An improved search for magnetization direction

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SUMMARY

Magnetic field interpretation is often conducted on an incorrect assumption that remanent magnetization is insignificant and that the resultant magnetization direction is in the local geomagnetic field direction. For compact anomalies various methods exist to test this hypothesis and return estimates of magnetization direction utilising trial reduction to pole (RTP) transforms. We have developed an analysis to return the magnetization direction which generates the most symmetric RTE anomaly and have shown that this approximately also matches input magnetizations of synthetic compact anomalies. Estimation of magnetization direction from elongate anomalies is more problematic and intrinsically less reliable, but nevertheless we found that we were able to recover approximate magnetization direction from these anomalies using cross-correlation of an analytic signal function computed from vertically integrated gradients (which we term the 'total vertically integrated gradient' or TVIG) with RTP and RTE grids computed for trial magnetization directions. The various methods are readily and automatically obtained from scanning TMI grids. The resulting magnetization direction estimates are empirical rather than analytic and are approximate. They are best used as initial estimates prior to application of more rigorous, manual methods.

Key words: remanent magnetization, RTP, RTE.

INTRODUCTION

Magnetic field interpretation is mostly conducted on the unqualified assumption that remanent magnetization is insignificant. From palaeomagnetic measurements of the Koenigsberger ratio of the strength of remanent to induced magnetization we know that this is commonly not the case. Some proportion of remanent magnetization (carried by low blocking temperature grains) may be viscous and aligned with the present geomagnetic field. However deviations of magnetization away from the local geomagnetic field orientation are more extensive than commonly recognised and are a widespread source of error in magnetic field modelling, inversion and interpretation studies. Magnetic moment analysis (MMA) provides a powerful means of recovering magnetization direction from magnetic field data (see for instance Schmidt and Clark (1998), Phillips (2005) and Foss and McKenzie (2011)) but MMA requires the analysis be located at the horizontal centre of magnetization. We would like to recover preliminary, approximate estimates of magnetization direction from an automated scan of magnetic field grids as a pre-cursor to MMA.

Fedi et al (1994) developed a method to recover estimates of magnetization direction by running a reduced-to-pole (RTP) operator across a range of trial declination and inclination values to select that magnetization direction which gave the largest value of the RTP grid minimum. Following Cordani and Shukowski (2009) we term this the 'maxi-min' method. This method provides reasonable results for compact anomalies. It requires that the minimum of the trial transformed grids are not determined by the regional gradient, interference of adjacent anomalies or by superimposed geological or measurement noise. Following an earlier study by Roest and Pilkington (1993), Dannemiller and Li (2006) developed a search for optimum magnetization direction by cross-correlation between the vertical gradient and total gradient (analytic signal amplitude or ASA) of reduced-to-pole TMI as transformed with trial magnetization estimates. For the correct magnetization direction the vertical and total gradients of RTP should be approximately symmetric, resulting in a high correlation. Both the Fedi et al (1994) and Dannemiller and Li (2006) methods have in common the application of RTP with trial magnetization estimates.

THE REDUCTION TO THE POLE (RTP) FILTER

The expression for the RTP transform operator $G_{rtp}(\mathbf{k})$ at a point $\mathbf{k} = (k_x,k_y)$ in the 2-D wave number domain is given by Blakely (1995) as follows :

$$G_{rtp}(\mathbf{k}) = |\mathbf{k}|^2 / [(\mathbf{g} \cdot \mathbf{f})\,(\mathbf{g} \cdot \mathbf{m})] \qquad (1)$$

where:
$\mathbf{g} = (ik_x,\ ik_y,\ |\mathbf{k}|)$ is the complex gradient vector in the 2-D wave number domain
$\mathbf{f} = (\cos D_f \cos I_f\ ,\ \sin D_f \cos I_f\ ,\ \sin I_f)$ is the unit vector of direction cosines for the local geomagnetic field vector **F** with declination D_f and inclination I_f
$\mathbf{m} = (\cos D_m \cos I_m\ ,\ \sin D_m \cos I_m\ ,\ \sin I_m)$ is the unit vector of direction cosines for the total magnetization vector **M** with declination D_m and inclination I_m

Here it is noted that the direction cosines are measured using the International Geomagnetic Reference Field (IGRF) coordinate system, namely, x-North; y-East, z–vertically down. The expression for the RTE transform operator $G_{rte}(\mathbf{k})$ in the wave number domain is given as follows :

$$G_{rte}(\mathbf{k}) = (\mathbf{g} \cdot \mathbf{n})^2 / [(\mathbf{g} \cdot \mathbf{f})\,(\mathbf{g} \cdot \mathbf{m})] \qquad (2)$$

where $\mathbf{n}$ is the unit vector of direction cosines for the specified true north direction.

The RTP transform has a well known instability in low inclination fields, predominantly due to the transform between near-orthogonal magnetization directions (rather than to the vector addition of the anomaly with near-orthogonal background fields). This instability of the transform therefore extends to RTP of low-inclination magnetizations in any geomagnetic field. To extend the search to low inclination magnetizations we have experimented with the reduction to equator (RTE) transform.

ANALYSIS OF COMPACT ANOMALIES

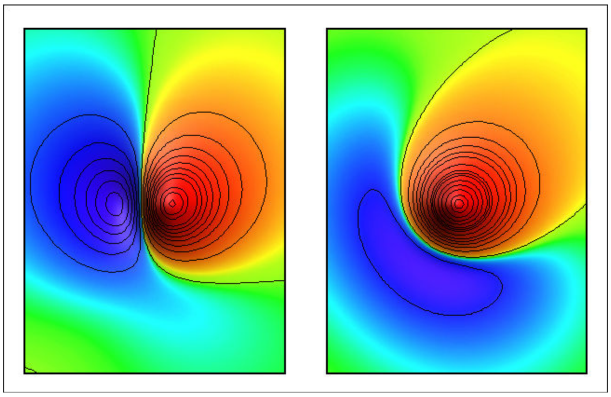

Figure 1. TMI Anomalies for magnetizations (declination, inclination): a) 90° -15° and b) 90° -65° in a geomagnetic field Declination 0°, Inclination -60°

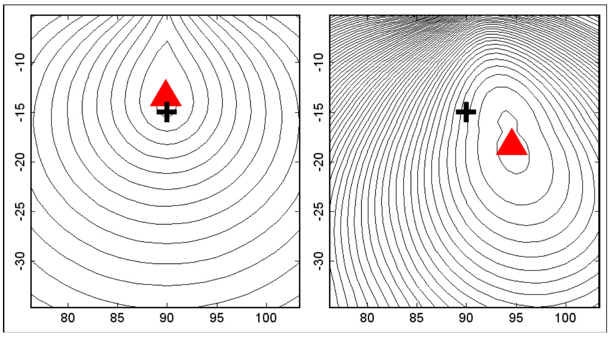

Figure 2. Declination-inclination plots for (left) Maxi-min and (right) RTP cross-correlation methods for low-inclination (-15°) dipole magnetization (cross – actual direction, triangle estimated)

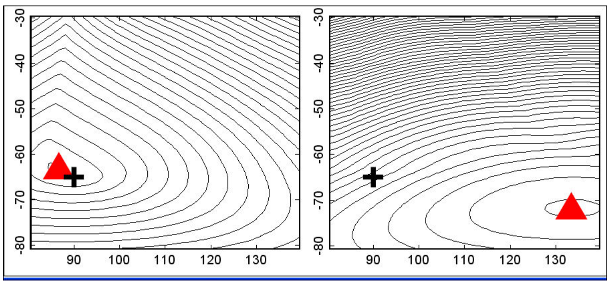

Figure 3. Declination-inclination plots for (left) Maxi-min and (right) RTP cross-correlation methods for high-inclination (-65°) dipole magnetization (cross – actual direction, triangle estimated)

Figure 1 shows example TMI anomalies from dipole sources of different magnetization direction. Figures 2 and 3 show the recovery of magnetization directions from these grids using the RTP maxi-min and RTP cross-correlation methods. As shown by Figures 2 and 3 the sensitivity of the methods depends on orientation of the magnetization. Table 1 lists the success of recovering these various magnetizations with the RTP maxi-min method of Fedi et al. (1994), with a symmetry method we have developed using RTE to search for low inclination magnetizations, and with RTP and RTE cross-correlation methods we have also developed. For the RTE there is no meaningful statistic corresponding to the maxi-min for the RTP. Rather we generate a symmetry statistic by summing differences within sample pairs on east-west and north-south expansions through the anomaly. The results in Table 1 show that both the RTP maxi-min and RTE symmetry methods recover reasonable magnetization estimates from these simple compact anomalies. For these compact anomalies the RTP and RTE correlation methods did not perform as well as the RTP mini-max and RTE symmetry methods, but still gave results mostly within 20°.

Method	Actual (dec, inc)	Estimated (dec, inc)	Angular error
Maxi-min	90, -15	90, -12	3°
Maxi-min	90, -40	89,-37	3°
Maxi-min	90,-65	87, -63	3°
RTE sym	90, -15	89, -28	13°
RTE sym	90, -40	89, -39	1°
RTE sym	90,-65	88, -62	3°
RTP corr	90, -15	95, -19	6°
RTP corr	90, -40	109, -56	21°
RTP corr	90,-65	128, -72	15°
RTE corr	90, -15	98, -30	16°
RTE corr	90, -40	126,-70	36°
RTE corr	90,-65	130, -72	16°

Table 1 Magnetization direction search results for a dipole source

ANALYSIS OF ELONGATE ANOMALIES

For thin '2d' bodies the inclination of magnetization in the plane perpendicular to the body cannot be uniquely determined without knowledge of the dip of the body, and any horizontal component of magnetization parallel to the axis of the body cannot be resolved. No geological body is truly 2d, nevertheless, for highly elongate anomalies the distal sections which may carry the most diagnostic information of magnetization direction represent only a small proportion of the anomaly and any analysis of such anomalies necessarily has only limited sensitivity to magnetization direction. While recognising this limitation we have experimented with new methods to extend analysis as best possible beyond the compact anomalies to which the standard methods are restricted.

For elongate anomalies the variation in sensitivity in recovering magnetizations of different direction is pronounced and is controlled primarily by orientation with respect to body elongation. With both the RTP maxi-min and RTE symmetry methods it is particularly difficult to recover reliable inclination estimates for magnetizations aligned approximately parallel to the anomaly axis. This limitation with the RTP maxi-min method is possibly because the location of the anomaly minimum is primarily constrained by the geometry of the body. In the case of the RTE symmetry method it is possibly because symmetry of the anomaly is also dominated by source geometry with only a secondary influence from the magnetization direction. Figure 4 shows two example anomalies for a highly elongate source body with magnetization parallel and perpendicular to its long axis. Figures 5 and 6 show cross-plots for the RTP maxi-min and RTP cross-correlation methods. For the magnetization parallel to the axis the uncertainty in the recovered magnetization is primarily in its inclination, with estimated values too steep. For the magnetization perpendicular to the body axis the uncertainty is mostly in declination. Errors in recovered magnetization for these two orientations and also an intermediate orientation are listed in Table 2.

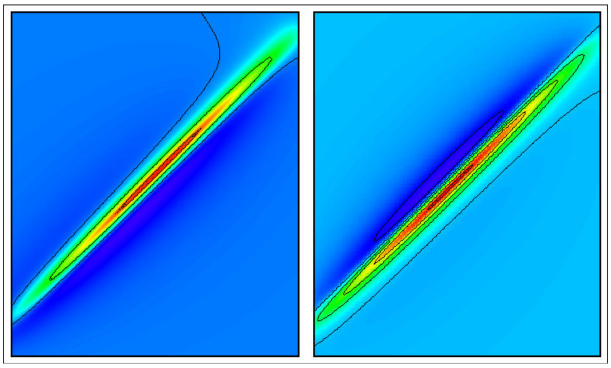

Figure 4. TMI anomalies for magnetizations (inclination, declination) a) parallel to axis: 45°,-30°, b) perpendicular to axis: 135°, -30°

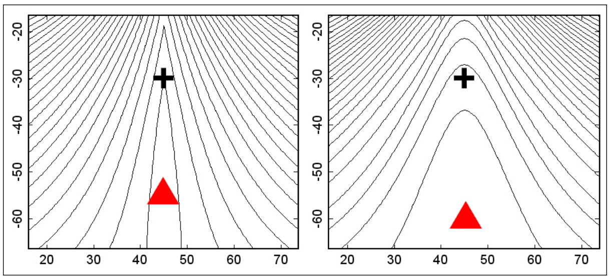

Figure 5. Declination-inclination plots for (left) Maxi-min and (right) RTP cross-correlation for along-axis magnetization (cross – actual direction, triangle estimated)

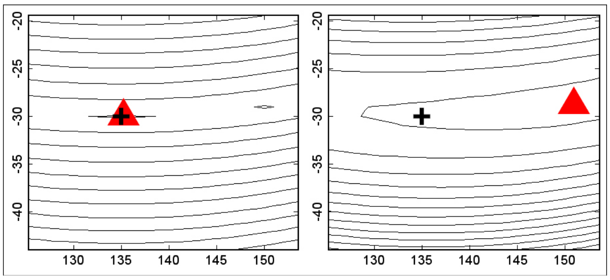

Figure 6. Declination-inclination plots for (left) Maxi-min and (right) RTP cross-correlation for across-axis magnetization (cross – actual direction, triangle estimated)

The low reliability of magnetization directions recovered from elongate anomalies with the RTP maxi-min and RTE symmetry methods led us to investigate the suitability of other methods for these anomalies. We were encouraged by the results of Dannemiller and Li (2006) using cross-correlation methods between a transform with limited sensitivity to magnetization direction (the analytic signal amplitude, ASA) which primarily maps the location and shape of the source body, and a second transform (the vertical derivative of RTP) which similarly maps the body only when the correct magnetization direction is used. Dannemiller and Li (2006) cross-correlated both gradients of RTP identically transformed with a range of trial magnetization directions. We preferred to avoid possible bias in the common application of the trial RTP to both grids and instead cross-correlated the ASA of TMI and the vertical derivative of RTP. In the 2D case the ASA is independent of magnetization direction but in the 3D case it does have some dependence on magnetization direction as reviewed by Li (2003). We found from experimentation with dipole sources that the distortion of the ASA according to magnetization direction does not substantially compromise recovery of the input magnetization direction, but that as expected this distortion is less for elongate (more 2d-like) anomalies. We also found success substituting the RTE transform for the RTP, requiring only a sign change to convert the negative RTE anomalies and produce positive correlations with the positive-signed ASA anomalies. Running the method twice, using both RTP and RTE provides a consistency check on the results (although the RTE results are generally less reliable).

method	Actual (dec, inc)	Estimated (dec, inc)	Angular error
Maxi-min	45, -30	45,-56	26°
RTE sym	45, -30	49 -31	3°
RTP corr	45, -30	45, -60	30°
RTE corr	45, -30	45, -54	24°
Maxi-min	90, -30	89, -36	6°
RTE sym	90, -30	89, -36	6°
RTP corr	90, -30	102, -34`	11°
RTE corr	90, -30	102,-34	11°
Maxi-min	135, -30	137, -30	2°
RTE sym	135, -30	141, -30	5°
RTP corr	135, -30	149, -29	12°
RTE corr	135, -30	154,-29	17°

Table 2 Magnetization direction search results for a highly elongate source with long axis azimuth 045°

We also found during experimentation that cross-correlation with the lower curvature RTP field rather than its vertical gradient often supplied marginally superior recovery of input magnetization directions. Dannemiller and Li (2006) point out that cross-correlation should be performed between grids of consistent curvature. We therefore looked to reduce the curvature of the ASA input grid through vertical integration. The ASA is not a true potential field and so an FFT process such as vertical integration is not strictly valid. To sidestep this problem we instead vertically integrated the individual gradient terms (the vertical term being replaced with the anomalous TMI value) and took the square root of the squares of the three terms. Consistent with the term 'total gradient' for the ASA we refer to this as the 'total vertically integrated gradient' (TVIG). To further reduce the (slight) influence of the distortion of the ASA we perform a second stage analysis using as input the RTP grid output for the initial magnetization direction estimate, setting the apparent geomagnetic field inclination to vertical. We also take the opportunity in this second pass to reduce the range of the trial magnetization directions and the step size, for a higher resolution search. We then accept either the first or second magnetization direction estimate according to which has the higher correlation coefficient.

ANALYSIS OF MULTIPLE ANOMALIES

For the various methods to be useful we would like to apply them to scan TMI grids in a moving window. Figure 7 shows a test grid with multiple anomalies, some at the margins of the grid. As illustrated in Table 3 the maxi-min method recovers very good estimates of magnetization direction. The cross-correlation methods are less successful but nevertheless give approximately correct direction estimates and we believe that these methods can be improved with continuing research.

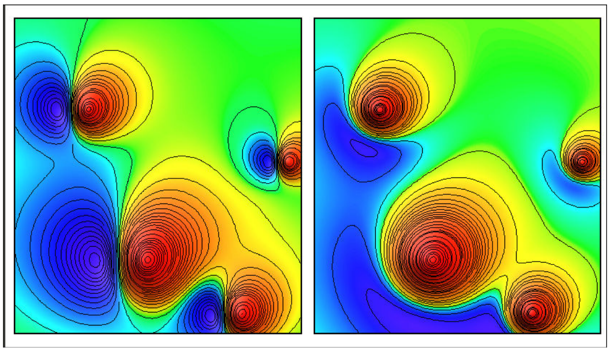

Figure 7. TMI anomalies for magnetizations (inclination, declination): a) -15° 90° and b) -65° 90°. Geomagnetic field: 0° -60°.

method	Actual (dec, inc)	Estimated (dec, inc)	Angular Error
Maxi-min	90,-15	89, -14	1°
Maxi-min	90, -40	84, -38	5°
Maxi-min	90,-65	83, -69	5°
RTP corr	90,-15	96, -19	7°
RTP corr	90, -40	109,-53	18°
RTP corr	90,-65	129, -69	16°
RTE corr	90,-15	97, -28	15°
RTE corr	90, -40	124, -61	29°
RTE corr	90,-65	131,-69	16°

Table 3 Magnetization direction search results for a multiple source model

CONCLUSIONS

We have confirmed that approximate magnetization direction estimates can be recovered from various methods that scan TMI grids using RTP and/or RTE computed for a range of trial magnetization directions. For compact anomalies and moderate to high inclination magnetizations the maxi-min method of Fedi et al (1994) produces reasonable results. We have found that we can also recover magnetization directions from such anomalies by finding the magnetization direction which generates the maximum symmetry RTE anomaly. For elongate anomalies estimates of magnetization direction are intrinsically less reliable, particularly for magnetizations parallel to the strike of the anomaly. In an extension of an existing method by Dannemiller and Li (2006) we have however recovered approximate magnetization directions from elongate anomalies using cross-correlation of an analytic signal derived from vertical integration of the orthogonal gradients of TMI and the RTP and RTE of TMI. Magnetization directions estimated from the various methods are all empirically rather than analytically derived and should only be used as approximations prior to more rigorous investigations such as MMA or staged inversion. The advantage of the methods reported here is that they can be automated to scan a TMI grid and supply local estimates of magnetization direction prior to the more rigorous, interactive studies.

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