The measurement of anisotropy with the generalized reciprocal method of seismic refraction interpretation

Derecke Palmer

Most geological processes of deposition and deformation indicate that anisotropy is more likely than isotropy. Seismic anisotropy, the variation of velocity with direction, can be caused by lamination, foliation, jointing, cleavage and cyclic layering. The resulting anisotropy is usually two dimensional, with the axis of symmetry commonly being vertical in sedimentary rocks.

Anisotropy is not readily detected from surface seismic surveys. Usually only the horizontal component of the velocity is measured. It is important to accommodate anisotropy for accurate depth and refractor velocity computations. In addition the anisotropy factor, the ratio of the horizontal velocity to the vertical velocity may be a diagnostic rock parameter.

Although earth anisotropy is commonly assumed to be elliptical, this form is applicable only to SH waves. P wave anisotropy is more complex, a property which makes the measurement of the anisotropy factor possible with the generalised reciprocal method (GRM) (Palmer 1980, 1981) of seismic refraction interpretation. This approach utilises the difference between the wavefront normal, which is used in the application of Snell's law and the ray direction, which is used in the computation of the optimum XY value.

Figure 1 demonstrates the relationship between the ray angle, ψ, and the wave normal direction, ϕ. The ray, which is the actual path of energy transport, starts at the centre of the ray surface or the Huygens' wavelet and finishes at the point of tangency. The velocity of the wavefront, the phase velocity, is measured in the direction of the wave normal. Since the medium is anisotropic, the ray surface is not circular, and

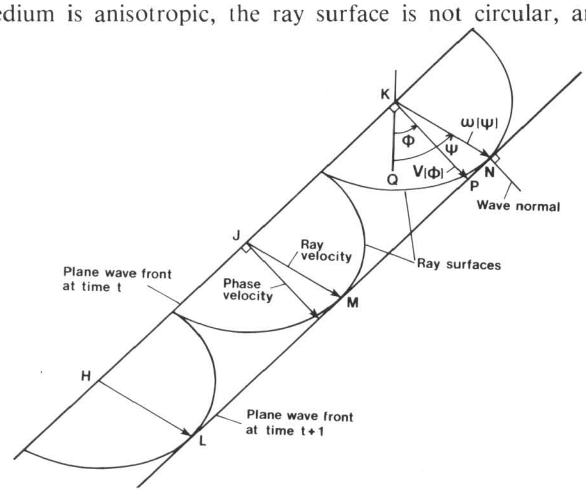

Fig 1

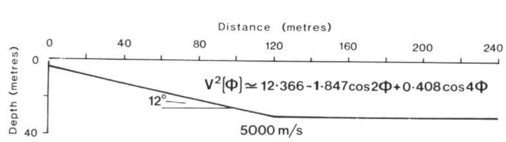

Fig. 2

hence the ray and the wavefront are not orthogonal. Therefore the ray and the wave normal generally are not coincident.

From the simple relationship between the phase velocity $V(\phi)$ and the ray velocity $\omega(\psi)$, viz.

$$V(\phi) = \omega(\psi) \cos(\psi - \phi) \tag{1}$$

it can be shown that

$$\tan\psi = \frac{V(\phi)\sin\phi + \dot{V}(\phi)\cos\phi}{V(\phi)\cos\phi - \dot{V}(\phi)\sin\phi} \tag{2}$$

Equation 2 can be used to compute c, where

$$\tan\psi = c\tan\phi \tag{3}$$

The phase velocity in a transversely isotropic medium is given in terms of the density, the elastic constants and the angle from the vertical by the Stoneley equation (1949):

$$V^2(\phi) = [1/(2\rho)][A\sin^2\phi + C\cos^2\phi + L + \{[(A-L)\sin^2\phi - (C-L)\cos^2\phi]^2 + (F+L)^2\sin^2 2\phi\}^{1/2}] \tag{4}$$

Although exact, this equation is not especially convenient, since elastic constants are not usually measured in conventional seismic prospecting. A more convenient expression is the Crampin equation (1977) which is a Fourier series expansion of the Stoneley equation for weak anisotropy:

$$V^2(\phi) = A + B\cos 2\phi + C\cos 4\phi \tag{5}$$

The Crampin equation can also be applied to strongly anisotropic materials such as the limestone–anisotropic shale medium of Levin (1979).

The Crampin equation can be re-written in terms of parameters which can be measured in the field, or assumed:

Horizontal velocity $= V_H = V(90°)$

Anisotropy factor $= k = V(90°)/V(0)$

$$a = -B/C \simeq 3.85 + 1.75\,k \approx 4$$

The ratio a probably warrants further examination, but a value of about 4 is probably sufficiently accurate for most purposes.

The modified Crampin equation is:

$$V^2(\phi) = [V_H^2/(2ak^2)]\ [(a-1)k^2 + (a+1) - (k^2-1)(a\cos 2\phi - \cos 4\phi)] \tag{6}$$

The critical angle for zero dip, i, can be obtained by solving a quadratic in $\cos 2\phi$, based on eqn (6), the refractor velocity,

V_2, and Snell's law, for a selected value of k, the anisotropy factor. Using this value, the phase velocity at critical refraction, $V(i)$, can be computed.

In addition the factor c (see eqn 3) can be computed, using eqn (7) which is obtained by substituting eqn (6) into eqn (2)

$$c \simeq \frac{k^2 - (4/a)(k^2-1)\cos^4 i}{1-(4/a)(k^2-1)\sin^4 i} \tag{7}$$

This value of c is then used to compute the average velocity $V(\mathrm{XY})$, using also the time-depth, t_G, the optimum XY value and the refractor velocity, V_2,

$$V(XY) = \left[\frac{XY\,V_2^2}{XY + 2c\,t_G V_2}\right]^{1/2} \tag{8}$$

Equation (8) is readily derived by combining eqn (3) above and eqns (9) and (10) below:

$$t_G = Z_G\,\frac{\cos i}{V(i)} \tag{9}$$

$$XY = 2Z_G \tan\psi \tag{10}$$

The value of k, for which the phase velocity $V(i)$ and the average velocity, $V(XY)$ are equal, is taken as the anisotropy factor for the upper layer. This value of k in turn specifies a, and therefore the phase velocity for any angle through the Crampin equation.

An example of this approach can be shown with the model in Fig. 2. The upper layer has the phase velocity function of the limestone–anisotropic shale material, in which the anisotropy factor is 1.157. This value is higher than the commonly accepted average value of 1.10. The refractor has a seismic velocity of 5000 m/s, and the refractor surface has a change in dip from 12° to zero. This change in dip produces changes in slope in the travel time curves from the refractor.

The refractor velocity analysis function, computed with eqn (11) below, is plotted in Fig. 3, for a range of XY values, in a reduced velocity format, to emphasize deviations from straight line segments.

$$t_V = \tfrac{1}{2}[t_{AY} - t_{BX} + AB] \tag{11}$$

An optimum XY value of about 75 m can be recovered. In addition, the apparent refractor velocities are 5195 m/s for the dipping interface segment and 5000 m/s for the horizontal interface segment.

The time-depths, computed with eqn (12) below, confirm 75 m as the optimum XY value.

$$t_G = \{t_{AY} + t_{BX} - [t_{AB} + (XY/V'_n)]\} \tag{12}$$

The phase velocity, computed with eqn (6), and the average velocity computed with eqn (8) are plotted in Fig. 4, for a

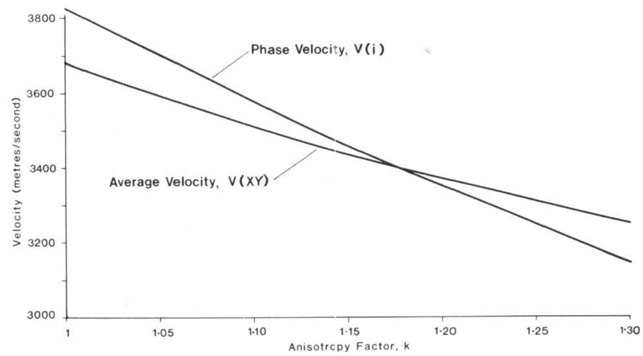

Fig 4

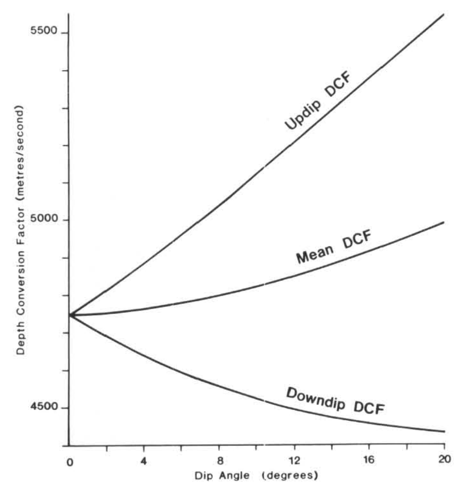

Fig 5

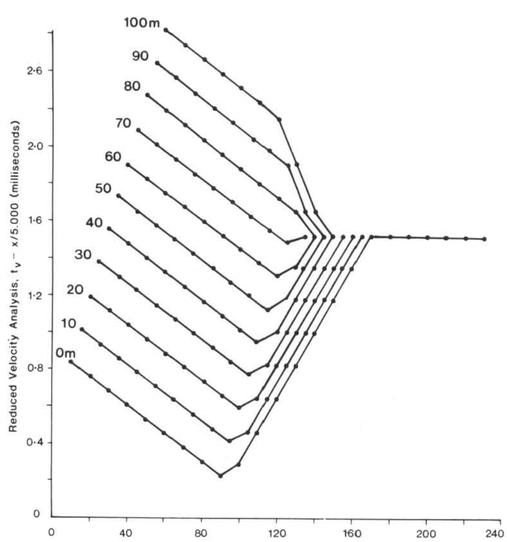

Fig 3

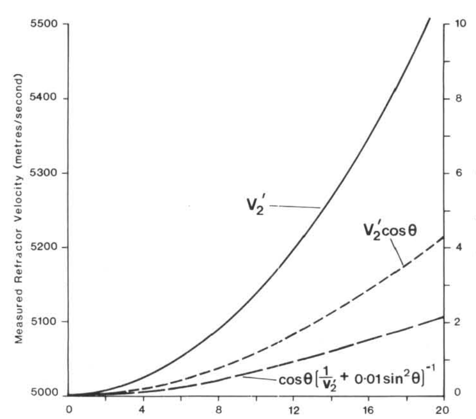

Fig 6

range of anisotropy factors. These velocities are equivalent for an anisotropy factor of 1.175, which is within 1.5% of the correct value.

The phase velocity at this anisotropy factor can be used to compute the depth conversion factor (*DCF*) at zero dip. For a single isotropic layer above the refractor, the *DCF* is $\cos i/V_1$, and it is independent of dip angle. However, when the upper layer is anisotropic, the *DCF* is a mean of the updip *DCF* and the downdip *DCF*, when forward and reverse travel time data are used:

$$DCF = \tfrac{1}{2}\left[\frac{\cos\alpha}{V_1(\alpha+\theta)} + \frac{\cos\beta}{V_1(\beta-\theta)}\right] \qquad (13)$$

where α and β are the updip and downdip critical angles and θ is the dip angle. While both of these *DCF*'s show pronounced dip effects, the mean is reasonably insensitive to dip, as is shown in Fig. 5.

The apparent refractor velocity, V_2', obtained from the inverse slope of eqn (11) is also strongly affected by dip. However, a correction factor which takes account of anisotropy can considerably reduce the errors related to dip. Figure 6 shows the improved refractor velocities obtained with eqn (14):

$$V_2 \simeq \cos\theta\,[(1/V_2') + b\sin^2\theta]^{-1} \qquad (14)$$

where

$$b \simeq V_H^2\,\frac{(k^2-1)}{k^2}\,\frac{\cos i\,\sin 2i}{V^3(i)} \qquad (15)$$

Acknowledgments

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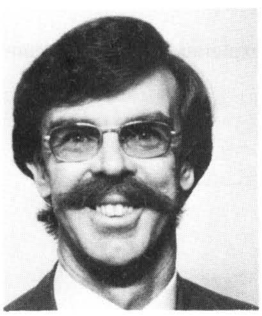

Derecke Palmer received his BSc in exploration geophysics and applied mathematics from the University of Sydney in 1966. From 1967 to 1971, he worked as a geophysicist with the Geological Survey of New South Wales on a wide range of mineral exploration, groundwater, and engineering projects. Since 1971, he has been a senior geophysicist, specialising in seismic methods, and in 1976 was awarded an MSc degree from the University of Sydney for work on refraction methods. He is the author of the book *The Generalized Reciprocal Method of Seismic Refraction Interpretation* published by the SEG in 1980, and he is currently completing another book entitled *Refraction Seismics* for Geophysical Press. His present interests include anisotropy, and seismic refraction and high resolution reflection methods. He is a member of SEG and ASEG.

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Central loop transient electromagnetic soundings

J. Peacock and A. King

Introduction

Geoterrex has been doing transient electromagnetic (TEM) soundings using the Geonics EM37 system in Australia for three years. This paper presents a brief summary of the equipment, field and interpretation procedures, and a few of the more interesting sounding case histories we have encountered.

System description

The EM37 TEM system was designed as a high powered, broadband TEM system capable of acquiring multi-component data quickly along profiles or doing broadband, quantitative soundings. The EM37 system transmits carefully controlled, nearly square pulses of alternating polarity, up to 30 A in amplitude, into a large loop which ranges in practice from 40 m² to 1000 m².

A small, 100 turn/m², air cored receiving coil measures the changing magnetic field in the off time between successive pulses. Transmitter base frequencies, ranging from 25 Hz down to 0.25 Hz, provide data at delay times from 80 μs to 800 ms over 4 decades of time. The very early times are important for resolving weak conductors or shallow layers in resistive environments, while the later times are required to penetrate thick conductive units.