ASEG-PESA 2015
Geophysics and Geology together for Discovery
24th International Geophysical Conference and Exhibition
15–18 February 2015 Perth, Western Australia

Linking electrical and hydraulic conductivity through models of random resistor networks

Alison Kirkby
University of Adelaide
North Terrace, Adelaide 5005
Alison.Kirkby@adelaide.edu.au

Graham Heinson
University of Adelaide
North Terrace, Adelaide 5005
Graham.Heinson@adelaide.edu.au

SUMMARY

We present models of random resistor networks to relate electrical resistivity to fracture permeability in the upper crust. In this approach, the upper crust is modelled as a network of resistors that are randomly assigned to be either electrically and hydraulically conductive or resistive based on a network-wide probability of connection. In the models presented here, the conductive resistors are assigned resistance values based on a constant fracture diameter of 1 mm and a fluid resistivity of 0.1 Ωm, with variable fault length distributions and probabilities of connection. We have found that the permeability is very sensitive to both of these parameters, increasing to 8.33×10^8 times the matrix permeability in the fully connected case. The resistivity is less sensitive, increasing by a factor of 1000.

Key words: resistivity, permeability, electromagnetic

INTRODUCTION

The ability to predict crustal permeability distribution is important for a number of resource industries, for example, geothermal energy and oil and gas. Geothermal energy targets require both elevated temperatures at accessible depths, and sufficient permeability to sustain adequate flow rates for commercial production. Likewise, understanding the permeability distribution is vital to accurately model the performance of an oil or gas reservoir (Babadagli et al, 2004). However, permeability often varies by orders of magnitude over short distances. Therefore, not only is it difficult to predict permeability from the surface, but also, even when drillholes are available these may not be adequate to characterise nearby targets.

Electromagnetic methods

Electromagnetic techniques have been applied extensively to exploration for conventional geothermal targets (e.g., Munoz, 2014; Pellerin et al., 1996). In conventional geothermal systems, the target is a strong electrical conductivity anomaly resulting from a clay cap caused by alteration of the host rock to electrically conductive clay minerals (Ussher et al., 2000; Wright et al., 1985}. However, unconventional geothermal energy targets such as those being investigated in Australia (e.g., Barnett and Evans, 2010,; Hogarth et al., 2013; Reid and Messeiller, 2013) are generally deeper and are located in a range of geological settings, making the application of a single exploration model difficult. In addition, the rocks in many of the sedimentary basins in Australia are highly electrically conductive (e.g., Peacock et al., 2013) and therefore high conductivities resulting from, for example, saline fluids or clay alteration may not produce strong conductivity anomalies. For these reasons, the application of electromagnetic techniques such as MT may be less straightforward in exploration for unconventional geothermal resources.

Time lapse MT monitoring of an enhanced geothermal system near Paralana, South Australia was performed in 2011 (Peacock et al., 2012; Peacock et al., 2013). In this experiment, MT data were collected pre- and post-injection of an electrically conductive fluid into a natural fault network at 3.6 km depth. Much stronger increases in electrical conductivity were observed parallel to the strike of the fault network than perpendicular to it, consistent with an increase in hydraulic (and electrical) conductivity. These observations show that the presence of fluid-filled fractures in a medium changes the effective electrical conductivity that is measured.

Electrical current and fluid flow

Fluid flow through porous media is, at low flow velocities, described by Darcy's Law:

$$Q = -\frac{kA}{\mu}\nabla p \qquad (1)$$

Where Q is the volumetric flow rate, k is the permeability, μ is the viscosity, p is the pressure and A is the cross sectional area of the sample. Ohm's law describes electric current flow:

$$I = -\frac{A}{\rho}\nabla v \qquad (2)$$

Where I is the current, A is the cross sectional area, ρ is the resistivity and v is the voltage.

Fractures are commonly approximated by the parallel plate model, where either side of the fracture is a smooth plate with separation d and width l_v, and therefore the cross-sectional area for fluid flow is $l_v d$. The steady state solution of the Navier-Stokes equations for laminar fluid flow leads to a cubic dependence of fluid flow on aperture (Brown, 1989):

$$Q = -l_y \frac{d^3}{12\mu} \nabla p \quad (3)$$

Comparison of this equation with Darcy's Law shows that the permeability of the fracture is equal to $d^2/12$. In contrast, the electrical current flow through such a fracture has a linear dependence on d:

$$I = -l_y \frac{d}{\rho_f} \nabla v \quad (4)$$

Where ρ_f is the resistivity of the fluid. As noted by Brown (1989) equations (3) and (4) have a similar form, with the permeability/viscosity being analogous to the electrical conductivity. Therefore, it may be possible to directly relate conductivity to permeability.

METHOD

Random resistor networks

Bahr (1997) proposed the use of random resistor networks to evaluate the bulk electrical conductivity of a medium (Figure 1). In this type of analysis, electrical current flow is assumed to occur through a network of resistors. Resistors within this network are defined as either open (i.e., high electrical conductivity) or closed (low conductivity). Fluid flow through the same network can be considered in terms of a network of pipes (or in 2D, flat plates) with varying apertures, corresponding to varying hydraulic conductivity. The open resistors can be compared to faults within a host rock filled with an electrically conductive fluid, whilst the resistive parts can be compared to the background host rock and/or faults that are closed or cemented with electrically and hydraulically resistive cement. Importantly, the conductivity is controlled not only by the total number of open bonds, but also on their position in the network.

This type of analysis can be performed in a probabilistic sense by considering a suite of different networks, each with the same probability *p* that any particular bond within the given network is open. By repeating this process at different probabilities of connection, and by modelling the current and fluid flow in different directions, the relationship between bulk electrical conductivity (and resulting electrical anisotropy) and the probability of connection in different directions, can be explored.

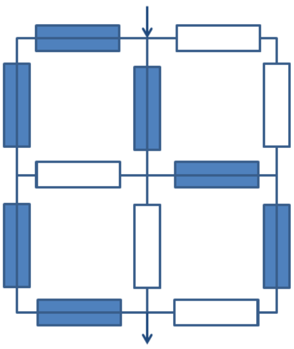


Figure 1. Simple 2x2 random resistor network. Blue bonds are connected (i.e., low resistivity) bonds and white bonds are broken (or high resistivity) bonds. Modified after Bahr (1997).

In order to replicate the behaviour of faults, a third variable can be introduced, defined here as the linearity factor. This factor biases the probability of connection of any given bond depending on whether the adjacent bond (in the direction of the bond) is open or closed. The linearity factor affects the relative probability of connection each bond in a network, so that the overall probability for a given network remains unchanged. For example, if a network has a linearity factor of two, the probability of connection of each bond that is adjacent to an open bond would have twice the probability of connection than one that was not adjacent to an open bond. This factor is included to make the networks more fault-like, with longer segments of high conductivity. High linearity factors are associated with longer average fault lengths, and low values are associated with short, segmented faults.

For a given network, the fracture porosity or total void space occupied by fluid-filled fractures can be estimated using the following equation, assuming the fracture diameter is small compared to the cell size:

$$\phi = \frac{(p_x + p_z)d}{c} \quad (5)$$

Where ϕ is the porosity, p_x and p_z are the probabilities of connection in the *x* and *z* directions, *d* is the fracture diameter and *c* is the cell size.

Modelling approach

We generated random resistor networks by first constructing a network of nodes. Bonds between nodes were then randomly assigned either a high or low value of both permeability and electrical conductivity, according to a network-wide linearity factor and probability of connection in each direction. Permeability and conductivity values assigned to the resistors were calculated based on fluid and matrix resistivities of 0.1 and 1000 Ωm, and a fracture diameter of 1 mm. The matrix permeability was set at 10^{-18} m^2. The probability of connection in the horizontal direction was set to be constant at 0.1. Electrical current and fluid flow was then modelled in two orthogonal directions across the network for linearity factors of 1, 5 and 20 for probabilities of connection in the vertical direction ranging from 0 to 1. In each model, a voltage and pressure difference of 1.0 was applied across the network. Through equation (2), the bulk resistivity is then equal to the inverse of the average current flow per unit length entering (and exiting) the network. Likewise, the bulk permeability is equal to the viscosity multiplied by the average fluid flow rate through the network.

RESULTS

An example of one model realization, with a probability of connection in the *z* direction of 0.5, is shown in Figure 2. The effective conductivity and permeability has been calculated for various probabilities of connection and linearity factors, and is plotted in Figure 3. The fault length distribution for various linearity factors, for a probability of connection of 0.4 in the vertical direction, is also shown in Figure 3.

The effective resistivity for the model shown in Figure 2 is 85 Ωm in the vertical direction and 590 Ωm in the horizontal direction; the effective permeability is 2.9×10^{-17} m^2 in the vertical direction and 2.1×10^{-18} m^2 in the horizontal direction.

The fluid flow tends to focus much more strongly in the longer fractures that connect all or most of the way across the network than the electrical current, which is more evenly distributed amongst all fractures and in the matrix (Figure 2). As a result, the fluid flow appears to follow a longer path across the network than the current flow, particularly in the less connected horizontal direction (Figure 2f). Consistent with this, the anisotropy in permeability (factor of approximately 14 in Figure 2) is in general higher than the electrical resistivity anisotropy (factor of 7).

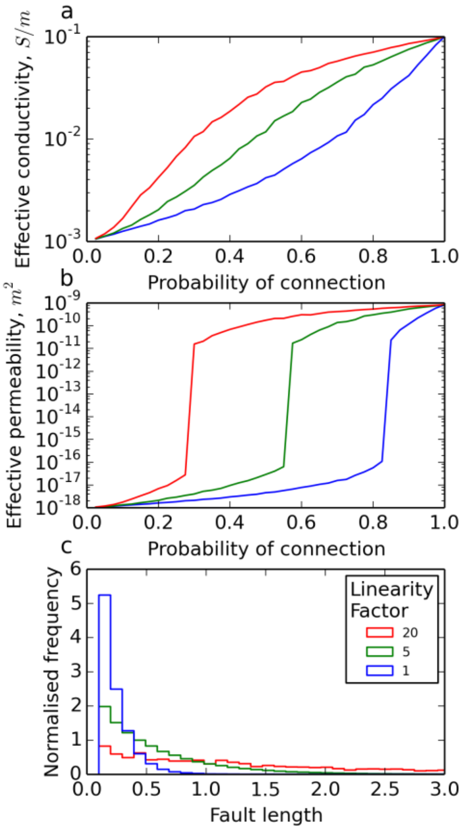


Figure 3. (a) Effective electrical conductivity, and (b) Permeability, vs. probability of connection for three linearity factors for a 50 × 50 cell random resistor network of an 5 × 5 m area of rock. (c) Histograms showing fault length distributions associated with each linearity factor, for a probability of connection of 0.5.

With both fluid flow and current, the relationship between the probability of connection and effective permeability or resistivity is not linear. As shown by Bahr (1997), at low probabilities of connection, there are not enough conductive resistors to make a connected path through the network, and therefore, increasing the probability of connection does not result in strong increases in either resistivity or permeability. However, as the percolation threshold is approached, both properties increase rapidly.

For fluid flow, the percolation threshold is well-defined (Figure 3b). It occurs at a vertical probability of connection of 0.25 for a linearity factor of 20, at about 0.55 for a linearity factor of 5, and at about 0.5 for a linearity factor of 1. The permeability increases by over 5 orders of magnitude over the percolation threshold, a change in probability of connection of 0.1. Increasing the linearity factor has the effect of shifting the percolation threshold to lower probabilities of connection, so that the probability of connection required to reach the percolation threshold is not as high. However, the rate of increase in permeability over the percolation threshold is similar (Figure 3b).

In contrast, the resistivity has a much less well defined percolation threshold, with effective conductivity only increasing by two orders of magnitude. As with fluid flow, the percolation threshold occurs at vertical probabilities of connection of 0.25, 0.55 and 0.85, occurring at lower probabilities of connection as the linearity factor is increased.

Comparison with Figure 3 shows that the model shown in Figure 2 is just below the percolation threshold.

CONCLUSIONS

We have presented here preliminary results of modelling of electrical current and fluid flow through simple 2D random resistor networks. Each bond in the networks was set to be either a hydraulically and electrically conductive, or resistive, based on a network-wide probability of connection in the horizontal and vertical directions. The permeability and resistivity values were set based on a fixed matrix permeability of 10^{-18} m^2 and resistivity of 1000 Ωm, a fluid resistivity of 0.1 Ωm, and a fracture diameter of 1 mm. The probability of connection in the horizontal direction was also fixed at 0.1 for all models. Modelling was carried out for probabilities of connection in the vertical direction ranging from 0 to 1.

The modelling shows that a percolation threshold can be defined for both fluid flow and current, below which the effective conductivity is close to the matrix value. At the percolation threshold, both permeability and electrical resistivity, but in particular the permeability, are highly sensitive to changes in probability of connection, increasing rapidly over the percolation threshold.

Further modelling will be undertaken to more fully explore the relationship between the different input parameters and the bulk electrical and hydraulic properties. In particular, we will look in more detail at the effect of changing the mean fracture aperture and fracture porosity, and will also investigate other parameters such as linearity factor, a proxy for mean fracture length, and the probability of connection each direction.

We will also expand the modelling to 3D, and incorporate more complex networks using fractal embedded networks (Bahr, 1997). With fractal embedded networks, each bond within the network can be considered as either conductive, resistive, or alternatively, embedded. In the case that the bond is embedded, the individual bond is replaced by a network of bonds with its own probability *p* of connection. The network can be embedded in this way multiple times, and as a result, connectivity is evaluated on smaller and smaller scales. An embedded geometry may be more realistic for modelling fracture networks, which can exist on the scale of several kilometres down to micro-scale cracks. Using embedded networks, a range of scales can be included in a single model.

Modelling the effect of open fractures on resistivity and hydraulic conductivity will allow us to develop an improved understanding of the relationship between electrical resistivities obtained from electromagnetic and subsurface fracture characteristics. This will help to improve the geological interpretation of electrical resistivities interpreted from magnetotelluric data, contributing to its utility as an exploration method for resource exploration.

ACKNOWLEDGMENTS

We thank the University of Adelaide for funding support and Geoscience Australia for providing a scholarship to do this work.

REFERENCES

Archie, G., 1942. The electrical resistivity log as an aid in determining some reservoir characteristics, Petroleum Transactions of the Australian Institute of Mining, Metallurgical, and Petroleum Engineers, 146, 54-62.

Babadagli, T., Al-Salmi, S., et al., 2004. A review of permeability-prediction methods for carbonate reservoirs using well-log data, SPE Reservoir Evaluation & Engineering, 7(02), 75-88.

Bahr, K., 1997. Electrical anisotropy and conductivity distribution functions of fractal random networks and of the crust: the scale effect of connectivity, Geophysical Journal International, 130(3), 649-660.

Barnett, P. & Evans, T., 2010. Exploration and assessment of Hot Sedimentary Aquifer (HSA) geothermal resources in the Otway Basin Victoria, in Proceedings of the 2010 Australian Geothermal Energy Conference, Record 2010/35, pp. 106-111, Geoscience Australia.

Brown, S. R.: Transport of fluid and electric current through a single fracture, Journal of Geophysical Research: Solid Earth, 94, (1989), 9429-9438.

Hogarth, R., Holl, H., & McMahon, A., 2013. Flow testing results from Habanero EGS project, in Proceedings of the 2013 Australian Geothermal Energy Conference.

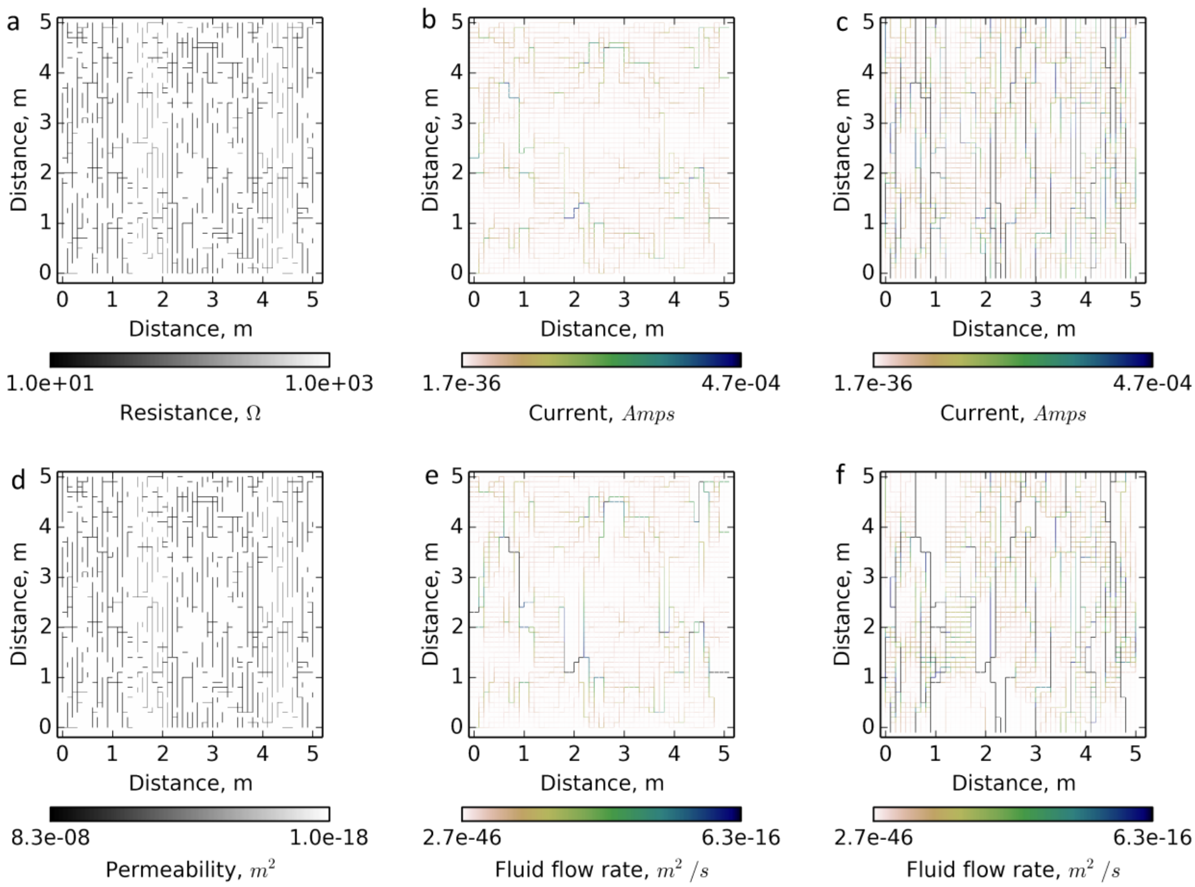
Muñoz, G., 2014. Exploring for geothermal resources with electromagnetic methods, Surveys in Geophysics, 35(1), 101-122.

Peacock, J., Thiel, S., Heinson, G., & Reid, P., 2013. Time-lapse magnetotelluric monitoring of an enhanced geothermal system, Geophysics, 78(3), B121-B130.

Peacock, J. R., Thiel, S., Reid, P., & Heinson, G., 2012. Magnetotelluric monitoring of a fluid injection: Example from an enhanced geothermal system, Geophysical Research Letters, 39(18), 1-5.

Pellerin, L., Johnston, J. M., & Hohmann, G. W., 1996. A numerical evaluation of electromagnetic methods in geothermal exploration, Geophysics, 61(1), 121-130.

Reid, P. & Messeiller, M., 2013. Paralana Engineered Geothermal Systems project 3.5MW development plan, in Proceedings of the 2013 Australian Geothermal Energy Conference.



A

Figure 2. Two dimensional electrical random resistor network representing a 5 × 5 m fractured rock mass with fault diameter 1 mm and fluid resistivity 0.1 Ωm. Probability of connection is 0.1 in the horizontal direction and 0.5 in the vertical

direction, linearity factor is 5. (a) resistivity, (b) horizontal current, (c) vertical current, (d) permeability, (e) horizontal fluid flow, (f) vertical fluid flow.