3D magnetic modelling and inversion incorporating self-demagnetisation and interactions

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SUMMARY

Self-demagnetisation can significantly reduce the amplitude and modify the shape of the magnetic response from highly magnetic bodies. For quasi-planar bodies, only the transverse component of magnetisation is reduced, with the result that the direction of magnetisation rotates towards the plane of the body.
Furthermore, when highly magnetic bodies are in close proximity, the assumption of uniform inducing field is violated. Rather, highly magnetic bodies can modify the local magnetic field appreciably, with the result that the magnetisation induced in one body is affected by the magnetisations induced in all the others. It is important to take such interactions between highly magnetic bodies into account.

Potential field modelling and inversion software "VPmg" has been upgraded to account for self demagnetisation and interaction between magnetic bodies. The algorithm computes H-field perturbations at the model cell centres in two stages: *initialisation* and *optimisation*. During initialisation, a demagnetisation tensor is estimated for each cell, from which a first estimate for the H-field perturbation is derived. During optimisation, the H-field field estimate is refined iteratively via an inversion procedure. Remanence can be taken into account.

The algorithm has been validated for homogeneous spheres, spheroids, slabs, and cylinders. It has also reproduced magnetic interactions between two horizontal cylinders for the case published by Hjelt (1973). Explicit verification for complex heterogeneous bodies requires a suitable independent algorithm for benchmarking.

The application to inversion in highly magnetic environments is illustrated on field data examples.

Key words: Magnetic modelling, high susceptibility, inversion, self-demagnetisation, interactions

INTRODUCTION

In conventional magnetic modelling, self-demagnetisation and interaction between magnetic bodies is ignored. This is perfectly acceptable for weakly magnetised rocks. However, in highly magnetised rocks, self-demagnetisation and interactions are important determinants of the measured magnetic response. Self-demagnetisation reduces the intensity of transverse magnetisation, and hence alters the orientation of resultant magnetisation. The effect of self-demagnetisation can be calculated analytically for homogeneous ellipsoids (Clark et al, 1986), but to the best of our knowledge no algorithm has hitherto been available for general 3D modelling and inversion of highly magnetic bodies. Accordingly, we have developed such an algorithm.

The new algorithm computes H-field perturbations at the model cell centres in two stages: *initialisation* and *optimisation*. During initialisation, a demagnetisation tensor is estimated for each cell, from which a first estimate for the H-field perturbation is derived. During optimisation, the H-field field estimate is refined iteratively via an inversion procedure. Remanence is taken into account as well as induced magnetisation. Once the H-field has been determined at the centre of each cell, modelling and inversion proceed using existing routines.

The new algorithm has been incorporated in the VPmg package, designed for geologically constrained 3D modelling and inversion (Fullagar & Pears, 2007; Fullagar et al., 2008). Geological models are categorical, insofar as the sub-surface is divided into rock type domains. Whereas most inversion programs operate on "property only" models, VPmg models are attributed with rock type as well as property. Inverting geological models is a natural way to integrate geology and geophysics, and also offer a number of practical advantages. In particular, the topological significance of geological boundaries is maintained, permitting geometry inversion as well as property inversion. In addition, the inversion can be restricted to selected geological units of prime interest.

The ability to accurately compute the magnetic responses of highly magnetic terranes is vital for quantitative interpretation, providing the link between geological model and geophysical data. In particular, modelling self-demagnetisation and interactions is essential in order to calculate the net magnetisation anywhere in the sub-surface since magnetisation depends on body shape, orientation, and placement relative to other magnetic bodies, in addition to local induced and remanent magnetisation.

Recently there has been considerable interest in inversion which generates "magnetisation models" rather than susceptibility models. Inverting for magnetisation is conceptually attractive insofar as it circumvents the need, *during computation*, (a) to determine remanence *a priori*, and (b) to account for self-demagnetisation and interactions. However, the effect of geometry on magnetisation can pose problems *during interpretation* of “magnetisation models”, since the inverted magnetisation cannot be validated against measurements of susceptibility and/or remanence. Net *in situ* magnetisation is not a rock property and therefore a magnetisation model is one step further removed from geology than a susceptibility model. This “ground truthing” issue would arise even if magnetisation inversion were unique; in fact magnetisation inversion is, like susceptibility inversion, subject to severe non-uniqueness (Figure 1).

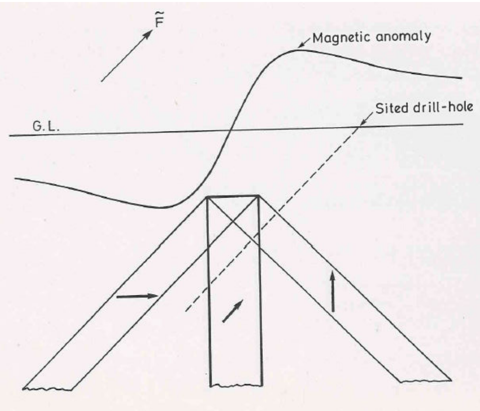

Figure 1: Three different tabular bodies, with magnetisations as indicated by the arrows, give rise to the same magnetic data (Clark et al, 1992).

In this paper the theoretical basis for the new algorithm, and its validation against analytic solutions, are described. It is then applied to the inversion of field data from Western Australia.

METHODOLOGY

The algorithm computes a vector adjustment, $\Delta\vec{H}$, for the H-field at the centre of each model cell, prior to computing the external magnetic field in the usual fashion. Magnetisation is assumed uniform within each cell; this “standard assumption” in magnetic modelling is reasonable if cells are sufficiently small. The H-vector adjustment proceeds in two stages: **initialisation** and **optimisation**. During initialisation, a demagnetisation tensor is estimated for each highly magnetised cell, assuming identical magnetisation in all such cells. This yields a first estimate for $\Delta\vec{H}$. During optimisation, the $\Delta\vec{H}$ vector is refined iteratively via inversion.

Inside a uniformly magnetised ellipsoid, the perturbation field, $\Delta\vec{H}$, can be related to the magnetisation, $\vec{J}$, via a demagnetisation tensor, $\tilde{N}$:

$$\Delta\vec{H} = -\tilde{N}\cdot\vec{J} \qquad (1)$$

Let $\vec{M}$ denotes the “intrinsic magnetisation”, i.e. sum of induced and remanent magnetisation vectors, $\vec{J}^I$ and $\vec{J}^R$ respectively. Then

$$\vec{M} = \vec{J}^I + \vec{J}^R = k\vec{H}_0 + \vec{J}^R, \qquad (2)$$

where k is the susceptibility and where $\vec{H}_0$ denotes the ambient geomagnetic field.

The net magnetisation when demagnetisation is considered is (Clark & Emerson, 1999)

$$\vec{J} = \left(\tilde{I} + k\tilde{N}\right)^{-1}\vec{M} \qquad (3)$$

The demagnetisation tensor concept has been extended to an arbitrary distribution of magnetised prismatic cells by Newell et al (1993). They separate the perturbation field into two parts, due to self-demagnetisation and mutual demagnetisation, i.e. interaction. Thus

$$\Delta\vec{H}_i = -\tilde{N}_{ii}\vec{J}_i - \sum_{\substack{j \\ j\neq i}} \tilde{N}_{ij}\vec{J}_j \qquad (4)$$

where $\tilde{N}_{ii}$ is the self-demagnetisation tensor for the ith cell, with magnetisation $\vec{J}_i$, and where $\tilde{N}_{ij}$ are mutual demagnetisation tensors between the ith and jth cells. The trace of $\tilde{N}_{ii}$ is 1, while the trace of any $\tilde{N}_{ij}$ is 0. Newell et al provide formulae for all the tensor components, but implementing their method directly for a large number of cells is prohibitively slow.

If the magnetisations of all cells are identical, then (4) reduces to

$$\Delta\vec{H}_i = -\vec{J}\sum_j \tilde{N}_{ij} = -\tilde{N}^i\vec{J} \qquad (5)$$

where $\tilde{N}^i$ denotes the demagnetisation tensor for the ith cell. Therefore, assuming a uniform magnetisation, the tensor components for each highly magnetic cell can be calculated if the perturbation fields, comprising both demagnetisation and interaction contributions, are known at the cell centres. The perturbation fields can be computed by forward modelling three times, assuming uniform magnetisation in the x-, y-, and z-directions in turn. The calculation for each coordinate direction defines a row of the tensor. Given the tensors $\tilde{N}^i$, the net magnetisation for each cell can then be determined by substituting into (3).

For heterogeneous bodies, this initialisation stage provides a useful first approximation to $\Delta\vec{H}$. During the subsequent optimisation stage the perturbation field estimates are updated via an iterative inversion procedure. The inversion terminates when the computed $\Delta\vec{H}$ are consistent with the net magnetisations, i.e. when

$$\vec{J} = \vec{M} + k\Delta\vec{H}. \qquad (6)$$

In order to avoid needless computation, the demagnetising field calculations are restricted to high magnetisation cells. At present the threshold is hardwired as $k(1+Q) \geq 0.1$ SI, where Q is the Koenigsberger ratio for the geological unit to which the cell belongs. If the magnetisation is identical in all high magnetisation cells, the tensor initialisation is virtually “exact”, and there is no need for optimisation.

Given the net magnetisation vector for each cell, forward modelling and inversion can proceed using existing algorithms, but with the difference that the inverse problem is non-linear: the resultant magnetisation depends on geometry in highly magnetic environments.

VALIDATION

The algorithm has been validated for homogeneous spheres, spheroids, slabs, and cylinders. The validity of the algorithm for ellipsoidal bodies has been established via comparison with the analytic solution (Clark et al, 1986). TMI responses computed with the new algorithm for a high susceptibility (1.5 SI) oblate spheroid model are compared with analytic values in Figure 2. The agreement is excellent.

The new algorithm has also reproduced the results of Hjelt (1973) for two parallel circular cylinders in close proximity (Figure 3). This represents a test of magnetic interaction as well as demagnetisation.

Validating the program for more complex situations has proved difficult because we have been unable to locate published examples.

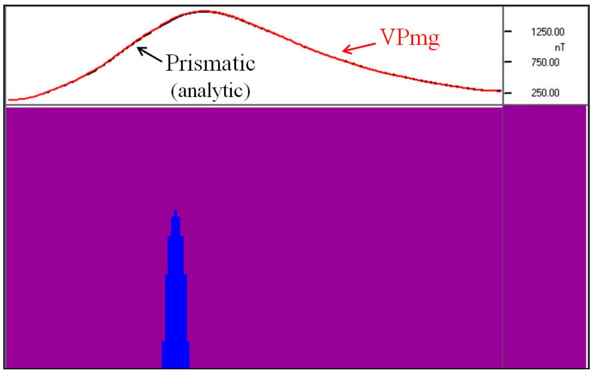

Figure 2: N-S TMI profiles over a vertical oblate spheroid, susceptibility 1.5 SI, in a non-magnetic host. The analytic curve (labelled "Prismatic") agrees closely with the curve generated with the new algorithm (labelled "VPmg"). Ambient field is 55000 nT with declination 45° and -60° inclination.

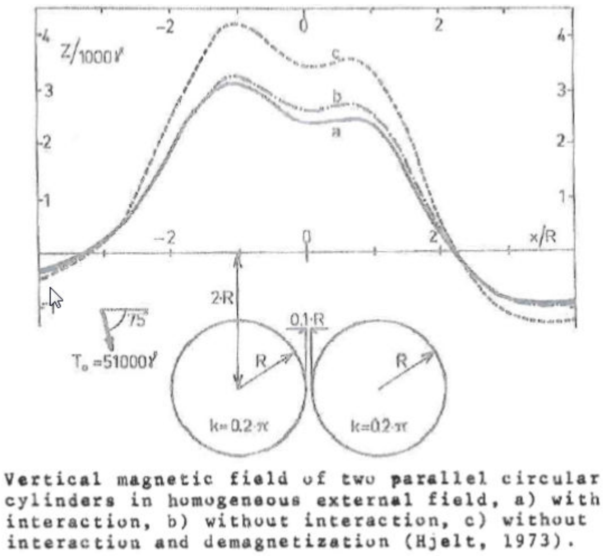

Figure 3a: N-S profiles for vertical component of induction, Bz, over two horizontal circular cylinders, susceptibility 0.628 SI, in a non-magnetic host (after Hjelt, 1973).

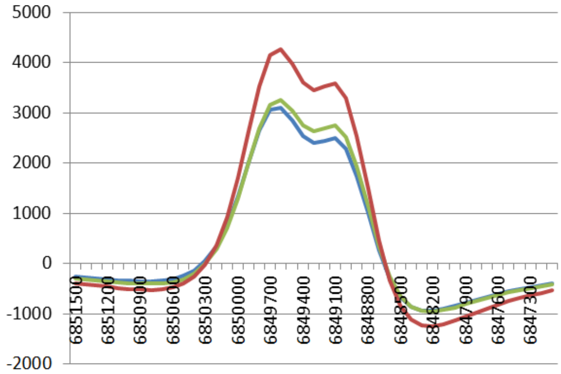

Figure 3b: Bz profiles computed over Hjelt's parallel cylinders using the forward algorithm describe here. The red curve takes no account of self-demagnetisation or interactions; the green curve accounts for self-demagnetisation; the blue curve takes both self-demagnetisation and interaction into account. Ambient field is 51000 nT with declination 0° and +75° inclination.

FIELD EXAMPLE

The new algorithm has been applied to field data recorded over magnetite-rich stratigraphy at Southdown, in SW Western Australia (Figure 4).

Figure 4: Location map of the South down area in Western Australia.

A starting model comprising 4 simple rectangular bodies (Figure 5) was presented to VPmg for forward modelling and inversion to update the model geometry while taking self demagnetisation (and interaction) into account. Three of the magnetic bodies has magnetic susceptibilities greater than 2SI.

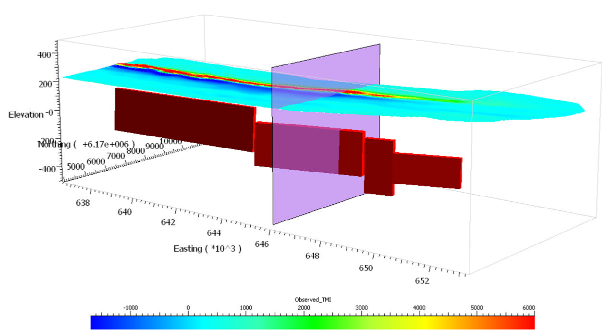

Figure 5: Perspective view of the 4 rectangular bodies (5x vertical exaggeration). Magnetic data is draped above the top of bodies for visualisation purposes. The purpose slice depicts the location of the section shown in Figure 5.

For a north-south profile across one of the magnetic units the fit is greatly improved after adjustment of the shape of a dipping slab (Figure 6). To better fit the data, VPmg geometry inversion has adjusted the parametric shape to be more elongated.

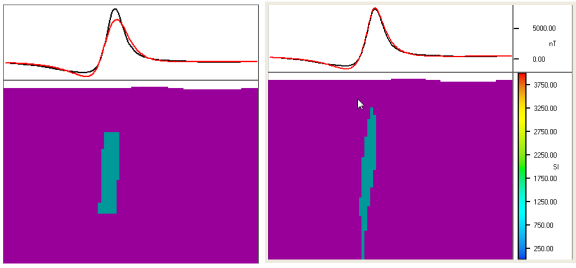

Figure 6: Observed (black) and VPmg calculated (red) magnetic responses of the starting model (left) and the inverted model (right). Magnetic body in the cross-section has a magnetic susceptibility of 2.34SI (5x vertical exaggeration).

CONCLUSIONS

Self-demagnetisation and interactions between magnetic bodies cannot be ignored when modelling in highly magnetic terranes. The resultant *in situ* magnetisation is a function of shape, orientation, and relative positions of magnetic bodies as well as their intrinsic (induced + remanent) magnetisations. We have developed a 3D algorithm which takes these non-linear effects into account during both forward modelling and geologically-constrained inversion. The algorithm provides a means for relating *in situ* magnetisation to measurements of susceptibility and remanence. The algorithm has been validated against analytical solutions, and its utility has been demonstrated via application to interpretation of field data from Western Australia.

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