Efficient Forward Modelling of Electromagnetic Surface Impedance for Coal Seam Assessment

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SUMMARY

The two-dimensional impedance method is used to calculate the electromagnetic surface impedance above subsurface structures at very low frequencies (VLF). The method was derived from Faraday's and Ampere's Laws and results in a single matrix equation where the right hand side corresponds to the source field introduced into the model as a fixed magnetic value. The left hand side corresponds to the impedance matrix determined by discretizing the solution space into pixels bounded by lumped impedance elements with values determined by the electromagnetic properties of the local media. Due to matrix sparsity and very large matrix dimensions, an iterative solver with a preconditioning technique was used to improve the speed, size and convergence of the solution. The improved method has been applied to the analysis of a coal seam with various structural anomalies and line of oxidation (LOX) along a line defined by 500m with 0.5m resolution. This paper reports a number of likely coal-seam scenarios relevant to surface mining operations.

Key words: Surface impedance, impedance method, iterative solvers, coal seam, electromagnetic geophysics.

INTRODUCTION

Surface impedance measurements have been used for many decades in geophysical surveying to map subsurface features of the earth. Initially the method was referred to as the Magnetotelluric Method (MT). It was developed during the 1950's as an electromagnetic geophysical method of imaging the earth's subsurface by measuring natural variations of electrical and magnetic fields at the earth's surface (Cagniard, 1953).

The earth's time-varying magnetic field is generated by both naturally occurring radiation sources, which include the Magnetotelluric Method (MT) and its variant, the Audiomagnetotelluric Method (AMT), and artificial sources which include Controlled Source Electromagnetic (CSEM), Very Low Frequency (VLF) and other radio sources.

Given the complexities of the earth's subsurface including horizontal layering, vertical faults, dislocations and folded structures, the forward modelling problem using analytical methods is limited to a few idealized cases (Porstendorfer, 1975).

The two-dimensional impedance method (James, 1999 and Thiel, 2001) was derived to solve quasi-static electromagnetic and radiation problems. The solution space is discretized into cells and using an impedance element at each edge to represent the size of the cell and the electromagnetic properties of the material enclosed. The current in each impedance element is calculated via matrix inversion.

Due to matrix sparsity and very large matrix dimensions, an iterative solver has been implemented for the matrix system solution. One of the challenges is that iterative solvers often fail to converge and at low frequencies the resulting matrix can be very ill conditioned. For that reason a preconditioning technique has been implemented in order to improve the speed, size and convergence of the solution (Saad, 2003).

ELECTROMAGNETIC SURFACE IMPEDANCE

Surface impedance measurements constitute a large part of frequency-domain techniques used in electromagnetic geophysics. In particular the Magnetotelluric (MT) (Cagniard, 1953) method use a simple expression to relate the apparent resistivity ρ_a to the surface impedance Z_s defined by

$$\rho_a = \frac{|Z_s|^2}{\omega\mu} \qquad (1)$$

where ω is the angular frequency of the radiation, μ is the magnetic permeability of the earth, and Z_s is the surface impedance defined as the ratio of the horizontal electric field component E_x measured on the surface of the earth and the horizontal magnetic field H_y, measured perpendicular to the electric field component (Wait, 1970). The magnetic field is the primary field and the energy source, while the horizontal electric field is the secondary reradiation from the earth. Thus

$$Z_s = \frac{E_x}{H_y} = \sqrt{\frac{j\omega\mu}{\sigma + j\varepsilon\omega}} \qquad (2)$$

where σ is the conductivity and ε is the permittivity. The surface impedance is a complex number, as the phase relationship between the two fields components will vary. If the earth is uniform and has a relatively high conductivity, then the phase of Z_s is 45°. If the earth is horizontally layered, then the surface impedance given by (2) is modified by a factor Q (Wait, 1970) defined as

$$Z_s = Q\,Z_1 \qquad (3)$$

where Z_1 is the intrinsic impedance of the upper layer and

$$Q = \frac{Z_2 + Z_1 \tanh(u_1 h)}{Z_1 + Z_2 \tanh(u_1 h)} \tag{4}$$

$$u_1 = \sqrt{\gamma_1^2 - \gamma_0^2 \sin^2(\varphi)} \tag{5}$$

where Z_2 is the surface impedance of the lower half-space, u_1 is the complex propagation coefficient for the top layer in the vertical direction, γ_0 is the free-space propagation coefficient, γ_1 is the propagation coefficient in the first layer, φ is the angle of incidence measured with respect to the surface normal and h is the depth of the upper layer.

The factor Q is dependent on the depth of the layer h and the conductivity of both media. The propagation coefficients are determined by

$$\gamma_n = \sqrt{\omega^2 \mu_n \varepsilon_n - j\omega\mu_n\sigma_n} \tag{6}$$

where μ_n, ε_n and σ_n are the permeability, permittivity and conductivity of layer n, respectively.

Based on this theory, VLF surface-impedance measurements have been used successfully to map the conductivity structure of the upper parts of the earth's subsurface, however, given its complexities including dislocations, folded structures, etc., the forward modelling problem using the analytical method is limited to few cases (Porstendorfer, 1975). The impedance method is implemented in order to analyse very large and complex problems.

THE IMPEDANCE METHOD

The impedance method requires the solution space to be discretized into two-dimensional rectangular cells (pixels or voxels) bounded by impedance elements with properties dependent on the local electromagnetic properties of the medium and the cell size (James, 1999 and Thiel, 2001). In the formulation, the magnetic field is assumed to be known at the source and calculated throughout the solution space. Figure 1 shows an inhomogeneous model where an applied magnetic flux induces cell current in each enclosed area.

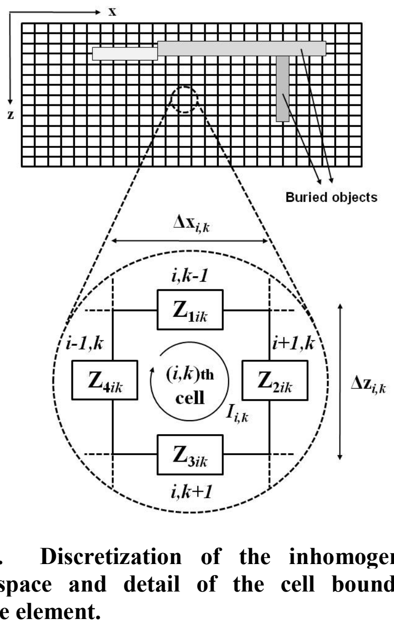


Figure 1. Discretization of the inhomogeneous 2-D solution space and detail of the cell bounded by an impedance element.

From Figure 1, $I_{i,k}$ is the circulating current in the (i,k)th cell and can be determined in terms of the four adjacent cells. The impedance of the (i,k)th element can be defined as

$$Z_{mik} = \frac{\Delta x_{i,k}}{(\sigma_{ik} + j\omega\varepsilon_{ik})\Delta y \, \Delta z_{i,k}} \tag{7}$$

where σ_{ik} and ε_{ik} are the conductivity and permittivity, respectively, of the material located in the (i,k)th cell, $\Delta z_{i,k}$ is the height of the first element and Δy is a constant width assigned to all elements throughout the solution space required by the 2-D formulation.

Using a combination of Ampere's Law, Faraday's Law and Kirchoff's Laws, the magnetic field $H_{i,k}$, in each element, can be calculated using the matrix equation

$$[\mathbf{S}]_{N\times N} [\mathbf{H}]_{1\times N} = [\mathbf{H}_0]_{1\times N} \tag{8}$$

where $\mathbf{H}$ is the vector of unknown magnetic field elements, $\mathbf{H}_0$ is the known source field vector (in general the number of non-zero elements in $\mathbf{H}_0$ will be small), N is the number of impedance elements in the solution space matrix and $\mathbf{S}$ is a sparse, square, matrix of size N^2, which, although dimensionless, represents the electrical properties and the physical dimensions of the pixels in the solution space.

The solution matrix $\mathbf{S}_{N\times N}$ is given in terms of the complex propagation coefficient γ_{ik} in the (i,k)th which is defined as

$$\gamma_{ik} = \sqrt{\omega^2 \mu_{ik}\varepsilon_{ik} - j\omega\mu_{ik}\sigma_{ik}} \tag{9}$$

Once the solution vector $\mathbf{H}$ from equation (8) has been determined, the surface impedance Z_s can be calculated from equation (2) as the ratio of the electric and magnetic fields at the surface of the earth modelled with an upper air layer (Thiel, 2001). Thus we can write

$$Z_s = \frac{E_x}{H_y} = \frac{(H_{i,k-1} - H_{i,k}) Z_{1ik}}{H_{i,k-1} \Delta y \, \Delta x_{i,k}} \tag{10}$$

where Z_{1ik} can be determined by equation (7). The same approach can be applied to the three remaining sides of the (i,k)th cell.

The formulation has been previously verified by comparison with the exact analytical solutions for a uniform half space and a horizontally layered half space and a vertical discontinuity (Thiel, 2001).

ITERATIVE SOLUTION WITH PRECONDITIONING

Krylov subspace methods are very useful for solving large systems of linear equations. In this paper we implement the GMRES (General Minimum Residual) method which approximates the solution by a guess vector in a Krylov subspace with minimal residual, where Arnoldi iteration is used to find the solution vector (Saad, 2003).

GMRES offers advantages in terms of speed and storage over other iterative solvers. The method requires only one matrix-vector product per step, and its residual error decreases monotonically, so it does not present any oscillation.

The convergence rate of GMRES is strongly related to the condition number of the linear system, at low frequencies the resulting matrix can be very ill conditioned. Hence, a preconditioning technique is necessary to improve the speed, size and convergence of the solution (Espinosa, 2006).

Preconditioning transforms the original linear system $Ax = b$ into an equivalent one which is easier to solve by GMRES. A preconditioner M of a matrix A of size $N \times N$ coefficients is a matrix such that $M^{-1}A$ has a smaller condition number than A. Three general types of preconditioning can be implemented (left, right and split). For our purposes we use the left preconditioned system given by $M^{-1}Ax = M^{-1}b$ where M^{-1} is chosen to improve the condition of the system and is "cheap" (computationally fast) to be computed and applied inside GMRES. M^{-1} should in some sense approximate the inverse of A.

Various preconditioning techniques such as Jacobi, SOR and SSOR, are well detailed in (Saad, 2003). One of the most popular is the *Incomplete LU factorisation*. The *ILU* can be achieved by setting values below a given threshold (drop tolerance) τ to zero, in this case, the *ILU* yields a sparse unit lower triangular $N \times N$ matrix L and a sparse upper triangular $N \times N$ matrix U, returned in the lower and upper triangular parts of the matrix LU.

In this paper we implemented the row-sum modified incomplete LU factorization (*MILU*) with pivoting (Benzi, 2002). Like ordinary *ILU*, *MILU* computes lower triangular L and upper triangular U by Gaussian elimination on the input matrix A, dropping some of the entries of L and U to keep them sparse. However, when *MILU* drops an element of either L or U, it compensates by adding the value of the dropped element to the diagonal of U in the same row providing higher accuracy in the solution. Once the modified *ILU* factorisation has been computed, it is applied with every step of the GMRES algorithm.

NUMERICAL EXPERIMENTS

Figure 2 shows a 2-D model which consists of a region defined by 500m length and 125m depth with 0.5m cell resolution in x and z direction, 1m upper air layer and host rock parameters defined by $\sigma = 10^{-2}$ S/m (conductivity) and $\varepsilon = 15\varepsilon_0$ F/m (permittivity). Various structural anomalies in the model include a line of oxidation (LOX) of size 5m×10m at 10m depth with $\sigma = 5\times10^{-3}$ S/m and $\varepsilon = 10\varepsilon_0$ F/m, three coal seams of sizes 2m×190m, 2m×100m and 2m×120m at 13m, 14m and 15m depth, respectively, with $\sigma = 10^{-3}$ S/m and $\varepsilon = 3\varepsilon_0$ F/m, and a vertical basalt intrusion extended from the upper air layer to the bottom layer, 10m×124m, with $\sigma=3\times10^{-2}$ S/m and $\varepsilon = 10\varepsilon_0$ F/m. The model is analysed at frequencies $f = 19.8$ kHz (VLF at North West Cape, Western Australia) and $f = 4$ kHz (frequency component where most energy has been found in the earth-ionosphere waveguide some distance from a lightning stroke, Thiel, 1988).

A comparison between the analytical solution with that obtained from the impedance method was performed for both frequencies considering the coal region limited by 150m to 170m in Figure 2 refered to as "Validation Area". The magnitude of the surface impedance compared with the analytical solution showed a very good agreement providing relative errors of 0.84% at 19.8kHz and 2.08% at 4kHz.

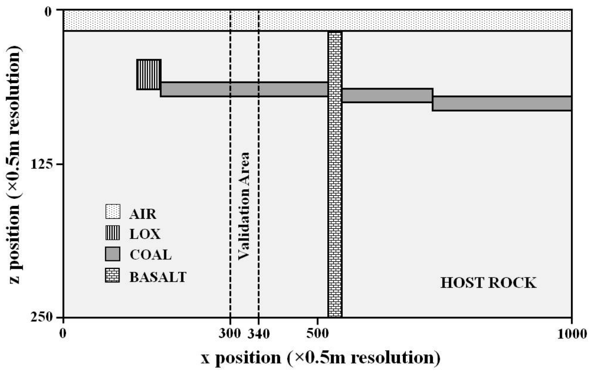


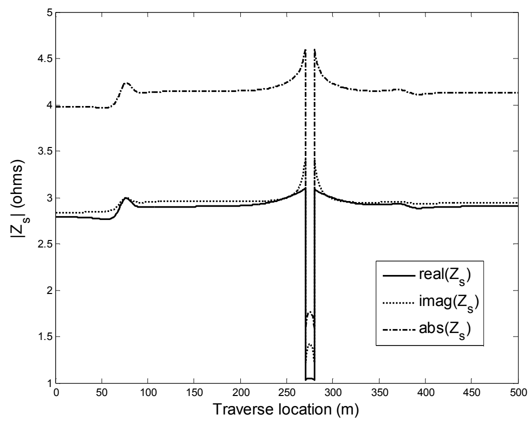
Figure 2. Two-dimensional region of 500m length and 125m depth with 0.5m cell resolution, various structural anomalies, LOX and a basalt intrusion.

The matrix $\mathbf{S}_{N\times N}$ to be solved has dimensions 250,000 × 250,000 and size 25MB. The computations were performed on an Intel Core i5 @ 3.20GHz PC with 4GB of RAM. For each frequency, the sparse matrix $\mathbf{S}_{N\times N}$ was computed in 75sec. For the solution of the linear system of equation (8), a drop tolerance of 10^{-3} for the modified incomplete LU factorization and a tolerance of 10^{-6} for the GMRES were used. The linear system was solved in 50sec with a relative residual of 10^{-8}.

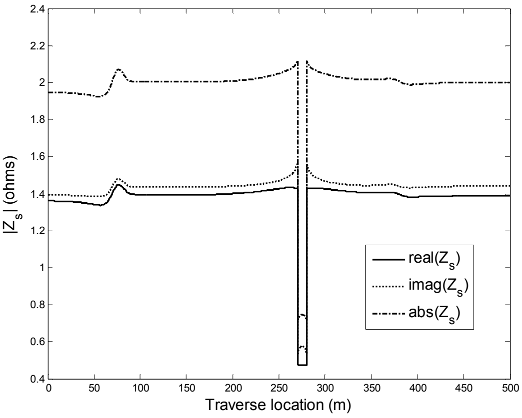
Figure 3 shows the surface impedance (real, imag and magnitude) for each frequency while Figure 4 shows the phase of the surface impedance. The apparent resistivity is computed in Figure 5 for each frequency and it is determined by equation (1).

In Figures 3, 4 and 5, the LOX, basalt intrusion and the step dislocation are clearly evident in the surface impedance data at both frequencies.

The phase in Figure 4 is approximately 53° on the basalt intrusion at 19.8kHz and 50° at 4kHz. This implies that in this area the VLF signal is attenuated in the uppermost layer. For the LOX and coal areas the phase remains constant around 45°. From Figure 5, the apparent resistivity approaches zero in the basalt intrusion because of its high conductivity. The system proved to be slightly more resistive for the lower frequency $f = 4$kHz.



(a)



(b)

Figure 3. Surface impedance for a two-dimensional solution space of 500m length and 125m depth at frequencies (a) 19.8kHz and (b) 4kHz.

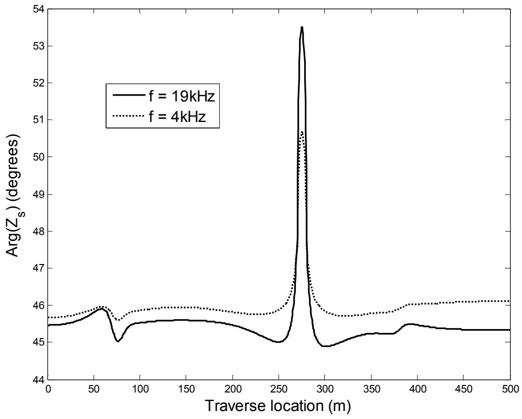


Figure 4. Surface impedance phase (degrees) for a two-dimensional solution space of 500m length and 125m depth at frequencies 19.8kHz and 4kHz.

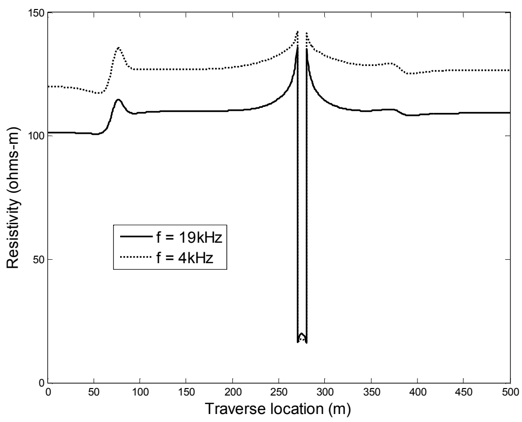


Figure 5. Apparent resistivity for a two-dimensional solution space of 500m length and 125m depth at frequencies 19.8kHz and 4kHz.

CONCLUSIONS

The impedance method enables the forward modelling of surface impedance profiles that satisfies surface impedance theory for horizontally layered sub-surface structures at very low frequencies (VLF). In the self-consistent formulation only the source field is known and the total magnetic field throughout the solution space is calculated. From this the distribution of the current and the electric field can be determined. The formulation allows for flexible cell size and shape, and for the ability to handle anisotropy.

The Impedance Method has the advantages of being conceptually simple and computationally efficient, since the matrix involved is square, symmetrical and sparse (tridiagonal with two off-side bands).

The implementation of the GMRES iterative solver and the MILU preconditioner allows for the analysis of very large and complex structures solving the matrix system in a reduced time producing a small relative residual, thus making the method a very efficient tool for realistic problems.

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