tr>
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<td>2030</td>
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<tr>
<td>gas sand</td>
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<td>1965</td>
<td>1310</td>
<td>1.80</td>
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<tr>
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<td>2030</td>
<td>830</td>
<td>2.07</td>
<td>.40</td>
<td>.085</td>
</tr>
</tbody>
</table>

\( V_p \) = p-wave velocity.  
\( V_s \) = s-wave velocity.  
\( \rho \) = bulk density.  
\( \sigma \) = Poisson's ratio.  
\( r \) = p-wave reflection coefficient at normal incidence.

**FIGURE 1**

Synthetic CMP gather generated by ray tracing through the model in Table 1, using the Zoeppritz equation to compute amplitudes, and convolving the resulting reflectivity series with a symmetric Ricker wavelet having a 24-ms dominant period. Traces a through e are formed from different linear combinations of the portions of traces in this gather for which θ < 20 degrees. The dashed lines are trajectories of constant incidence angle.

Here we do the matching instead with the observed p-wave stack, trace b of Fig. 1, and estimate acoustic impedance as a function of reflection time. Starting with an initial model (Fig. 2, trace a) with no contrasts in acoustic impedance, the SLIM method produced the acoustic-impedance model and associated synthetic trace shown in Fig. 2 (traces b and e). That derived model exhibits the essential characteristics of the two sand layers — a high value of \( \rho V_p \) for the shallow one, and a low value for the deep one.

For the second stage of the extended SLIM process, we match a synthetic with the gradient stack. The initial model for \( V_s \) is such that \( V_p/V_s = 2 \), where \( V_p \) is obtained from the

### FIGURE 2

P-wave velocity structure and associated traces:  
a. initial \( \rho V_p \) model for the first stage of the SLIM process.  
b. final SLIM-derived \( \rho V_p \) model.  
c. true \( \rho V_p \) structure.  
d. synthetic p-wave stack traced based upon the initial \( \rho V_p \) model.  
e. synthetic p-wave stack trace based upon the final \( \rho V_p \) model.  
f. p-wave stack obtained from the CMP gather in Figure 1.
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Summary

Vertical electrical sounding is applied to a wide variety of objectives, including mineral resources, thermal energy, and water resources. Thus, there is a great need for electrical exploration instruments that minimise the effects of noise from stray currents or electromagnetic coupling, as well as from anomalies such as local variations in shallow underground soil or changes in topography. In addition, there is a demand for more rapid measurement capability, cost reduction and instruments that can conduct in-situ measurement automatically.

Methods that have been proposed for improvement of S/N ratio include the Offset Wenner method, proposed by Barker (1981), and the synthetic potential method using the