Uncertainty in Near-Surface Refraction Inversion

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SUMMARY

Uncertainty in the tomographic inversion of near-surface seismic refraction data can be separated into aleatory variability, which describes the misfit errors and epistemic uncertainty, which describes the suite of acceptable models. Common implementations of refraction tomography usually focus on reducing aleatory variability and frequently disregard epistemic uncertainty.

In this study, the tomograms generated with three models of the seismic velocities in both the weathering and in the sub-weathering, using the generalized reciprocal method (GRM), are consistent with the traveltime data. However, only one tomogram is consistent with the optimum XY value and the attributes derived from the head wave amplitudes and seismic velocities. This study demonstrates that epistemic uncertainty can be explicitly addressed with the GRM, because the most probable tomogram can be selected objectively from a number of acceptable alternatives.

The GRM-based tomogram successfully detects, defines and differentiates narrow regions with low seismic velocities which represent shear zones and a massive sulphide ore body. None of these zones is detected with the tomogram generated with the default starting model using smooth vertical velocity gradients.

It is concluded that minimizing epistemic uncertainty through the use of the most appropriate starting model is more important than minimizing aleatory variability.

Key words: refraction, uncertainty, GRM, non-uniqueness, tomography.

INTRODUCTION

Refraction tomography, which is widely used to invert seismic refraction data recorded for geotechnical investigations, is inherently non-unique. This study demonstrates that non-uniqueness can be explicitly addressed with the generalized reciprocal method (GRM) (Palmer, 1980).

ALEATORY AND EPISTEMIC UNCERTAINTY

Non-uniqueness in the inversion of geophysical data can be equated with uncertainty, which in most engineering treatments of risk, is generally separated into aleatory variability and epistemic uncertainty. Aleatory variability is the natural randomness in a process, and it can be equated with misfit errors in refraction tomography. Epistemic uncertainty is the scientific uncertainty in the model of the process, due to limited data and knowledge, and it is characterized by alternate models (Palmer, 2011).

The concepts of aleatory variability and epistemic uncertainty correspond with precision and accuracy. Precision refers to how closely individual measurements agree with each other. Accuracy refers to how closely a measured value agrees with the correct value. Figure 1 presents a common dart board analogy which illustrates the difference between aleatory variability and epistemic uncertainty and its relevance to the tomographic inversion of near-surface seismic refraction data. The bull's eye is defined in Figure 9.

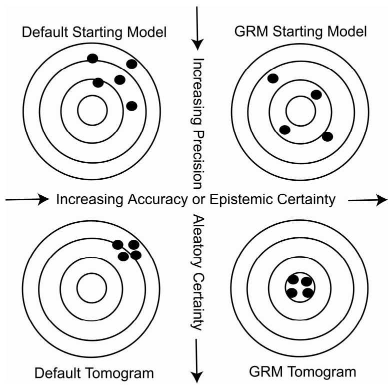

Figure 1. A common dart board analogy which illustrates the differences between precision or aleatory variability and accuracy or epistemic uncertainty.

Despite the reality of non-uniqueness, most strategies for the inversion of near-surface seismic refraction data fail to separate aleatory variability from epistemic uncertainty. In fact, a common strategy has been to assume incorrectly that minimizing misfit errors and therefore aleatory variability, demonstrates that epistemic uncertainty somehow has been minimized also. Often, it can result in featureless tomograms.

In this study, epistemic uncertainty is explicitly addressed by generating refraction tomograms for three models of the seismic velocities in the weathering and three models of the seismic velocities in the sub-weathering with the GRM. All tomograms produce misfit errors which are comparable to, but slightly larger than the default starting model of smooth vertical velocity gradients. However, only one model comprises seismic velocities which are consistent with the optimum XY value, and with the *a priori* refraction attributes derived from the head wave amplitudes and seismic velocities.

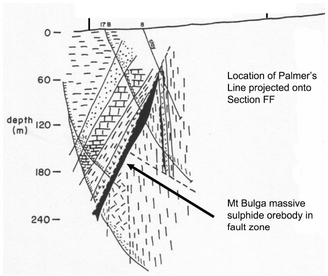

Figure 2. Geological cross-section showing the massive sulphide ore body and the adjacent shear zone.

This study uses data recorded over a narrow massive sulphide ore body at Mt Bulga (Palmer, 2006, 2010a, 2011). A representative geological cross section is shown in Figure. 2. The ore body does not constitute a common target for seismic refraction investigations. Nevertheless, it provides a crucial test of the spatial resolution of refraction tomography, and therefore, the importance of epistemic uncertainty, because it is a narrow vertical feature with quite distinctive petrophysical properties. This study demonstrates that the GRM is able to generate good estimates of the lateral extent of the ore body and to differentiate it from the adjacent shear zone.

REFRACTION TOMOGRAMS

Figure 3 presents the wavepath eikonal traveltime (WET) (Schuster and Quintus-Bosz, 1993) tomogram derived from the default starting model of the smooth vertical velocity gradients. There are no indications of any distinctive features which might correspond with the steeply dipping massive sulphide ore body or any associated shear zones.

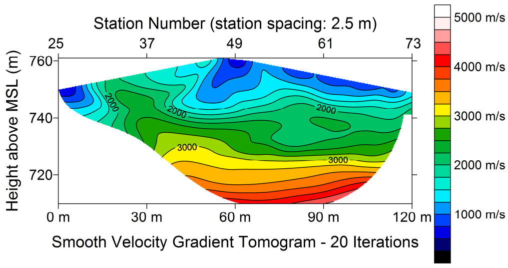

Figure 3. Tomogram generated from the smooth vertical velocity gradient starting model.

Figure 4 presents three GRM WET tomograms which represent disparate models of the seismic velocities in the weathering. The seismic velocities in the sub-weathering have been derived with the optimum XY value of 3.75 metres. There is little ambiguity in recognizing the base of the weathering in the GRM WET tomograms, unlike the smooth vertical velocity WET tomogram. There are large gradients in the seismic velocities in the vicinity of the 2000 m/s contour.

The shear zone between stations 45 and 49 with a seismic velocity of ~3000 m/s can be differentiated from the adjacent massive sulphide ore body between stations 49 and 53 with a seismic velocity of ~2500 m/s. This separation is best recognized with the tomogram which accommodates the velocity reversal and poorest with the tomogram which employs vertical velocity gradients. Furthermore, a major shear zone with a low seismic velocity of ~2500 m/s between stations 61 and 67 is readily apparent. None of these features can be recognized in the smooth vertical velocity WET tomogram in Figure 3.

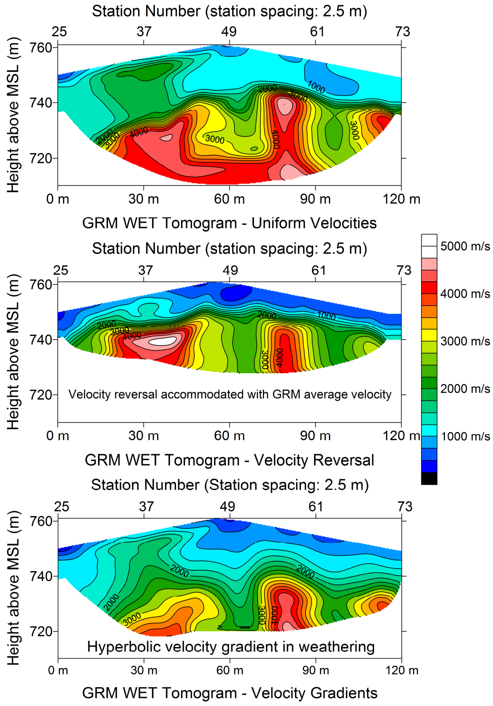

Figure 4. Three GRM WET tomograms which represent (i) uniform seismic velocities derived from uncritical acceptance of the traveltime data, (ii) the average vertical velocities which accommodates a likely velocity reversal, and (iii) hyperbolic velocity gradients derived from the uniform seismic velocities, after 5 iterations.

Although the default WET tomogram is comparable to the GRM WET tomogram, which includes velocity gradients in the weathering, above an elevation of 740 m, this is not the case in the sub-weathering below the same elevation. Palmer (2010b, 2010c) demonstrates that common default starting models, such as the smooth vertical velocity gradients, generate artefacts consisting of high seismic velocities in the sub-weathering. Automatic inversion methods do not parameterize the traveltime data into arrivals from different layers and therefore, they are unable to explicitly resolve the fundamental ambiguity of updip and downdip apparent velocities into true seismic velocities and dip and/or structure. As a result, the higher updip seismic velocities are usually but incorrectly assumed to represent another valid refractor, and inevitably, they are included within the final tomogram. The issue is not resolved with various measures of misfit errors, because it relates to epistemic rather than aleatory uncertainty.

The significance of the undetectable artefacts with high seismic velocities is that they can conceal regions with genuine low seismic velocities which are indicative of faults and shear zones. Two regions with low seismic velocities, which are delineated with the GRM in Figure 4, are neither detected nor defined using the default starting model of smooth vertical velocity gradients in Figure 3. It follows that any WET tomogram obtained with starting models, which are NOT generated with the GRM or equivalent inversion algorithms which explicitly identify forward and reverse traveltimes, are likely to contain artefacts consisting of high seismic velocities in the sub-weathering (Palmer, 2011).

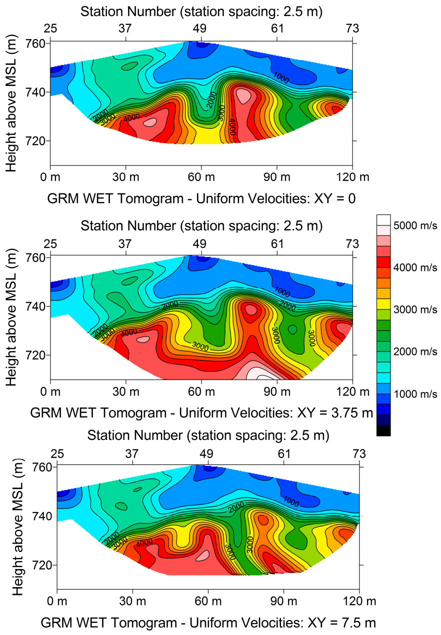

Figure 5. Three WET tomograms generated from the seismic velocities in the sub-weathering using the GRM refractor velocity function with XY values of zero, 3.75 m, and 7.5 m after 10 iterations.

Figure 5 presents WET tomograms generated from three GRM starting models using the seismic velocities in the sub-weathering computed with XY values of zero, 3.75 m and 7.5 m, that is, the optimum XY value of 3.75 ± 3.75 m, and uniform spatially varying seismic velocities in the weathering. The exact location and lateral extent of the narrow region with the low seismic velocity in the vicinity of station 49 varies with the selection of the XY value of the starting model. In particular, the differentiation of the shear zone between stations 45 and 49 with a seismic velocity of ~3000 m/s from the adjacent massive sulphide ore body between stations 49 and 53 with a seismic velocity of ~2500 m/s, is critically dependent on the XY value. By comparison, the shear zone with a low seismic velocity of ~2500 m/s between stations 61 and 67 is unambiguous.

Many implementations of refraction tomography frequently recommend increasing the number of iterations from the default, commonly 20, to 50 or 100, in order to improve resolution and reduce artefacts, even if the RMS error does not decrease. However, Figure 6 demonstrates that the resolution is significantly reduced after 50 iterations, while the tomograms generated after 100 and 200 iterations are virtually identical to those generated with the smooth vertical velocity gradient starting model in Figure 3. In effect, over processing has converted the known vertical structure to horizontal layering, and therefore, has generated artefacts, rather than reduced them. It demonstrates that refraction tomography is a smoothing operation which rarely improves spatial resolution.

Figure 7 is a cross-section which presents the scaled density ratio computed from the head coefficient derived from the head wave amplitudes and the seismic velocities. In this model, the density of the weathered layer has been assumed to be uniformly one. In the absence of any density values measured on discrete samples, no account has been taken of the likely lateral variations in the densities in the weathering. Nevertheless, Figure 7 clearly shows the increased density of the massive sulphide ore body between stations 49 and 53, as well as a decrease in density of the shear zone between stations 45 and 49.

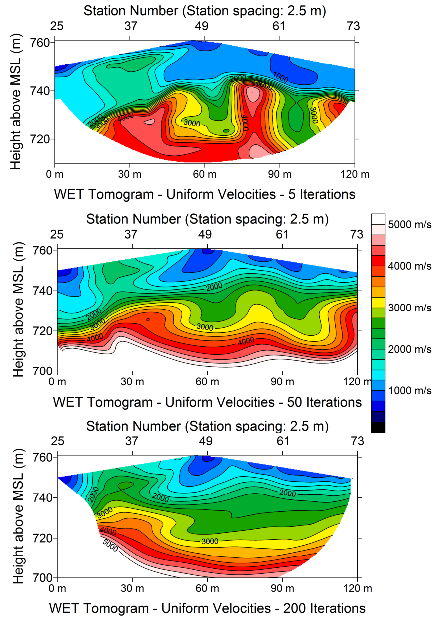

Figure 6. Three tomograms generated from the GRM starting model for the optimum XY value of 3.75 m which uncritically accepts the traveltime data, after 5, 50 and 200 iterations.

The head coefficient and the seismic velocities can also be combined to obtain a model of the P-wave modulus (Palmer, 2010a, 2010d), which is shown in Figure 8. The P-wave modulus is quite high for the region of high density between stations 49 and 53, where the massive sulphides occur. The inclusion of the density with the head coefficient has effectively counteracted the effect of the low seismic velocity, and generated a more realistic measure of the geotechnical strength of the massive sulphides.

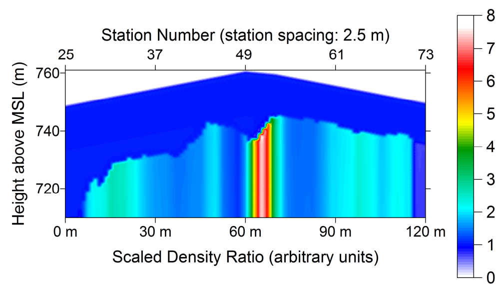

Figure 7. The scaled density ratio computed from the head coefficient and the seismic velocities.

Figure 9 presents a conceptual model of GRM model space. All of the tomograms presented in Figures 4 and 5 are represented in Figure 9 as white to black gradational circles. Although not shown, it is a simple task to generate additional models which incorporate vertical velocity gradients or velocity reversals in the weathering with other models of the seismic velocities in the sub-weathering, derived with for example, XY values of zero and 7.5 m.

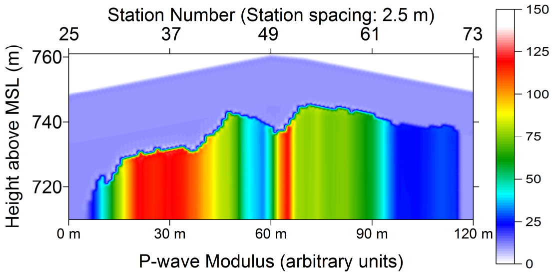

Figure 8. The P-wave modulus computed from the head coefficient and the seismic velocities.

All of these tomograms have comparable misfit errors. The centre or crosshairs is defined by the optimum XY value. The shaded region or "bull's eye" is defined by the uncertainty in the optimum XY value of plus/minus half the station interval, the latter being Δx. (The two opaque circles represent two of the 2D shear zone tomograms (Palmer, 2010a, 2010d.))

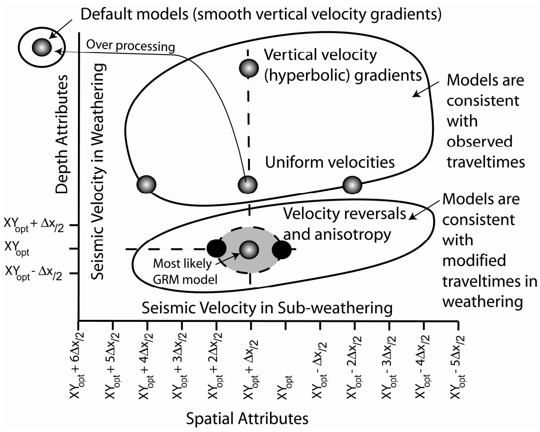

Figure 7. GRM global model space. The crosshairs are defined by the optimum XY value, and the bull's eye is defined by the uncertainty in the optimum XY value of plus/minus half the station interval.

Furthermore, it is possible to derive the default WET tomogram using smooth vertical velocity gradients in Figure 3, simply by over processing a detailed GRM starting model, as shown in Figure 6. This model space has been placed outside of that for the GRM, because both model studies and Figure 6 demonstrate that the use of default starting models can generate undetectable artefacts in the seismic velocities in the sub-weathering. They do not constitute viable or useful models, despite being consistent with the traveltime data.

Figure 9 demonstrates that the GRM algorithms are able to generate starting models and in turn, WET tomograms, which (i) explicitly identify forward and reverse traveltimes, and therefore, exclude a major class of artefacts, (ii) are consistent with the traveltime data to sufficient accuracy (Palmer, 1980, p.50), and (iii) include all geologically reasonable seismic velocities. It can be concluded that the GRM can investigate a useful global model space in which the XY spacing is a key variable, and that the GRM algorithms satisfy both necessary and sufficient conditions for the inversion of seismic refraction data (Palmer, 2011).

CONCLUSIONS

Uncertainty in the tomographic inversion of near-surface seismic refraction data can be separated into aleatory variability, which describes the inherent random variations associated with misfit errors, and epistemic uncertainty which is due to a lack of knowledge of acceptable starting models for inversion. This study demonstrates that minimizing epistemic uncertainty through the use of the most appropriate starting model is more important than minimizing aleatory variability. While tomograms which are both accurate and precise are always preferable, imprecisely accurate tomograms are invariably more useful than precisely inaccurate tomograms.

Only the GRM based tomograms successfully detect, define and differentiate narrow regions with low seismic velocities which represent shear zones and a massive sulphide ore body. Refraction tomograms generated from low resolution starting models, such as those employing smooth vertical velocity gradients, are susceptible to the generation of undetectable artefacts. Therefore, they are not able to provide a reliable measure of the occurrence or otherwise of virtually any significant spatial variations in the seismic velocities in the sub-weathering. This study demonstrates the necessity of employing inversion algorithms, which generate detailed starting models and which avoid artefacts, though explicitly parameterizing forward and reverse traveltimes. It can be concluded that the use of the GRM algorithms is essential for the majority of routine near-surface refraction investigations.

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