

# Joint inversion of spatial autocorrelation curves with HVSR for site characterization in Newcastle, Australia

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## SUMMARY

In order to investigate site characterization of Newcastle affected by the 1989 Newcastle Earthquake ( $M_L = 5.6$ ), we conducted microtremor array measurements. The spatial autocorrelation (SPAC) method was applied to estimate the S-wave velocity structure from observed microtremor data. Although the inversion of spatial autocorrelation curves is effective for estimating shallow S-wave velocity structures within the sedimentary layer, it is usually difficult to estimate the boundary between the sedimentary layer and bedrock due to a lack of amplitude of vertical components of microtremors at low frequencies. On the other hands, it is well known that horizontal to vertical particle motion spectral ratios (HVSR) have information which assists in resolving the bedrock depth and velocity. Thus, we have applied joint inversion of the spatial autocorrelation curves with HVSR to observed microtremors. Since observed HVSR curves are subject to fluctuations due to unknown Love and body wave contributions or other noise effects, there is difficulty in fitting absolute values of observed HVSR. Therefore, we evaluate observed HVSR and theoretical H/V spectra by zero-lag cross-correlation to fit the shapes of HVSR. The observed HVSR curve has accurate information for the deeper velocity structure and therefore, the estimated velocity model by joint inversion differs in velocity estimations at depth. It is concluded that the joint inversion of the spatial autocorrelation curves with HVSR is useful in practice for obtaining improved estimates of S-wave velocity models down to bedrock.

**Key words:** joint inversion, SPAC method, HVSR, microtremors

## INTRODUCTION

Newcastle was devastated by the Newcastle earthquake with  $M_L = 5.6$  in 1989. The risk assessment for the potential of subsequent earthquake is an important task. Within this framework, estimation of near surface S-wave velocity structures is necessary to construct a site response model which will affect the ground shaking (Dhu and Jones, 2002).

The microtremor survey method (Okada, 2003) has been widely used in recent years because it is a non-destructive method and easy to conduct for estimating S-wave velocity structures. In the array technique of the microtremor method,

the spatial autocorrelation (SPAC) method (Aki, 1957) is commonly used to estimate near surface S-wave velocity structures. Asten *et al.*, (2004) proposed the direct fitting method of the spatial autocorrelation (SPAC) curves which can avoid interpretation errors of phase velocity estimations in the SPAC method.

On the other hands, the horizontal to vertical particle motion spectral ratios (HVSR) from a single station have useful information for bedrock depth and velocity. Although the values of HVSR curves are unstable due to Love waves, body waves or other types of noises (Zor *et al.*, 2010; Endrun, 2011), fitting of the shape of HVSR curves has still meaningful as Zor *et al.* (2010) did by using zero-lag cross correlation.

In order to investigate site characterizations down to bedrock in Newcastle, Australia, we perform microtremors array measurements. The SPAC and HVSR methods are applied to observed microtremors. S-wave velocity structures are evaluated by joint inversion of SPAC curves with HVSR curves using zero-lag cross-correlation.

## METHOD

### SPAC method

The basic theory of the SPAC method is summarized in Okada (2003). Suppose microtremors are obtained by a circle array with a radius  $x$  and surface waves of fundamental mode are dominant in microtremors. The azimuthal average of complex coherencies  $COH$  between a central and a circumferential receiver goes to the Bessel function, referred to as SPAC curves  $\rho$ ,

$$\rho(x, f) = \frac{1}{2\pi} \int_0^{2\pi} COH(x, f, \theta) d\theta = J_0 \left( \frac{2\pi f}{c(f)} x \right) \quad (1)$$

where,  $f$  is frequency,  $\theta$  is azimuthal angle,  $c$  is phase velocity, and  $J_0$  is the Bessel function of the first kind of zero order. Phase velocities can be obtained by fitting observed SPAC curves to the Bessel function. In this study, observed SPAC curves are directly compared with theoretical ones for assumed layered media without interpretation of phase velocities proposed by Asten *et al.* (2004).

The misfit function for SPAC curves  $F_{sp}$  is defined as follows (Wathelet *et al.*, 2008),

$$F_{sp} = \sqrt{\frac{1}{\sum_{k=1}^{n_R} n_{Fk}} \sum_{i=1}^{n_R} \sum_{j=1}^{n_{Fj}} \frac{(\rho_{obs}(x_i, f_j) - \rho_{theo}(x_i, f_j))^2}{\sigma^2(x_i, f_j)}} \quad (2)$$

where,  $\rho_{obs}$ ,  $\rho_{theo}$ , and  $\sigma$  are observed and theoretical SPAC curves, and the standard deviation at frequency  $f_j$  and for  $i$  th receiver spacing, respectively.  $n_R$  is the number of receiver spacing and  $n_{Fi}$  is the number of frequency samples for  $i$  th receiver spacing.

### HVSR method

Following Arai and Tokimatsu (2005), the HVSR of microtremors at frequency  $f$  can be defined as follows,

$$HV(f) = \sqrt{\frac{P_{NS}(f) + P_{EW}(f)}{P_{UD}(f)}} \quad (3)$$

where,  $P_{UD}$  is the Fourier power spectrum of the vertical motion and  $P_{NS}$  and  $P_{EW}$  are those of the two orthogonal horizontal motions.

In inversion, observed HVSR curves are evaluated by zero-lag cross-correlation (ZLC) in log scale. The misfit value for HVSR curves  $F_{HV}$  is defined as follows,

$$F_{HV} = ZLC^{-1} = \left( \frac{1}{N^2} \frac{\sum_{i=1}^N HV_{obs}(f_i) HV_{theo}(f_i)}{\sqrt{\sum_{i=1}^N HV_{obs}(f_i)^2 \sum_{i=1}^N HV_{theo}(f_i)^2}} \right)^{-1} \quad (4)$$

where  $N$  is the number of frequency samples,  $HV_{obs}$  and  $HV_{theo}$  are observed and theoretical HVSR in log scale, respectively. Theoretical HVSR values are defined as the Rayleigh waves ellipticity assuming a value of 0.7 of the Rayleigh-to-Love wave amplitude ratio for horizontal motions.

### Joint inversion scheme

In order to apply joint inversion of SPAC curves with HVSR curves, the following misfit function is proposed,

$$F = \alpha F_{SP} + (1 - \alpha) F_{HV} \quad (5)$$

where,  $\alpha$  is the fraction of two misfit functions. In this study,  $\alpha$  is given as 0.5.

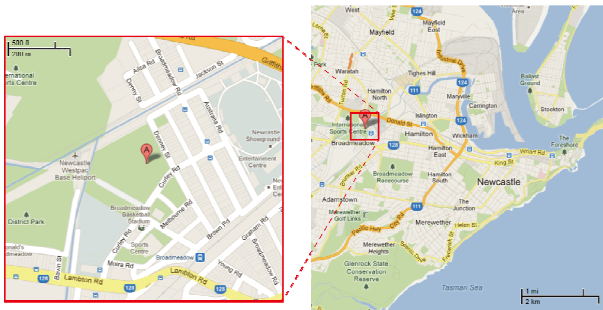


Figure 1. Location of the survey site (BRD02) (modified from google map). "A" indicate the location of the central receiver in the array.

## RESULTS

### Field data acquisition

Figure 1 shows the location of the survey site (BRD02) in Newcastle, Australia. 3-component microtremors are observed by 2 triangle arrays with a central receiver (Figure 2). Sampling time is 5 msec and data length is about 50 min. We divided observed data into 100 data sets. Each data set is about 30 sec with 30 % overlap. Standard deviations of each observed values are computed by 10 sub-data sets defined by the mean values of 10 data sets.

### Data analysis

Figures 3 and 4 show the observed HVSR curve, dispersion curve and SPAC curves. In the estimation of HVSR curves, the Konno-Ohmachi smoothing function (Konno and Ohmachi, 1998) was applied. Note that the observed dispersion curve is not included in the inversion. The frequency range of observed HVSR is lower than that of the dispersion curve or SPAC curves. This shows the shape of the observed HVSR curve in low frequencies has the information for S-wave velocity structures at a deeper part.

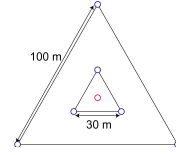


Figure 2. Array shape used in the survey.

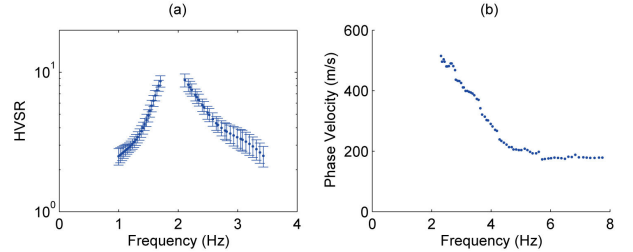


Figure 3. (a) Observed HVSR and (b) dispersion curves.

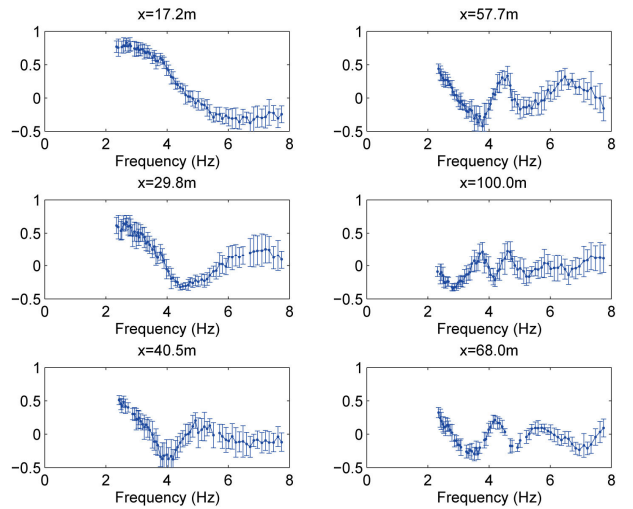
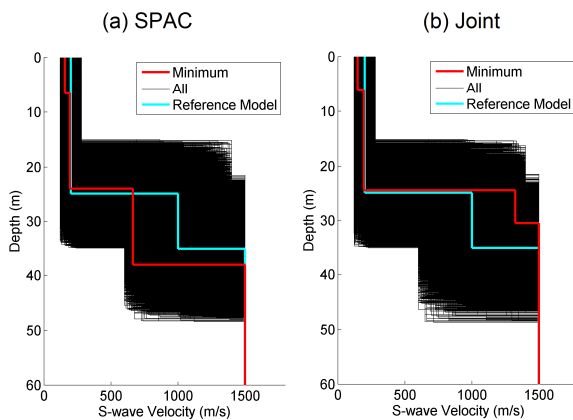


Figure 4. Observed SPAC curves for each receiver spacing.

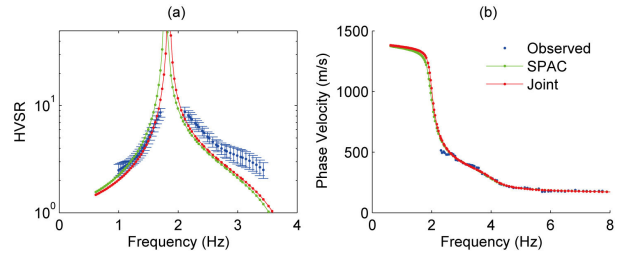
Then, S-wave velocity structures were estimated by inversion. As an inversion method, genetic algorithm with elite selection and dynamic mutation (Yamanaka and Ishida, 1996) was employed. The four-layered model was assumed in inversion. The search range of S-wave velocity and thickness is  $\pm 40\%$  for the reference model, which was created by trial and errors. The S-wave velocity for an infinite half-space is fixed as 1500 m/s. 20 trials with randomly seeded initial populations were performed. For comparison, joint inversion of SPAC curves with HVSR curves and inversion only considering SPAC curves were performed.

Figure 5 shows inverted S-wave velocity structures when inversion considering only SPAC curves and joint inversion were performed. There is no significant difference between both inverted models for 1st and 2nd layers. However, the depth and velocity for the 3rd layer become respectively shallower and higher when performing joint inversion.

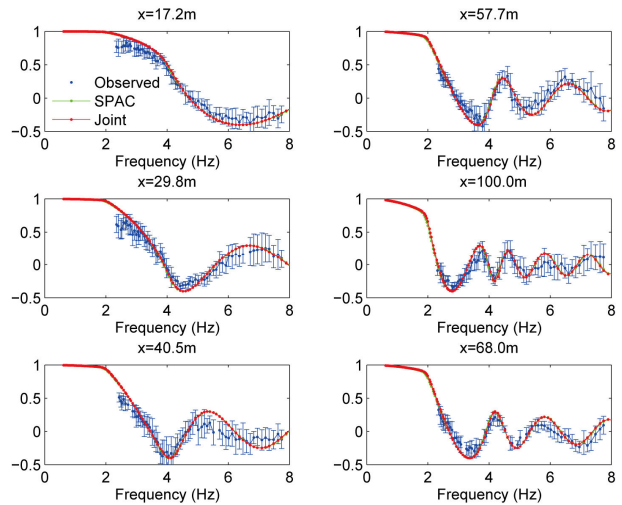
Figures 6 and 7 are the comparison of observed values with theoretical ones for inverted velocity models with the minimum misfit values. Theoretical SPAC curves for both inverted models are almost identical. The difference exists at low frequencies below 2 Hz where very few points on the SPAC spectrum were estimated. Moreover, at low frequencies, the probability of unrealistic values for SPAC curves is high due to insufficient energy level of vertical component data (Wathelet *et al.*, 2008). On the other hands, the estimated velocity models from both inversions generate a difference in the peak frequency and the shape of theoretical HVSR. By including HVSR in a joint inversion, the resultant theoretical HVSR curve is in good agreement with the observed one.



**Figure 5.** S-wave velocity structures by genetic algorithm inversion considering (a) only SPAC curves and (b) SPAC curves and HVSR curve. Red and cyan lines are the inverted velocity model with the minimum misfit value and the reference model, respectively. Black lines are all the computed models in inversion.



**Figure 6** Comparison of observed values with inverted ones for (a) HVSR curves and (b) dispersion curves. Green and red lines are theoretical values for the inverted velocity models for inversion considering SPAC curves and joint inversion, respectively.



**Figure 7** Comparison of observed values with inverted ones for SPAC curves. Green and red lines are theoretical values for the inverted velocity models for inversion considering SPAC curves and joint inversion, respectively.

## CONCLUSIONS

In this study, we investigated site characterization of Newcastle, Australia by microtremors array measurements. Joint inversion of SPAC curves with HVSR was performed to obtain the S-wave velocity structure.

The inverted velocity structure at depth obtained by joint inversion was different from that obtained by inversion considering SPAC curves only, although including HVSR in the inversion didn't generate a significant change for shallower velocity estimations. Since SPAC curves at low frequencies have high probability of unrealistic values due to low signal amplitude, the inverted velocity model incorporating the HVSR in addition to SPAC curves is a more reasonable velocity model.

It is concluded that the joint inversion of the spatial autocorrelation curves with HVSR is useful for obtaining improved estimates of S-wave velocity models down to bedrock.

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