# Multimodal direct fitting of SPAC coefficients using amplitude response


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## SUMMARY

We usually analyse microtremors without any considerations of higher modes of surface waves. However, recent studies have demonstrated that the use of higher modes would improve estimated velocity models. In this study, we apply multi-mode analysis with amplitude response to the direct fitting method in SPAC method. The proposed method has the advantage that we don't need to identify observed modes, which can avoid mode misidentification. The proposed method was applied to synthesized microtremors. Estimated SPAC coefficients were good agreed with theoretical ones proposed in this study, even if in the frequency where two modes have about the same power fractions. The S-wave velocity was then successfully inverted by the direct fitting of SPAC coefficients. The proposed method also works well in the analysis of field data acquired in Tsukuba City, Japan. In the inversion performed in this study, we do not use prior information about velocity structure. It is concluded that the multi-mode analysis proposed here is very robust in the multimodal analysis in SPAC method.



**Key words:** multi-mode inversion, higher modes, SPAC method, microtremors


## INTRODUCTION

The microtremor method (Okada 2003) has been applied to investigate engineering geotechnical site characterization. The spatial autocorrelation (SPAC) method (Aki, 1957) is one of the most effective array techniques in the microtremor method. Asten *et al*. (2004) proposed the direct fitting method in the SPAC method without interpretation of the dispersion curve. Recent studies demonstrated that higher modes of surface waves cannot be ignored (e.g., Ohori *et al*., 2002; Asten and Robert, 2006). Xia *et al*. (2003) also pointed out that the use of higher modes improves the S-wave velocity estimation.

In order to investigate multi-mode analysis, the work by Aki (1957) becomes starting point. Aki (1957) proposed SPAC method including partial waves having different phase velocities. This regards observed SPAC coefficients as a superposition of each component weighted by its power fraction. Some authors have developed multi-mode analysis methods by extending Aki's approach to higher modes of surface waves. Asten (1976) separated two modes of surface waves by solving phase velocities and energy fractions from observed data. Asten's multi-mode method has been extended a direct fitting approach, called multi-mode SPAC (MMSPAC) (Asten *et al*., 2004). Tokimatsu *et al*. (1992) also proposed the multi- mode method in which energy fractions of each mode are calculated by dispersion curves and amplitude responses (Harkrider, 1964, 1970) of higher modes.

The latter approach using amplitude response is superior in the point that we don't need to identify observed modes and therefore, we can avoid mode misidentification. Despite its simplicity, however, the multi-mode analysis using amplitude response has not been extended to the direct fitting method.

In this study, we proposed the multi-mode analysis using amplitude response in the direct fitting method of SPAC coefficients. The applicability of proposed method was then investigated by applying simulated microtremors. The proposed method was also applied to field data acquired in Tsukuba City, Japan.

## THEORY OF SPAC METHOD

The basic theory of SPAC method is summarized in Okada (2003). Suppose microtremors are obtained by a circle array with a radius $r$ and surface waves of fundamental mode are dominant in microtremors. The azimuthal average of complex coherencies $COH$ between a central and a circumferential receiver goes to the Bessel function, called SPAC coefficient $\rho$,

$$\rho(r,\omega) = \frac{1}{2\pi}\int_0^{2\pi} COH(r,\omega,\theta)d\theta = J_0\left(\frac{\omega}{c(\omega)}r\right) \tag{1}$$

where, $\omega$ is angular frequency, $\theta$ is azimuthal angle, $c$ is phase velocity, and $J_0$ is the Bessel function of the first kind of zero order. The phase velocity is obtained by fitting SPAC coefficients to the Bessel function. The direct inversion method using SPAC coefficients (Asten *et al*., 2004) is to compare observed SPAC coefficients with theoretical ones for an assumed medium in an inversion.

Aki (1957) proposed the description of the SPAC coefficient including partial waves having different velocity. In this case, the SPAC coefficient is described as,

$$\rho(r,\omega) = \sum_i \frac{P_i(\omega)}{P(\omega)} J_0\left(\frac{\omega}{c_i(\omega)}r\right) \tag{2}$$

$$P(\omega) = \sum_i P_i(\omega) \tag{3}$$

where, $P_i$ and $c_i$ are the power and velocity of $i$ th component, respectively.

Harkrider (1964) derived the vertical displacement of $i$ th Rayleigh mode for a harmonic vertical point force, called amplitude response (Harkrider, 1970). Tokimatsu *et al*. (1992) developed the multi-mode analysis method by extending Aki's method to higher modes using amplitude response. Tokimatsu *et al*. (1992) extracted the contribution of vertical displacement of each mode depending on only a frequency by applying an asymptotic expansion. Finally, Tokimatsu *et al*. (1992) regarded its square value as a power fraction in Eq. (2) and it can be written as,

$$P_i(\omega) = c_i(\omega) A_i^2(\omega) \tag{4}$$

where, $A_i$ is the amplitude response of $i$ th mode.

Since theoretical phase velocity and amplitude response of each mode can be calculated for an assumed layered medium, we can obtain theoretical SPAC coefficients from Eq. (2)~(4). Therefore, we can easily introduce multi-mode inversion using Eq. (2)~(4) to the direct fitting method of SPAC coefficients. We will show the results of the proposed multimodal inversion in the following sections.

## SIMULATION STUDY

To evaluate the effectiveness of the multi-mode inversion method using amplitude response, simulation study was conducted. Figures 1 and 2 show the four-layered model used for simulation study and theoretical dispersion curves and power fractions of each mode. The power fraction of each mode was defined by normalizing a power of each Rayleigh mode (Eq. (4)) by a summation of power fractions of considered modes. Figure 2 indicates that the power fraction of 1st higher mode is grater that of fundamental mode in the frequency rage from 5 to 7.5Hz. It is expected that 1st higher mode of surface waves couldn't be neglected in this range.

Simulated microtremors were synthesized by the following procedure. We assume the triangle array with 10 receivers (Figure 3). Discrete Wave-number Integral (DWI) method (Bouchon and Aki, 1977) was employed for waveforms calculation. The source is a vertical force with 8Hz Ricker wavelet. 1000 sources are randomly distributed on the surface and then each waveform is calculated, separately. By superposing 50 waveforms chosen from 1000 waveforms randomly, simulated microtremors of about 30 seconds were synthesized. Likewise, total of 100 data sets were made.

SPAC coefficients were obtained from simulated data by averaging those of 100 data sets. Figure 4 shows obtained SPAC coefficients and theoretical ones calculated from Eq. (2)~(4) corresponding to $r$ = 25m and 50m. Blue and green solid lines correspond to the theoretical SPAC coefficient of fundamental mode and 1st higher mode, respectively. Meanwhile dash lines show power fractions of each mode. We can see the transition of observed SPAC coefficients from fundamental mode to 1st mode with the increase of the power fraction of 1st mode, and SPAC coefficients are good agreed with theoretical ones, even if power fractions of two modes are about the same.

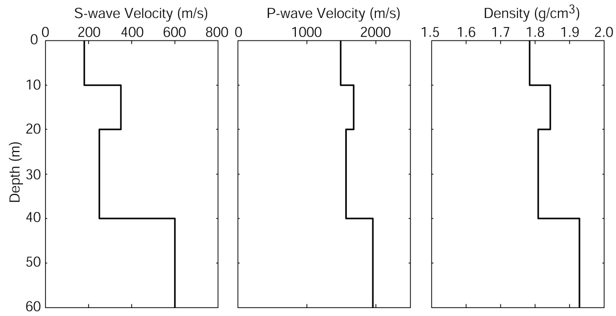


**Figure 1. Simulated layered model.**

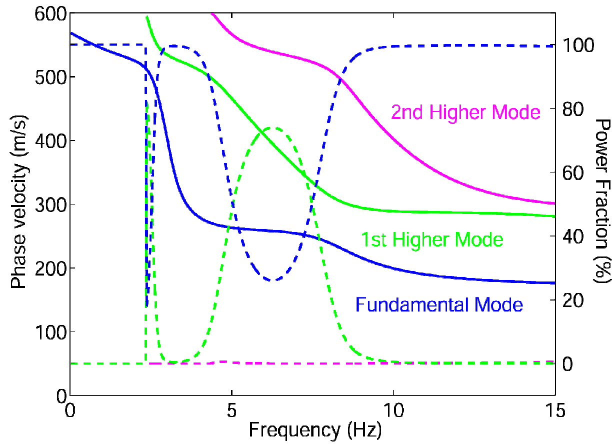


**Figure 2. Theoretical dispersion curves (solid lines) and power fractions (dash lines) calculated from Eq. (4) of each mode for the simulated model (Figure 1).**

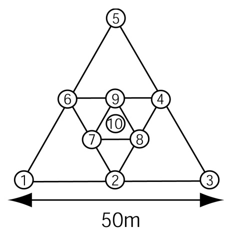


**Figure 3. Array shape.**

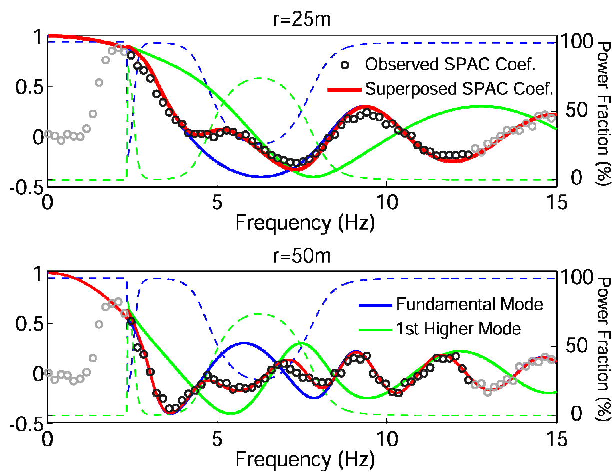


**Figure 4. Comparison of observed SPAC coefficients with theoretical ones (red lines) corresponding to r = 25m, 50m. Dash lines show power fraction ratios calculated by Eq. (4). Only black circles of observed SPAC coefficients are used in an inversion.**

Furthermore, S-wave velocity profiles are directly inverted by the direct fitting of SPAC coefficients using amplitude response. To chose some parameters in the inversion processing, the dispersion curve was obtained from ESPAC method (Okada, 2003), though the estimated dispersion curve was not used in inversion at all. Observed phase velocities were selected by the following relation.

$$2r_{\min} < \lambda < 4r_{\max} \quad (5)$$

where, λ is the wavelength of the observed phase velocity and $r_{min}$ and $r_{max}$ are the minimum and maximum receiver separation distances in the observed array. The frequency range of SPAC coefficient used in inversion was same as that of the observed dispersion curve. The reference model related to the model search range in inversion is determined by transforming a third of observed wavelength vs. frequency to S-wave velocity vs. depth.

Genetic Algorithms (GA) with elite selection and dynamic mutation (Yamanaka and Ishida, 1996) was employed for the inversion processing. Six-layered model was assumed in the inversion. The search range of S-wave velocity and thickness is ±50% for the reference model. 20 trials were performed with the random seeds of an initial population. The final inverted model was constructed by averaging the S-wave velocity and thickness for each layer of 20 trials.

The S-wave velocity profile estimated by multi-mode inversion is shown in Figure 5. We can see the final model obtained by inversion is good consistent with simulation model though the standard deviations near the infinite half-space are relatively large due to a lack of observed values of in the lower frequency. Especially, the reversal of S-wave velocity, which is usually difficult to obtain without using higher modes, is well retrieved.

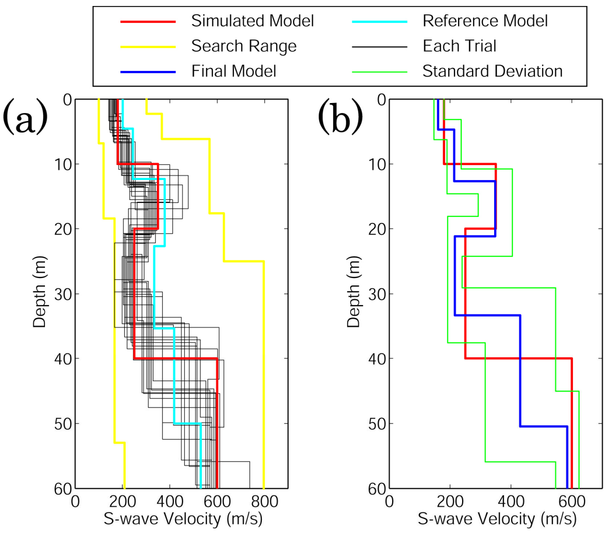


**Figure 5. Results of the direct fitting inversion using amplitude response. (a) Simulated model (red), reference model (cyan), inverted models for each trial (black) and search range in GA inversion (yellow). (b) Final inverted model (blue) obtained by averaging S-wave velocity and thickness for each layer of 20 trials and its standard deviations (green).**

## FIELD EXAMPLE

The proposed method is also applied to the field data. The survey site is located in the Tsukuba City, Japan. The array shape is similar to Figure 2 and the largest length of the array is 30m. Sampling time is 2msec and data length is about one hour and 20 minutes. In this site, P-and S-wave velocity structures were known by PS-logging (Suzuki and Takahashi, 1999). Figure 6 shows theoretical dispersion curves and power fractions of each mode calculated from logging data. We can see the 1st higher mode is predominant near 7.5Hz from its power fraction.

Figure 7 shows observed SPAC coefficients using acquired field data and theoretical ones constructed by the logging data, corresponding to $r$ = 15m and 30m. Although the power fraction of 1st mode is relatively high near 7.5 Hz, observed SPAC coefficients are agreed with theoretical ones. The S-wave velocity was then estimated by GA inversion. The reference model was also constructed by a waveforms transformation. Other parameters in GA inversion are same as simulation study. Figure 8 shows the final inverted S-wave velocity. This estimated model by the inversion is good agreed with logging data.

## CONCLUSIONS

In this paper, we proposed the multimodal analysis using amplitude response of Rayleigh mode in the direct fitting method of SPAC coefficients. Use of amplitude response to obtain power fractions of each mode is superior in the point that we don't need to solve the power fractions from observed data. Therefore, we can avoid mode misidentification of observed phase velocities. Practical application point of view, this point becomes important since it may solve uncertainty problem by experimental knowledge of engineers.

We applied the proposed method to synthesized microtremors in order to conduct quantitative evaluation of the method. SPAC coefficients obtained from simulated data were good consistent with theoretical ones proposed in this study. The inverted model by GA is good agreed with simulated model without prior information of simulated model. Our multi-mode method also worked well in field data obtained in Tsukuba-City, Japan. We could demonstrate the efficiency of the proposed method.

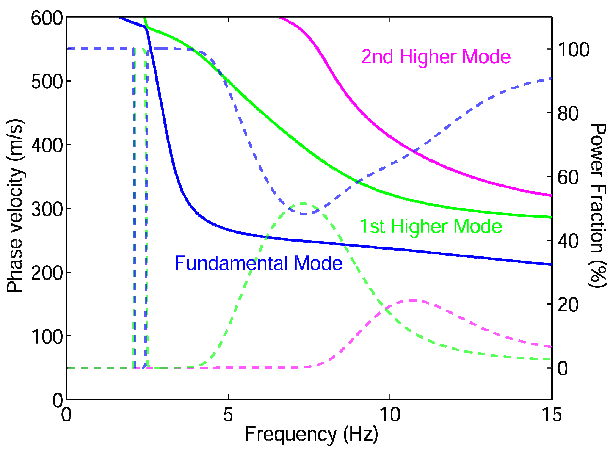


**Figure 6. Theoretical dispersion curves (solid lines) and power fractions (dash lines) calculated from Eq. (4) of each mode for a model constructed from logging data.**

## ACKNOWLEDGMENTS

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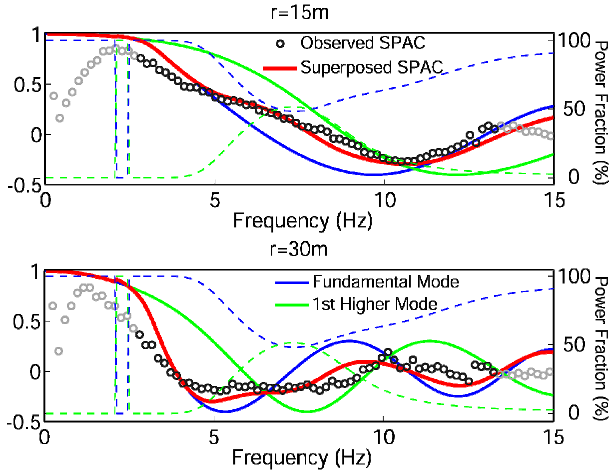


**Figure 7. Comparison of observed SPAC coefficients with theoretical ones (red lines) corresponding to r = 15m, 30m. Dash lines show power fraction ratios calculated by Eq. (4). Only black circles of observed SPAC coefficients are used in an inversion.**

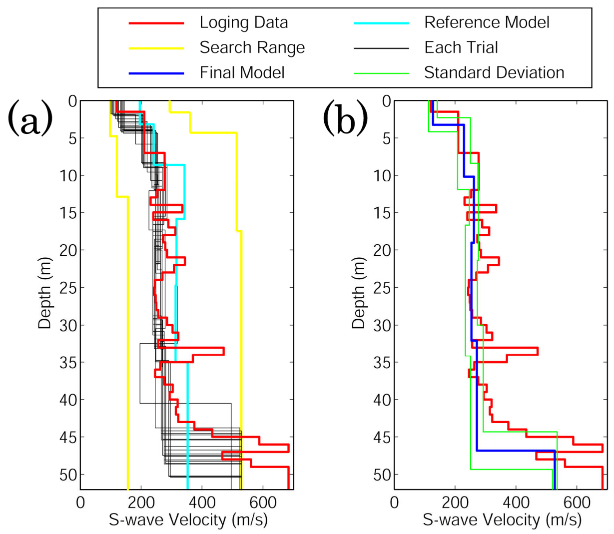


**Figure 8. Results of the direct fitting inversion using amplitude response. (a) Simulated model (red), reference model (cyan), inverted models for each trial (black) and search range in GA inversion (yellow). (b) Final inverted model (blue) obtained by averaging S-wave velocity and thickness for each layer of 20 trials and its standard deviations (green).**

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