A New Idea of NREGS and its Application to Oil Exploration, Case Study: Detecting Geological Salt Structures in Northern Coasts of Persian Gulf

Yahya AllahTavakoli
National Iranian Oil Company (NIOC), Exploration Directorate, Survey Department
ytavakoli@ut.ac.ir

1 Introduction
Herein we are going to extract all information about lateral variation of Density Anomaly of underground geological structures from gravitational data and we are interested to make Gravitational Anomalies uncorrelated with topography and to make it correlated with Mass Density Anomaly for exploration purposes. Therefore, we have done some definitions and propositions leading us to the intention. The paper comprises seven sections, the first is introduction and motivation, the second and third are about the required mathematical definitions; in fourth and fifth section, the employed definitions are suggesting our Simple Idea of Nearest Reference Equi-Gravitational Surface (NREGS) for prospecting lateral density variations in under ground; in sixth section methodologies for upward continuation has been discussed and finally in last section, the idea is applied in a case study to detect presence of any salt structures in a prospected region for exploration studies and it has been shown that correlation of gravitational data with topography has minimum value on NREGS.

2 Equi-Gravitational Surface

Suppose a vector function \( g : \mathbb{R}^3 / \Omega \rightarrow \mathbb{R}^3 \), Earth Gravitational Vector Field, as follow, Vanícek Petr, Krakiwsky E. (1986)

\[
g(r) = G \int_{\Omega} \frac{\rho(r')}{|r-r'|} d\beta'
\]

(1)

Where the function \( \rho \) is Earth Interior Mass Density Distribution Function and the space \( \Omega \) is Earth Interior Space. In addition, norm of gravitational vector field is a scalar field \( g : \mathbb{R}^3 / \Omega \rightarrow \mathbb{R}^+ \) defined in earth exterior space.

The gravitational field norm \( g \) is not an injective functional with respect to Geodetic Height \( h \) but in earth exterior space, there exists a continuous surface \( \Sigma \) where norm of gravitational field will be Strictly-Decreasing Function and consequently will be Injective Function with respect to \( h \) (Coordinate \( \varphi, \lambda, h \) are Geodetic Coordinates and surface \( h = H_{\text{Topographic}}(\varphi, \lambda) \) is Earth Topographic Surface). If we assume that

\[
\frac{\partial g}{\partial h} \frac{\partial H}{\partial H} < 0
\]

(3)

Which exterior is a subspace where norm of gravitational field will be Strictly-Decreasing Function and consequently will be Injective Function with respect to the \( h \) (Coordinate \( \varphi, \lambda, h \) are Geodetic Height, then according to following inequality for gravitational field norm, Hofmann. B., Moritz. H. (2006) (The positive number \( J \) is mean curvature of level surface)

\[
\Sigma = \{(\varphi, \lambda, h_{\text{ref}}) | h_{\text{ref}} = \inf_{\xi} H_{(\varphi,\lambda)} \}
\]

(2)

Where \( h = H_{\text{Topographic}}(\varphi, \lambda) \) is Earth Topographic Surface.

SUMMARY

If Correlation of Gravitational Data with Topography is minimized correctly and coherently, Interpretation of Land-Based Gravitational Data will be a valuable clue leading us to detecting Lateral Variations of underground Mass-Density Anomalies particularly for Oil Exploration Purposes. For this reason, the author recommends a New Idea based on new mathematical definition of “Nearest Reference Equi-Gravitational Surface (NREGS)” for making uncorrelated gravitational data with topography. NREGS is Reference Equi-Gravitational Surface in exterior of earth where the gravitational data will be upward-continued to and interpreted. The Mathematical Theory of the Idea explains that gravitational data on NREGS should be the most uncorrelated with topography. Hence, gravitational data on it can easily illustrate lateral density variation of geological substructure with a lot of clear contrasts being so useful for horizontally detecting underground density anomalies e.g. salt structures. In a case study based on 6350 gravitational observations in Coastal Fars of Iran, the author applied NREGS Idea for exploration of salt structures. There, NREGS idea detects all outcropped salt structures and also, it shows some other Low Anomalies which can be underground hidden salt structures. In addition, the case study shows that the gravitational data on NREGS is statistically more uncorrelated with topography than other data e.g. Bouguer Anomaly, Free-Air Anomaly, etc. It means that the case study practically proves the theory of the New Idea, too.

Key words: Equi-Gravitational Surface, NREGS, Geological Salt Structure
A New Idea of NREGS and its Application to Oil Exploration

Y. AllahTavakoli

\[ \frac{\partial g}{\partial H} = -2gJ + k \pi G \rho < 0 \]  

(4)

Earth topographic surface can approximate the surface \( \Sigma \). It means

\[ h_{\text{ref}} = h^{\text{Topo}} \]

(5)

Hence, Equigravitational Surface is defined as follow

Definition: a surface in exterior space of \( \Sigma \) \( (\Sigma_{\ref{g}} = \{ r(\varphi, \lambda, h) | h \geq h_{\text{ref}}(\varphi, \lambda) \}) \), that satisfies

\[ g(r) = c \]

(6)

is named Equi-Gravitational Surface where the earth gravitational norm is equal constant \( (0, c) \) and the constant \( c_0 \) is infimum value of gravitational norm on the surface \( \Sigma \).

\[ c_0 = \inf \{ g(\Sigma) \} \]

(7)

By above definition, Equi-Gravitational surface will be Well-Defined and for every \( (0, c) \) the Equi-Gravitational surface is a Unique Continuous Surface in \( \Sigma_{\ref{g}} \).

3 Reference Gravitational Field and Reference Equi-Gravitational Surface

Suppose \( E_{\ref{g}} \) is interior space of reference ellipsoid \( E \), so the earth gravitational vector field can be decomposed to two parts as

\[ g(r) = \gamma(r) + g^{\text{Topo}}(r) \]

(8)

Where

\[ \gamma(r) = G \int_{\Omega_{\ref{g}}} \frac{\rho(\varphi')}{|r' - r|} (r' - r) d\beta' \]

(9)

\[ g^{\text{Topo}}(r) = G \int_{\Omega_{\ref{g}}} \frac{\rho(\varphi')}{|r' - r|} (r' - r) d\beta' \]

(10)

The vector field \( \gamma(r) \) is gravitational effect of earth masses in interior of reference ellipsoid, which can be approximated by Normal Gravitational Field, Hofmann, B., Moritz. H. (2006) and the vector field \( g^{\text{Topo}} \) is gravitational effect of masses in the Topographic Space \( \Omega_{\ref{g}} - E_{\ref{g}} \), the space between reference ellipsoid surface and the Topographic Surface and also it can be approximated by gravitational topographic effect as follow

\[ g_{\text{Topo}}^{\ref{g}}(r) = G \int_{\Omega_{\ref{g}}} \frac{\rho_0}{|r' - r|} (r' - r) d\beta' \]

(11)

The earth topographic surface \( h = h^{\text{Topo}}(\varphi, \lambda) \) and mass density are known functions. It should be mentioned that herein, the mass density function of topographic masses has to be supposed as constant function \( \rho = \rho_0 \). The constant \( \rho_0 \) can be equal Mean Earth Topography Mass-Density. Thus, Reference Gravitational Vector Field \( g^{\ref{g}} \) models the earth gravitational vector field \( g \) as follow

\[ g^{\ref{g}} = \gamma + g^{\text{Topo}} \]

(12)

Moreover, Gravitational Anomaly \( \delta g \) is defined as difference of real earth gravitational vector field \( g \) and its model \( g^{\ref{g}} \).

\[ \delta g = g - g^{\ref{g}} \]

(13)

The Gravitational Anomaly \( \delta g \) is directly depends on the target, Density Anomaly \( \delta \rho = \rho - \rho_0 \)

\[ \delta g(r) = G \int_{\Omega_{\ref{g}}} \frac{\delta \rho}{|r' - r|} (r' - r) d\beta' \]

(14)

Definition: For Reference Gravitational Vector Field \( g^{\ref{g}} \) according to the definition of Equi-Gravitational Surface in pervious section, Nearest Reference Equi-Gravitational Surface (NREGS) to the Earth Surface Topography be defined by

\[ g^{\ref{g}}(r) = \| \delta g^{\ref{g}}(r) \| = \inf \{ \| g^{\ref{g}}(\Sigma_{\ref{ref}}) \| \} \]

(15)

Where the surface \( \Sigma_{\ref{ref}} \) be approximated by earth topographic surface.

In next sections, aforementioned definitions will motivate us to propose the idea of using NREGS in detection of lateral variations of underground Density Anomalies.

4. Flat Area Idea

This Idea is a simple old thought to find Lateral Density Variation by Land-Based Gravitational Observations for exploration studies. In a Flat Area, Lateral Variations of Gravitational Norm \( g \) and Gravitational Anomaly \( \delta g \) are Directly Correlated with Lateral Variation of Density \( \delta \rho \) and Density Anomaly \( \delta \rho \). Schematically, this old easy idea is illustrated in figure 1.

Reference Equi-Gravitational Surface

Earth Surface

\[ g_{\ref{g}} > g_1 \]

\[ g_2 > g_{\ref{g}} \]

\[ g_2 > g_1 \]

Figure 1 Lateral Gravitational Norm is directly depend upon Lateral variation of Density
5. The Idea Based on Gravitational Anomaly Variation on Nearest Reference Equi-Gravitational Surface (NREGS)

If the Topographic Mass Density is equal Mean Density $\rho_0$, then the gravitational field $g$ shall be identical to reference gravitational field $g^{\text{Ref}}$ and it will be expected that $g^{\text{Up}}$ - the Upward Continued land Gravitational observation to NREGS - be equal the constant $\rho_0$ and the real Density of Topography $\rho$. In other hand, the definition of reference gravitational field takes into account topographic effect; so we expect that main part of gravitational variations on Nearest Reference Equi-Gravitational Surface (NREGS) will be the result of Lateral variations of Density Anomaly when the density anomaly is not trivial.

Also, according to aforementioned definition of NREGS, its disparity with earth surface is Minimum and it is close enough to Earth surface. Hence, we expect that the gravitational anomaly in each point of NREGS depends mostly on its underground Density Anomaly.

Essentially, by this procedure, we achieve gravitational data having minimum correlation with topographic height and Maximum correlation with Density Anomaly. The figure 2 has illustrated this idea, schematically. In this figure, we see that the gravitational observation on a geological structure with high density can be less than the gravitational observation on a geological structure with low density, occurring because of the topography correlation. It means

On Topography: $\rho_2 > \rho_1 \Rightarrow g_2 > g_1 \quad (16)$

However, on NREGS we have most wanted result for exploration studies as follow

On NREGS: $\rho_2 > \rho_1 \Leftrightarrow g_2^{\text{Up}} > g_1^{\text{Up}} \quad (17)$

6. Upward Continuation of Gravitational Data to the NREGS

Computation of the EquiGravitational Surface is a Well-Posed Direct Problem and it will numerically be done by relations (11), (12) and (15) but otherwise Modelling of real gravitational data in earth exterior space and their upward-continuation to the Reference EquiGravitational Surface is an Inverse Problem, which is typically an Ill-Posed Problem. Today, there are many Local Gravitational Field Modelling Methods based on Land or Airborne Gravimetry, precise levelling, Remove-Compute-Restore methods by Solution of Geodetic Boundary Value Problem (GBVP), Astrogravimetric methods, precise geoid computation etc. Every Local Gravitational Field Modeling Method can perform upward Continuation procedure, Ardalan and Grafarend (2004), Hoffmann and Moritz (2006), Moritz (1980), Vanicek and Krakiwsky (1986).

7. Case Study

As case study, we applied the idea to extract all information about Lateral Variation of Density Anomaly for Hydrocarbonate (Oil or Gas) exploration purposes. Our case study was on 6350 Gravitational observations in Coastal Fars of geographic region of Iran. Zagros region includes many hidden and noticeable salt structures, Bahrroudi A. (2003), Where many of its anticlines and salt structures are in doubt to be Hydrocarbonate Trap or not. Gravitational observations and their precise processed results are clues to reach extra knowledge about geological substructures. Local Gravitational Field Modelling Methods by land-based gravitational observations will be so fruitful specially to seek hidden salt structures. Prospecting salt structures is so important in oil exploration projects since they can be hydrocarbonate-trap, or vice versa they can cause damage of hydrocarbonate-field structure to release hydrocarbonate fluid from the trap to earth surface in long geological periods. Hence, in our case study, land-based gravitational data on the region have been applied and upward continued to Nearest Reference Equi-Gravitational Surface (NREGS) where the data is expected to be constant but according to lateral density variations, the upwarded gravitational data has lateral variation on the NREGS. According to definition of this surface being near earth surface, the upward-continued gravitational data is directly more dependent to earth surface lateral density variations than topography.

The figure3 illustrate the geological map of the case study in northern coasts of Persian Gulf, where the black spots are the outcropped salt structures. The computed NREGS in coastal Fars of Iran above Persian Gulf has been shown in figure4 and its Upward-Continued Gravitational data is illustrated in figure 5. The areas with low Gravitational Data which are showed by arrows, demonstrate areas containing substructures with low-density anomalies that may be outcropped or hidden salt structures. In table1, the correlation between Topographic Heights and Gravitational observations, Free-Air Anomalies, Gravitational Anomalies on Earth Surface, Complete Bouguer Anomalies and the Upward Continued Gravitational data of NREGS in Deng region have been compared together and the results shows that the Upward Continued Gravitational Anomaly on the NREGS is most uncorrelated with Topographic Heights.
Table 1 Correlation with Topography

<table>
<thead>
<tr>
<th>Data</th>
<th>Gravitation</th>
<th>Free-Air</th>
<th>Gravitational Anomaly</th>
<th>Complete Bouguer</th>
<th>Upwarded Gravitational Anomaly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation with Topography</td>
<td>-0.95</td>
<td>0.94</td>
<td>0.50</td>
<td>0.58</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Figure 3 Geological Map: the Positions of the outcropped salt structures have been showed by black spots in Northern Coasts of Persian Gulf

Figure 4 NREGS in the Case Study Region

Figure 5 the upwarded Gravitational Data to NREGS, illustrates accurate positions of outcropped salt structures in depth and shows some other low anomalies suspected to be hidden underground salt structures

References
