

# Velocity Model Determination for Accurate Location of Mining-Induced Seismic Events

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## SUMMARY

Determining the location of mining-induced seismic events is strongly dependent on having an accurate velocity model. However, such a model is seldom available. This paper describes the determination of a velocity model for seismic event location, using the seismic events themselves as sources whose location is to be determined along with the parameters of the velocity model (a simultaneous inversion of event locations and velocity structure). Seismic monitoring of a mine in Colorado is used as an example, with an array of geophones installed both on the surface and in underground roadways. Velocity models of increasing complexity are considered, starting with a homogeneous velocity, moving to a (slightly dipping) layered-earth model, and eventually including static time shifts to account for the effects of a weathered, near-surface, low-velocity layer on arrival times at geophones mounted on the surface. This series of increasingly complex models obviously shows increasingly better fits to the data, but also shows more plausible event locations, and with more realistic elevation spans. Examination of spatial patterns in the residuals indicates that there are likely mining-induced changes in velocity that are not accounted for in the model.

**Key words:** mining-induced seismicity, velocity model, inversion

## INTRODUCTION

Monitoring of mining-induced seismicity provides a way to observe the response of the rockmass to mining activity. The locations and magnitudes of these seismic events, coupled with a geomechanical model of how the stresses caused by mining are evolving, can be used for the detection of potentially hazardous zones of anomalous seismicity and rockmass response. (See Gibowicz and Kijko, 1994, for background material on mining seismology.) In many hardrock mines, daily seismicity maps and reports are used for hazard management purposes. This work describes seismic monitoring in coal mines, where these techniques are not yet routinely used due to more-complex velocity structures and faster mining rates.

The most important parameter associated with a seismic event is its location, without which other source parameters such as magnitude, stress drop, and moment tensor cannot be

obtained. Accurate locations require knowledge of the velocity structure of the rock between the seismic events and the sensors, but sufficiently detailed models are seldom available. In hard rock environments, the velocity variation is small enough that a homogeneous velocity model can yield meaningful event locations, but this is not the case in a sedimentary coal environment. Particularly when seismic sensors are distributed between surface and underground locations, the velocity structure is likely to contain significant variation, so that a homogeneous velocity model will produce poor results. To address these issues, this paper describes work using the mining-induced seismic events themselves as sources, so that their location is determined simultaneously with the parameters of the velocity model.

Simultaneous velocity structure and earthquake location inversion was first done by Crosson (1976), followed by more work at the crustal scale. More recently, the idea has been applied to estimating 3-D velocity models using hydraulic fracturing events (see Zhang and Thurber, 2003), Block *et al.* 1994, and references). The work presented here is on a mine scale of a few kilometers, and focuses on the event locations rather than on the velocity model.

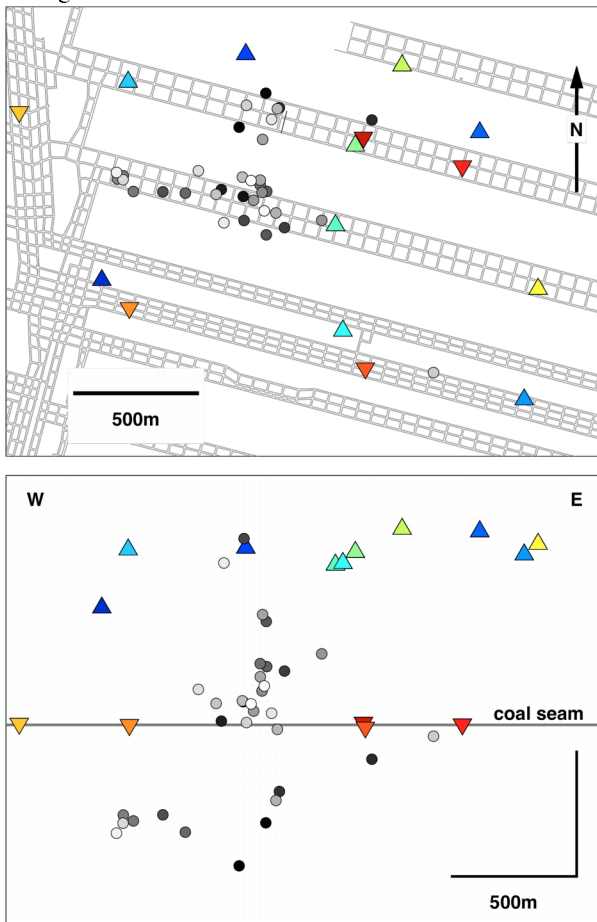
## METHOD AND RESULTS

In contrast to hard rock mines, where homogeneous velocity models have proven adequate for locating seismic events, the sedimentary environment of a typical coal mine contains a much larger variation of seismic velocity between rock types, and so the simple homogeneous model results in very poor location results. Figure 1 shows an example of event locations from a mine in Colorado. These were determined by a commercial seismic system using a homogeneous velocity model. While the plan view of the event locations looks reasonable, the elevations seen on the section view are clearly incorrect, spanning an elevation range of almost 2 km (with some events occurring up in the air), versus a seismogenic zone of a few hundred metres. In addition, the fit to the data shown by the residuals is extremely poor, indicating that the model is inadequate.

The most likely reason for both the poor data fit and the unrealistic locations is that the actual velocity is much more complex than this simple model represents. Two likely corrections suggest themselves: First, the geology of the area consists of gently dipping sedimentary rocks, so a layered velocity model would be appropriate. Second, many of the geophones are on the ground surface, so static time corrections

may be needed to correct for near-surface low-velocity zones due to weathering.

There are three sources of knowledge available about the velocity structure: First, a sonic velocity log conducted in the vicinity of the coal seam down a gas-vent borehole includes a few metres of the rock overlying the seam. Second, an underground weight-drop source in a known location was recorded on a surface array of exploration geophones, so the traveltimes vs offset curve was used to determine a first-pass velocity. (Unfortunately, the source was not sufficiently powerful to trigger the mine seismic monitoring system.) Third, a refraction survey carried out on this same surface array gives some information on the velocities within a few tens of metres of the surface. This information is included as a (rather uncertain) prior in a Bayesian inversion for both the velocity model and, simultaneously, the locations of the mining-induced events.



**Figure 1. Original, homogeneous velocity model, event locations in (a) plan view and (b) section view. The event elevations are obviously implausible. Geophone and event colours are the same in figures 1-4 to facilitate comparison. See legend in figure 2.**

The data consist of a set of arrival-time picks,  $d_{ij}^{obs}$ , representing the arrival time at the  $i$ 'th sensor from the  $j$ 'th seismic event, and their associated uncertainties  $e_{ij}$ , represented here as the standard deviation of a Gaussian probability. Only P-picks were used, since S-arrivals tend to be rather ambiguous in this environment.

The forward model is a set of computed times for the first arrivals from each event, with location and origin time

$(x_j, y_j, z_j, t_j)$  to each of the sensors,  $d_i^{pre}(x_j, y_j, z_j, t_j | \mathbf{v})$ , where  $\mathbf{v}$  describes the velocity model. In a layered earth, most of these arrivals will be bent rays refracted at the layer boundaries, but some will be head wave arrivals. Arrival times are computed by ray tracing, through a velocity model consisting of layers, each of constant velocity. The geological dip is accounted for by using a rotated coordinate system, with the  $z$ -axis perpendicular to the layering.

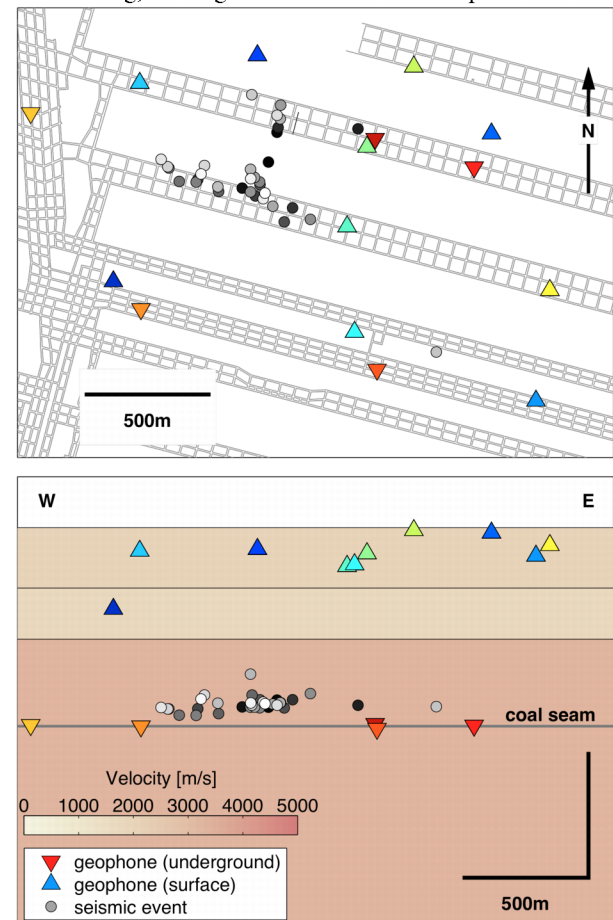
The event locations and velocity model are determined by minimizing an objective function (Tarantola, 2005)

$$\phi(\mathbf{m}) = (\mathbf{f}(\mathbf{m}) - \mathbf{d}^{obs})^T \mathbf{C}_D^{-1} (\mathbf{f}(\mathbf{m}) - \mathbf{d}^{obs}) + (\mathbf{m} - \mathbf{m}_{pr})^T \mathbf{C}_M^{-1} (\mathbf{m} - \mathbf{m}_{pr})$$

where the pick uncertainties are collected into the data covariance matrix  $\mathbf{C}_D$ , and  $\mathbf{f}(\mathbf{m})$  represents the forward traveltimes computation. The model parameter vector  $\mathbf{m}$  consists of  $N_{evt}$  seismic event locations and time, and  $N_{vel}$  velocity model parameters:

$$\mathbf{m} = [x_1, y_1, z_1, t_1, \dots, x_{N_{evt}}, y_{N_{evt}}, z_{N_{evt}}, t_{N_{evt}}, v_1, \dots, v_{N_{vel}}]^T$$

The prior model,  $\mathbf{m}_{pr}$ , contains an initial guess at the event locations, taken from the homogeneous model results but with elevations at the mining level, and a velocity model deduced from the sonic log, underground weight drop, and refraction survey results. The prior model covariance matrix,  $\mathbf{C}_M$ , representing the uncertainty in the prior information, has a small uncertainty for the lower half-space that was sampled by the sonic log, and larger uncertainties for other parameters.



**Figure 2. Simultaneous inversion of velocity plus event locations. The velocity model consists of two layers over a**

### half space, with static time shifts to account for the weathered zone at each geophone.

Two different classes of velocity model were tested, one with layer thicknesses and velocities as parameters, and one with fixed and equal layer thicknesses but varying velocities. In the latter case, the model covariance matrix  $\mathbf{C}_M$  can be used to smooth the resulting velocity model by including off-diagonal elements.

The inversion is improved by putting sensible bounds on the velocity model parameters. The layer thicknesses must be greater than zero, and variance within geological units means that it makes no sense to invert for layers thinner than, say, ten metres. Velocities were constrained to lie between 500 and 5000 m/s. To solve this bound-constrained minimization problem, a trust-region Gauss-Newton method was used (Morini and Porcelli, 2010).

A result for a velocity model consisting of two layers over a half space is shown in figure 2. Each geophone has a static time shift to account for changes in the shallow weathered layer. The seismic event elevations are now tightly clustered in the vicinity of the mine workings, which is much more plausible than the original set of locations, and the plan view locations are also more tightly clustered, in the vicinity of the tailgate and close to the mining face. The observed and best-fit predicted data for this model are shown in figure 3.

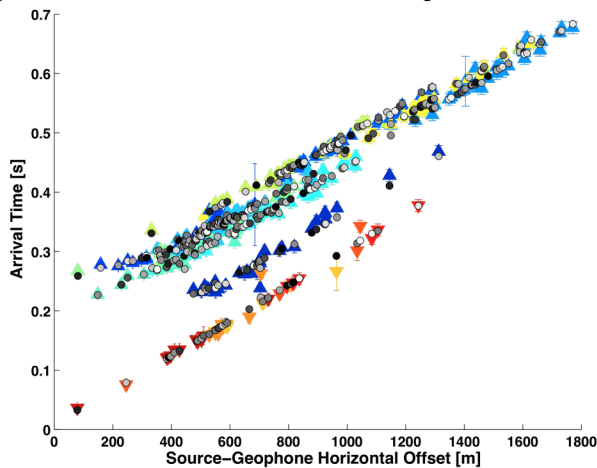


Figure 3. Observed and predicted data, plotted as a function of inverted source-to-geophone horizontal distance.

### Discussion

Two questions emerge regarding the accuracy of the velocity model and that of the seismic event locations. The first concerns the resulting velocity model: How does one assess the validity of the result? Certainly the homogeneous velocity model does not come close to fitting the data, and the event locations are not plausible. The addition of either a single extra layer to the model or a set of static time shifts has the effect of both significantly reducing the residuals and of collapsing the elevation range of the event locations to a realistic value. However, neither addition on its own allows the residuals to decrease to the expected  $\chi^2 = N_{\text{data}}$  level, nor does the addition of extra layers without the static shifts. Therefore, both a layered earth and geophone statics to model velocity changes due to weathering are required to obtain a good enough fit to the data.

Some confidence in the velocity model is obtained by examining the results from inverting for models with different parameterizations. For example, the two-layer (i.e. single layer over a halfspace) result looks similar to the three-layer case in figure 2, when the top two layers are replaced by a single layer with average velocity. The fixed-thickness result also looks the same, provided the model covariance contains off-diagonal elements to regularize the result.

Another way to test the velocity result is to redo the inversion, omitting some of the data points. For example, the middle layer in the result shown has a lower velocity than the other two layers. It also contains a single geophone, raising the question of whether the low velocity is an artefact caused by some problem with this one sensor. This is not the case, however, since omitting all arrivals at this geophone still results in a low-velocity layer.

Another clue to the validity of the model comes from examining the residuals for spatial patterns. Any spatial correlations between the residuals would suggest that there is an aspect to the data that has not been captured by the model. An example is shown in figure 4. The residuals for the model in figure 2 are shown for all arrivals at one chosen geophone, plotted by colouring the event from which the arrival came. There is an obvious spatial pattern to the residual polarities, with the arrivals from events behind the face showing observed times that are later than expected, and events further forward showing earlier than expected times. This is probably due to velocity changes caused by mining—i.e., the rock behind the face has been fractured and de-stressed, and so has lower velocity than the virgin rock. The rock ahead of the face has higher velocity due to increased stresses in this area.

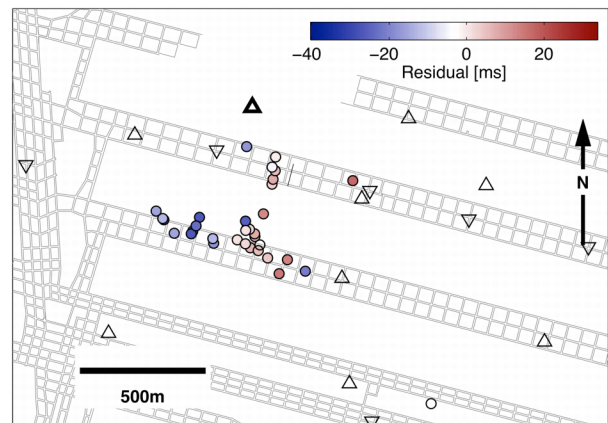


Figure 4. The residual,  $d^{\text{pre}} - d^{\text{obs}}$ , plotted for arrivals at one geophone. The systematic spatial pattern indicates that something is missing from the model, probably changes in velocity caused by mining.

The second question, more important for the present purpose of obtaining accurate seismic event locations, is how do the event locations change as the velocity model changes? For a set of models with the same hyperparameters (number of layers etc.), this question could be answered, at least for small changes in the velocity parameters, by examining the posterior model covariance matrix. The posterior probability of a model  $\mathbf{m}$  is approximated by (Tarantola, 2005)

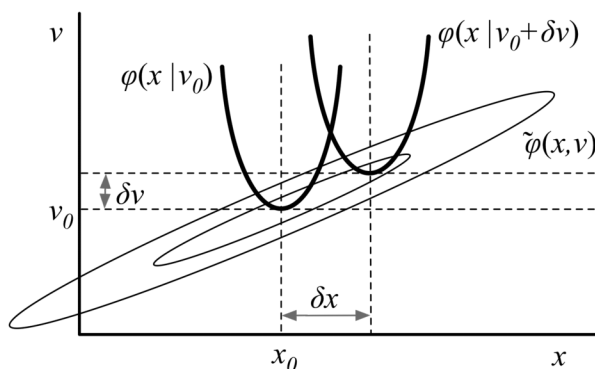
$$p(\mathbf{m}) = \text{const} \cdot \exp\left(-\frac{1}{2}(\mathbf{m} - \tilde{\mathbf{m}})^T \tilde{\mathbf{C}}_M^{-1}(\mathbf{m} - \tilde{\mathbf{m}})\right) = \text{const} \cdot \exp(\tilde{\varphi}(\mathbf{m}))$$

where  $\hat{\mathbf{m}}$  is the best-fit model, and  $\tilde{\mathbf{C}}_{\mathbf{M}}$  is the posterior covariance matrix,

$$\tilde{\mathbf{C}}_{\mathbf{M}} = (\mathbf{J}^T \mathbf{C}_D^{-1} \mathbf{J} + \mathbf{C}_M^{-1})^{-1}.$$

The best-fitting set of event locations for a given velocity model in the vicinity of  $\hat{\mathbf{m}}$  is that set that minimizes  $\tilde{\phi}(\mathbf{m})$  on the “slice” through model parameter space with the velocity parameters held constant. The principal eigenvector of the covariance matrix is aligned in the direction of least change in  $\tilde{\phi}$ ; thus, as the velocity changes, the location part of the model parameter will move along this direction. The sensitivity of location parameters with respect to velocity parameters is therefore given by the slope of this eigenvector.

Figure 5 illustrates the above situation schematically, for the case where there is only one velocity parameter,  $v$ , and one event location parameter,  $x$ . The solution to the simultaneous velocity and location inversion lies at the point  $(x_0, v_0)$ . If the velocity is changed to  $v_0 + \delta v$ , the best-fit location moves to  $x_0 + \delta x$ . For small changes in  $v$ , the ratio  $\delta x / \delta v$  is given by the principal eigenvector of the posterior covariance matrix.



**Figure 5.** Schematic illustration of the sensitivity of a location parameter,  $x$ , to a velocity parameter,  $v$ . The solution to the simultaneous velocity and location problem  $(x_0, v_0)$  lies at the minimum of the objective function  $\phi$ . A small change  $\delta v$  in the velocity parameter results in a change  $\delta x$  in the location parameter, and the ratio  $\delta x / \delta v$  is given by the slope of the first eigenvector of the covariance matrix.

A second technique allowed by the Bayesian methodology would be to integrate over all possible velocities (and velocity models) yielding a marginal probability distribution for the event locations. The results of these methods are not presented here due to lack of space.

This example has used data from a single day's events. For future planned work, a better result could be obtained by using data from events spanning the largest-possible range, which would require data gathered over weeks or months.

## CONCLUSIONS

In the sedimentary environment described here, accurate location of mining-induced seismic events requires the use of a layered velocity model. In particular, the event elevations inferred from a homogeneous model turn out to be wrong, even though the plan-view locations are surprisingly good, at least for events in the middle of the sensor array. If geophones are installed on the ground surface, then static shifts to account for changes in velocity due to weathering are also required to obtain reasonable locations. These velocity parameters can be obtained from the arrival times of the seismic events, by simultaneously inverting for velocity and a set of event locations. A Bayesian approach allows prior information, such as that obtained from sonic logs down nearby boreholes, to be incorporated into the inversion.

Plotting the residuals in a manner that allows their spatial correlation to be visualised, can show likely shortcomings in the model being used. In the example shown here, a systematic pattern in the polarity of the residuals depending on the event location relative to the mining face indicates a possible change in velocity due to the mining process.

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