

# Ultra-Fast 3D parameterised AEM inversion using spectral methods

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## SUMMARY

One of the first usages of spectral methods in EM was by Peter Annan in development of the algorithm for EM modelling program "PLATE", popular in the days of mainframe VAX computers. Using recent developments in spectral methodology and parallel computing on GPU boards, it is predicted to be possible on a desktop to run geologically plausible AEM forward models in microseconds, and inversions in milliseconds. In practice, the limitations of noise, imperfectly known waveforms, coupled with the effects of dielectric permittivity, viscous magnetism (superparamagnetism) and induced polarization conspire to limit the accuracy and speed of the process.

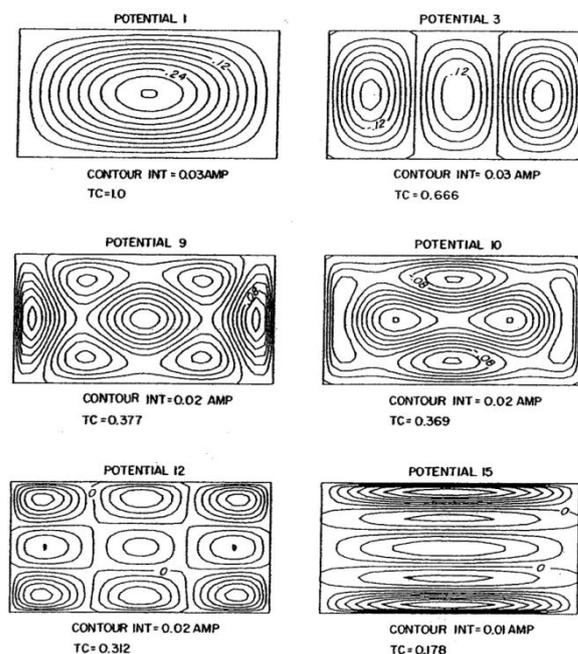
**Key words:** Plate, Disc, AEM, Inversion, 3D, Fast, Spectral

## INTRODUCTION

Airborne EM data sets generally consist of millions of amplitude values as a function of time, sorted and averaged into amplitude decays as a function of easting and northing on sets of survey lines. In mineral exploration "State of the art" interpretation has progressed from "manual bump finding and conductor quality estimation" in the early days (Palacky and West, 1991) to full 3D interpretation (Macnae et al., 2012). Both approaches have difficulties in that they take significant time and the results depend significantly on the modeller and interpreter (choice of modelling code e.g. Maxwell, UBC, TechnoImaging, incorporation of geological constraints etc.). In hydrogeophysics, the prevalence of horizontal conductors has led to 1D inversions, potentially with lateral or spatial constraints as the "State of the art" for AEM modelling (Auken et al. 2005, Viezzoli et al., 2008).

There have in parallel been significant efforts made into fast and automated modelling/imaging of AEM data. These started with "Conductivity Depth Imaging" introduced for the Spectrem system by Macnae et al. (1991), with conductance-depth estimation based on the 19th century receding image theory of Maxwell (Macnae and Lamontagne, 198. After transform, single point soundings were stitched together to get 2D conductivity-depth values. Many other researchers advanced 1D inversion, and other approximate conductivity-depth transforms were developed post the 1987 work of Macnae and Lamontagne. Some of the more interesting but to date "commercially unsuccessful" were the introduction of fast 3D anomaly fitting methods by Macnae et al (1998); Sattel and Reid (2006) and Schaa and Fullagar (2010).

This paper looks at spectral methods as a way to speed up forward EM modelling and inversion. To my knowledge, the first use of spectral methods in EM modelling was the PhD thesis of Peter Annan (1974). Program "Plate" developed at the University of Toronto was for many years an approximate EM model used in forward modelling of EM induction in rectangular plate. Figure 1 presents contours of eigenpotentials derived by Annan showing current streamlines in plates. With the limitations of VAX computers in the 1970's, the accuracy of Plate was intentionally limited to just a few eigenpotentials (15) due to speed considerations. Since then, computers have improved several orders of magnitude in speed and available memory, and coding greatly simplified. The equivalent to Annan's eigenpotentials can now be created in 5 or 6 lines of MATLAB code, with disk conductors as easy to compute as plates (Trefethan, 2000), and a modelling profile calculated with simple coupling and reciprocity.



**Figure 1: Subset of Annan's (1974) eigenpotentials**

## METHOD AND RESULTS

EM induces current in a confined conductor within a surrounding medium through two mechanisms: 1) vortex induction where currents are confined to the target conductor and circulate around changing magnetic fields and 2) galvanic pickup where host currents are channelled into and then out of the target conductor.

Programs Plate, MultiLoop I and II and Maxwell all provide quick and approximate forward models for vortex induction,

with the approximations from discretisation in the case of Plate, and the over-simplified geometry of the ribbons in MultiLoop and Maxwell. Any forward model can be used in an inversion, and this has been implemented for Maxwell.

The starting point of the advance reported here is implementation of Annan's solution in MATLAB as a forward model, based on the modifications of the code of Trefethan for eigenpotentials on confined surfaces obeying the Laplace equation. Note that, under the quasistatic approximation that speed of light delays of the order 3 to 10  $\mu$ s per km can be ignored, and that the vortex currents obey Laplace in 2D, with 0 current crossing the boundary.

Figs 2 (the slowest decaying) & 3 (the first 25) present the current eigenpotentials from Trefethen (2000) program example 23 over a square plate.

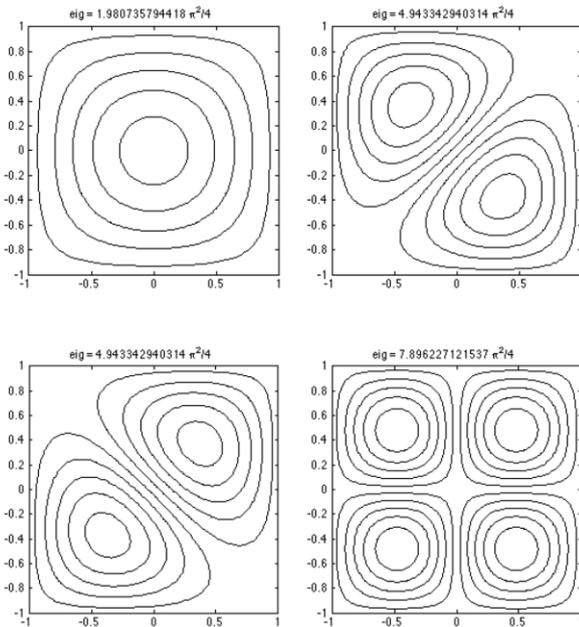


Figure 2: The current eigenpotentials with slowest decay in a square plate. Note that the second and third current patterns are degenerate with the same eigenvalue.

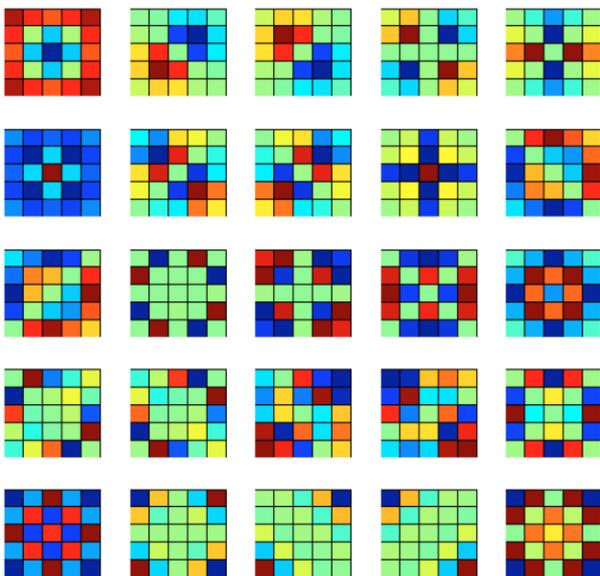


Figure 3: Current eigenpotentials as in Fig 2, presented as a colour amplitude image on an equispaced grid. The

potentials were calculated at 225 Chebychev collocation points and then interpolated (sub-sampled) to the images shown.

Based on the MATLAB implementation, calculation of the eigenpotentials and eigenvalues (related to decay rate or characteristic frequency) takes of the order of a millisecond, and only needs to be done once for any specific aspect ratio of length/width of the plate. It is trivial to compute eigenpotentials for a disk by the same method, and this computation is illustrated in Trefethan (2000). Figure 4 presents the eigencurrents for a disk for comparison with those in a square plate.

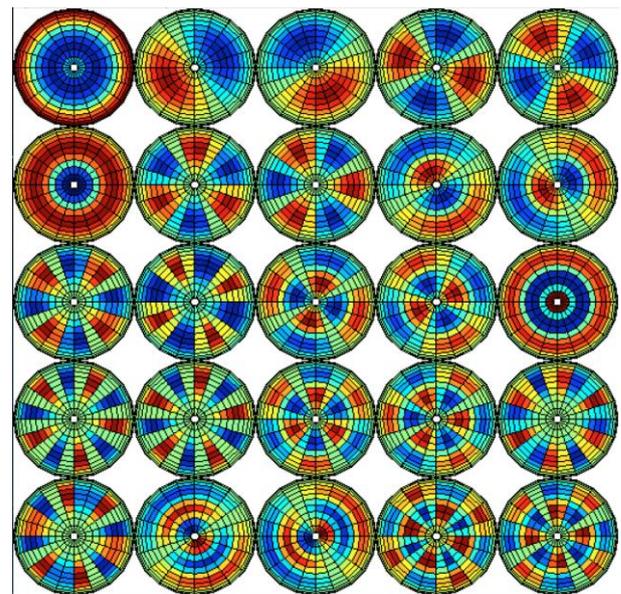
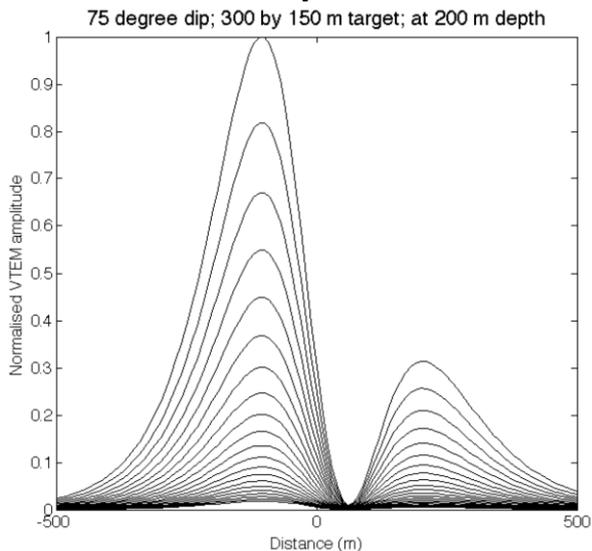


Figure 4: Eigenpotentials for a conducting disk. Many but not all of these are similar to those in the square plate (Fig 3). Note while potentials 1 to 4 (top rows) look very similar, the sign of an eigenpotential is arbitrary; eigenpotential 6 has reversed sign between the plate and the disk, with colours reversed.

There are several additional steps in calculating an EM response from eigencurrents.

- 1) Choose a (small) set of collocation points where the primary field is to be evaluated on the plate (Chebychev, Galerkin, equispaced), and calculate the eigenpotential at these points, and the time constant for that eigenpotential.
- 2) At each station:
  - a. Take the dot product of the primary field normal to the plate with the eigenpotential at each collocation point.
  - b. Take the dot product of the reciprocal receiver field normal to the plate with the eigenpotential at each collocation point (redundant for coincident loop systems).
  - c. Multiply the Tx-Rx factors for each eigenpotential.
  - d. Multiply each result by the decay of its eigenpotential, and sum.
- 3) Plot a profile.

Figure 5 presents a typical forward model for the VTEM response at 101 fiducials, calculated in about 1 ms, or a few tens of microseconds with a GPU, using the eigenpotentials shown in Figure 8 (at the end of the paper). This model is of course one for a resistive host with no conductive cover. It is easy to modify this response for a target located beneath a conductive overburden, using Maxwell's receding image theory. A forward model in this case can be calculated with a small time increase, from an additional 1 to 10 ms depending on the relative contrast of the overburden conductance and the target conductance. Disc models are as quick to calculate as plate models. I anticipate the Sphere and ellipsoidal models should be equally easy (Smith and Wasylechko, 2012) but this has not been attempted at this time



**Figure 5. Forward model using spectral methods of the off-time dB/dt decay over a conductive plate in a resistive host, with a VTEM loop overflying at 40 m altitude.**

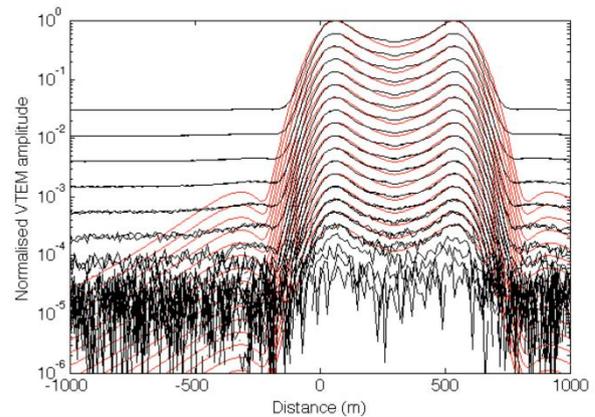
The next step in a more general 3D forward model is to allow for current gathering. Annan (1974) established the spectral method methodology for this to be added for conductors within one uniform layer, but it has not yet been implemented in the MATLAB code.

Standard target detection methodologies can be run based on say the criteria of Sattel and Reid (2006), and there are many off-the-shelf inversion methods available. For the purposes of illustration, I have used `fmincon` from the MATLAB optimisation toolbox, an algorithm allowing simple constraints to fit synthetic and field data using the spectral forward models. To speed up inversion, the predicted inductive and resistive limits (King and Macnae, 2001, Schaa and Fullagar, 2010) of the data over the target are first extracted and inverted, with further speed up from the use of spatial moments (Macnae et al, 1998)

Examples of an inversion of a synthetic decaying response plus noise shown in Figure 7. The target has a horizontal disc, representing for example the weathered top of a kimberlite. A background response plus noise was added before running the inversion. Testing on field data has commenced, but is not reported in this extended abstract.

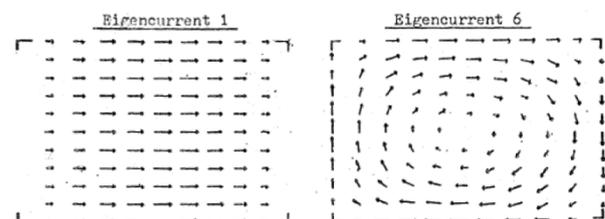
Limitations to the efficiency of this code are that it does not take into account SPM or IP effects, which should be stripped from data prior to inversion. The quasi-static assumptions in

the spectral method imply that the model is not useful at frequencies over a few hundred kHz.



**Figure 6: Test inversion of synthetic VTEM data over a 300 m wide disc target. A background response was included in the synthetic data, but not the inversion model.**

The eigenpotential method of Annan (1974) also permitted the modelling of current gathering effects when the plate was located within a conductive background. (Figure 7). Because the current gathering eigencurrents are orthogonal to the vortex currents, forward modelling and inversion should be rapid for these more complex models. The process to be tested will involve 1) inverting using vortex currents as illustrated in Figure 6 in an initial inversion, 2) using 1D methods to predict a background conductivity at the depth of the fitted target and 3) refitting with current gathering eigenpotentials permitted, coupled to the electric primary field.



**Figure 7: Annan (1974), p 140-141 sample eigencurrents modelling the effects of current gathering (Eigencurrent 1) as well as vortex induction (Eigencurrent 6).**

## CONCLUSIONS

The spectral method for the solution of EM vortex and galvanic current gathering, first introduced by Annan (1974) is easy to implement in MATLAB in a few lines of code using existing library functions and published recipes. Forward models run in milliseconds, of tens of microseconds on an inexpensive GPU. Canned MATLAB optimisation packages duplicate the functionality of existing geophysical inversion codes, and a virtually complete 3D inversion package can be implemented in a couple of pages of code. The approximate inversion of a single anomaly using a GPU card can take a few milliseconds.

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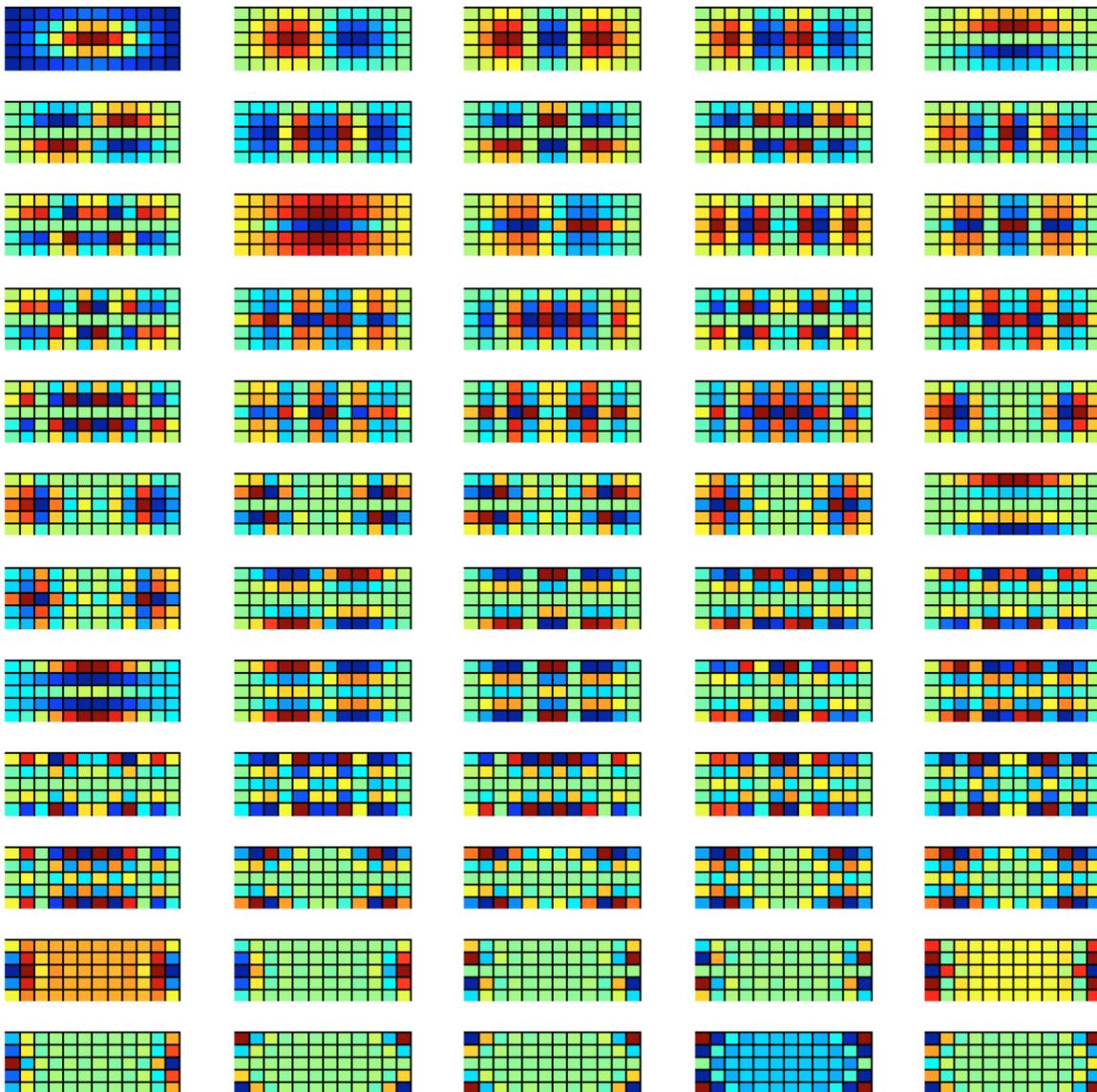


Figure 8. The full set of 60 eigenpotentials used for forward and inverse modelling of a plate with 2:1 aspect ratio. Program PLATE was restricted to 15 eigenpotentials.