Time-Domain Electromagnetic Responses of a Polarizable Target

Lindsay Thomas  
*University of Melbourne*  
Parkville, Victoria 3052  
Australia

Terry Lee  
*Bureau of Mineral Resources*  
G.P.O. Box 378  
Canberra ACT 2601  
Australia

**Summary**

The time-domain electromagnetic response of a polarizable sphere in a non-polarizable host rock is influenced more heavily by the chargeability \(m\) than by the time constant \(\tau\), in the range of delay times commonly observed. These results are obtained from a new mathematical model of the sphere and halfspace system, based on the earlier work of Lee (1983).

**Introduction**

Prediction of sign reversals in the time-domain electromagnetic transients (TEM) observed with a coincident-loop system near media which have dispersive conductivity dates back at least to Bhattacharyya (1964). Various authors, including Bhattacharyya (1964), Lewis and Lee (1984), and Rachie *et al.*, (1985), have computed representative responses for geometrically simple models which include the effects of dispersion. Weidelt (1982) showed that coincident-loop TEM experiments would not exhibit negative transients unless there were polarizable elements in the subsurface.

Observations of negative transient responses have also been known for some time, and recent examples have been shown by Elliott (1987) and Flis (1987). In the paper by Elliott (1987) the coincident Induced Polarization (IP) responses were also shown, but the IP responses do not completely explain the TEM responses (or vice versa). Flis (1987) reports the results of a brief numerical model study in which the effects of dispersive conductivity were described by the Cole-Cole model. Although the model shown there did not exhibit negative transients, Flis conjectured that an appropriate choice of Cole-Cole parameters would produce negative transients.

The observation that not all numerical models will produce negative transients, and that field observations can be equivocal in their association with other IP observations, suggests that the effect of polarizable media in real situations may be more complicated than the simple models so far used can allow. In addition, we observe that the time scale for conventional IP responses is of the order of one second, while the observation of sign reversals in TEM models and field observations occurs during the first few milliseconds after the current cutoff. This suggests that the TEM phenomenon may be qualitatively different in origin to the IP response.

Accordingly, we have constructed an analytical model, in which a sphere with dispersive conductivity is embedded in a conducting halfspace, to study the effect of the various components of the Cole-Cole model on the TEM response, and the way in which a polarizable sphere couples to a non-polarizable ground. Figure 1 shows an example of the results from our model; the target sphere has the dispersive properties of massive sulphide targets as reported by Pelton *et al.* (1978).

![Figure 1](image)

**Theoretical basis**

Lee (1983) showed that the TEM response of a coincident loop system (e.g. SIROTEM) in the vicinity of a sphere in a halfspace could, for low host-rock conductivity, be approximated by

\[
V_d(t) = V_{rd}(t) + V_{rd}(t) + V(t)
\]

where the subscripts refer to the total response, and the response of the halfspace, of the sphere in air, and from interactions between the sphere and the halfspace, respectively. The geometry of the system is shown in Fig. 2. We have extended this formulation by allowing the sphere to have a complex conductivity, described by the Cole-Cole model as introduced by Pelton *et al.* (1978).

We incorporated the complex conductivity property by allowing the sphere conductivity to be complex, in equation 1
of Lee (1983), and following many of the steps of that paper. The sphere impedance was represented by the impedance of the equivalent circuit shown in Fig. 2 (see Pelton et al., 1987, equation 1). The result in this case is a contour integral of two parts, a pole contribution and a branch cut contribution, and we observe that the effect on the final observed response depends on the relative magnitudes of these two components, both of which are complicated functions of the Cole-Cole parameters.

As well, the final observed response depends on the relative magnitude of the response of the polarized sphere and that of the halfspace.

At the time of writing, we have one significant limitation in that we are limited in our analysis to the case where the frequency dependence $c = 0.5$. While this leads to IP decays which are faster than many reported, it is at least a value in the field of those observed.

Because of the complexity of this problem, we first study the effect of the sphere-in-air contribution, so as to be better able to understand the significance of the complex interaction terms. The conclusions presented here include interaction terms which are computed from the DC asymptote of the sphere impedance.

**Results**

Figure 3 shows the decay curves observed directly over the sphere for the same set of parameters as were used in Fig. 1. The halfspace, sphere-in-air and interaction contributions are shown separately, dotted where negative. In this case the $V_s$ term changes sign at 10 milliseconds, and the total response goes negative when this term dominates the halfspace response.
FIGURE 5
Change in transient decay voltage for a 10% change in the quantities shown. Curves are dotted where negative.

Figure 4 is a similar display for another set of parameters, from Fig. 13 of Pelton et al. (1978), this time from the high-sulphide-concentration region of the diagram. Here the sphere-in-air response is everywhere negative, and the sign change in the total response would occur when the sphere response eventually dominated the halfspace response.

We have used our algorithm to compute numerical partial derivatives, and Fig. 5 is an example showing the effect on the transient of a 10% change in each of the Cole-Cole parameters $R_0$, $m$ (chargeability), and $\tau$ (time constant). It can be seen that the effects are highly correlated. The response is nonlinear in these parameters, and in Fig. 6 we show curves for four extreme cases, which demonstrate the range of effects caused by the chargeability and time constant parameters.

Discussion

Subject to the limitation that the exponent $c = 0.5$, our models allow us to see the effect, in a three-dimensional case, of changing the major parameters in the complex impedance model we have used.

The effect of the parameter $\tau$ is more evident at late times. If $\tau$ is small, the response is controlled by the size of $m$, and at late times the sphere response decays with an exponent of 2.5. If $\tau$ is large, we expect and observe that the late time decay will still be approximately exponential. Increasing $\tau$ has the effect of causing polarization effects to be observed at earlier times, if they are to be observed at all.

The parameter $m$ (chargeability) has the more profound effect on the shape of the response curves. We view the system as relaxing between high-frequency and low-frequency impedance states, and $m$ describes the separation between those impedances, so we expect and observe that increasing $m$ increases the early-time response. If $m$ is sufficiently large, the transient response will later become negative. We have not yet drawn definite conclusions about the time at which this occurs.

References

Bhattacharyya, B. K. (1964)—Electromagnetic fields of a small loop antenna on the surface of a polarizable medium, Geophysics 24, 814–831.


