Quasi-3D inversion of full size AEM datasets

**SUMMARY**

We present a new algorithm for quasi-3D inversion of airborne transient electromagnetic data (AEM). The algorithm uses a 3D voxel grid for the model domain while the forward response and derivatives are calculated using interpolated “virtual” 1D models collocated at the AEM measurement points. The algorithm efficiently decouples the model domain from the practical spatial sampling which are often dictated by the landscape, ground installations etc., rather than the optimal model resolution. The algorithm uses sparse storage with efficient solvers and it scales linearly both with the number of CPU’s and with the survey size. This means that it can be used to invert entire surveys of thousands of kilometres on small multiprocessor servers priced less than 5000 Euro. The algorithm allows inputting a-priori information from boreholes, joint inversion with ground-based electrical/ electromagnetic data and modelling of flight altitude and shift parameters.

**Key words:** 3D inversion, airborne electromagnetic, AEM, transient electromagnetic, TEM

**INTRODUCTION**

Inversion of electromagnetic data has developed from single site inversions, over profile oriented inversion using lateral constraints (Auken et al., 2005) to inversions including entire surveys using spatial constraints (Viezzoli et al., 2008). In these inversion methods the model domain is linked to the actual observation points and for e.g. airborne surveys the spatial discretization of the model domain will reflect the flight lines and the spatial sampling of the decays as illustrated in Figure 1A.

We have developed a new quasi-3D inversion code which efficiently decouples the model domain from the data space and which can invert efficiently entire surveys in one go. Because of the decoupled model domain it becomes straightforward to perform joint inversion of e.g. airborne data with ground-based data or to add a-priori information to the domain. The latter can be used to include geological information into the inversion allowing better resolution of equivalent layer sequences or layers which are at the edge of detection and resolution. Finally the 3D model domain is straightforward to integrate in geological or hydrostratigraphic models.

**METHOD AND RESULTS**

The algorithm works directly in a 3D voxel grid disconnected from the actual measuring points as illustrated in Figure 1B. The voxel model domain, \( M \) defines the subsurface model parameters like resistivity, \( \rho \), on a set of nodes, \( \rho \), and the distribution of the properties is computed everywhere by means of an interpolation function \( f \) which can e.g. be inverse distance, nearest neighbour or kriging. This is expressed as

\[
\rho(x,y,z) = f(\rho_i) \tag{1}
\]

The position of the nodes is fixed during the inversion and is chosen considering topography, geological variations and resolution capabilities of the geophysical data. Typical it will be a regular grid which also allows to include topography.

**Figure 1 A)** Typical set of 1D models reflecting the flight lines of an airborne survey. **B)** Schematic of the quasi-3D inversion grid.

Given this definition of \( M \), the 1D forward responses are computed by at each dataset location we create a “virtual” 1D model. This model discretizes the subspace into typical 20 or 30 layers where the resistivity of the layers is interpolated from the surrounding voxel node points. It is denoted formally as \( m = f(M) \) and the process is illustrated in Figure 2.

The Jacobian of the data with respect to the voxel model domain \( M \) and with respect to the “virtual” model space \( m \) is linked by the interpolation function \( f \) through the chain rule:

\[
\frac{\partial d_i}{\partial M_k} = \sum_j f \left( \frac{\partial d_i}{\partial m_j} \right) \frac{\partial m_j}{\partial M_k} \tag{2}
\]

where \( d_i \) represents a specific datum, \( M_k \) represents the resistivity value of a specific node and \( m_j \) indicate all the resistivity values of the “virtual” 1D model connected to \( M_k \).
through the interpolation function $f$. Finally $(\partial M)/(\partial M_i )$ is the derivative of the interpolation function $f$.

![Figure 2 A) Reconstruction of a specific 1D model for computing the 1D forward response of the corresponding airborne sounding. The black dots represent the interpolated resistivities in the virtual model. The grey dots represent the interpolation nodes used to reconstruct the resistivity values of the 1D model. B) Interpolated resistivity value (black dot) and interpolation nodes (grey dots) for a specific layer of the virtual model.](image)

The Jacobian of the entire model domain $M$ is computed using equation (2) looping over all the voxel in the grid. In this way classical 1D, 2D or 3D Jacobian computations, also from different data types in a joint process, can be used to generate the Jacobian $M$. It is worth to note that a 3D Jacobian is obtained for $M$ also from 1D forward responses through the interpolation function $f$.

Auxiliary parameters like flight height or data shifts for AEM data is included separately in a designated block in the Jacobin as these parameters need to be inverted for at the exact sounding location.

The performance of the inversion scheme has been measured on AEM surveys of size up to approx. 100 000 dual moment soundings corresponding to about 1500 line kilometer of data. The computation time and the memory requirement of the inversion scale linearly with the survey size as seen in Figure 3. In addition, the forward and Jacobian computation has been parallelized and the computation time is almost inversely proportional to the number of CPU’s. This means that the quasi-3D inversion scheme can be effectively used to invert entire surveys of thousands of kilometres on small multiprocessor servers priced less than 5000 Euro.

The new quasi-3D inversion scheme has been tested on several SkyTEM AEM surveys, and the inversion results have been compared to the Spatially Constrained Inversion proposed by Viezzoli et al. (2008): the two approaches give equivalent data misfit and comparable inversion models.

The definition of the quasi-3D model space allows for a straight forward incorporation of the geophysical data into geological and/or hydrological models. This also simplifies the propagation of the uncertainty of geophysical parameters into the (hydro)geological models. Furthermore, a priori information from boreholes like resistivity logs can be applied directly to the model domain, even if the borehole positions do not coincide with the actual observation points. In fact, the a priori information is constrained to the model parameters through the interpolation function at the borehole locations.

The presented algorithm is a further development of the AarhusInv program package (Auken et al., 2014), which manages both large scale AEM and ground-based surveys. In particular, AarhusInv supports airborne TEM data (Auken et al., 2008) and FEM data (Semon et al., 2009) and several ground based methods, i.e. EM (Christiansen et al., 2007), DC (Auken et al., 2005; Auken and Christiansen, 2004), Induced Polarization (Fiandaca et al., 2012; Fiandaca et al., 2013), Nuclear Magnetic Resonance (Behroozmand et al., 2012b; Behroozmand et al., 2012a), Surface waves (Wisén and Christiansen, 2005) and magnetotellurics (Christiansen and Auken, 2009).

![Figure 3 Computation time and memory requirement of the quasi-3D inversion scheme as varying the number of dual mode TEM soundings. The computations were performed on a 32 CPUs AMD OPTERON 6168 1.9 GHz server.](image)

**CONCLUSIONS**

The definition of the quasi-3D model domain decouples the geophysical model from the position of acquired data, allowing for straight-forward integration of different data types in a joint inversion scheme. The computation time and the memory requirements of the inversion algorithm scales linearly with the size of the problem, and the computation time is almost inversely proportional to the number of CPUs. We believe that this new approach will facilitate the integration of geophysics, geology, and hydrology for improved groundwater and environmental management.

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**REFERENCES**


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