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Remanent magnetisation inversion

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SUMMARY

Remanent magnetisation is an important consideration in magnetic interpretation. In some cases failure to properly account for remanence can lead to completely erroneous interpretations. In general the strength and orientation of remanence are unknown. Two main strategies have been pursued for "unconstrained" inversion of large data sets. One strategy is to invert quantities, such as total magnetic gradient (3D analytic signal), which are insensitive to magnetisation direction. The inverted property is then magnetisation amplitude. Another strategy is to invert for the magnetisation vector, allowing its three components to vary freely. These approaches are useful, but the resulting magnetisation models are highly non-unique.

When interpreting magnetic data in tandem with geological modelling there is greater potential to infer remanence parameters. Non-uniqueness is reduced if the shape of magnetic domains is constrained, especially if the susceptibility is known and if remanence can be assumed uniform. Accordingly, inverting for the remanent magnetisation of individual homogeneous geological units of arbitrary 3D shape is the subject of this paper. Our remanent magnetisation inversion (RMI) approach can be regarded as a generalisation of parametric inversion of simple geometric bodies.

If susceptibility is known, the optimal remanent magnetisation vector within each selected unit is determined via iterative inversion. Sensitivity to change in magnetisation is determined in the x-, y-, and z-directions, and the perturbation vector is found via the method of steepest descent. If the susceptibility is unknown, the optimal susceptibility of each unit (subject to bounds) can be determined via a similar inversion procedure. The geological units can carry remanent magnetisation, but it is fixed during this stage. The susceptibility and/or remanence inversions can be repeated, if necessary, to refine the magnetic parameters. Self-demagnetisation and interactions are taken into account when susceptibilities are high.

The application of the RMI algorithm is illustrated in examples for both known and unknown susceptibility.

Key words: remanent magnetisation, 3D magnetic inversion, constraints, non-uniqueness

INTRODUCTION

Remanent magnetisation is an important consideration in magnetic interpretation. In some cases failure to properly account for remanence can lead to completely erroneous interpretations. Accounting for remanence introduces some computational complexity, but the main issue is that the strength and orientation of remanence are usually unknown.

Two main strategies have been pursued for "unconstrained" inversion of large data sets. One strategy is to invert quantities which are insensitive to magnetisation direction. The inverted property is then magnetisation amplitude. Magnetic quantities which lend themselves to this approach include the vertical integral of the analytic signal (Paine et al, 2001), the analytic signal itself (equivalent to the total magnetic gradient of Shearer & Li, 2004), and the magnetic anomaly vector amplitude (Li et al, 2010). The second strategy is to invert for the magnetisation vector, allowing its three components to vary freely throughout model volume, e.g. Lelièvre & Oldenburg (2009), Ellis et al (2012). Both strategies are useful when the shape(s) of the magnetic source(s) is/are unknown, but the resulting models are highly non-unique.

When interpreting magnetic data in tandem with geological modelling there is greater potential to infer remanence parameters. Non-uniqueness is reduced if the shape of magnetic domains is constrained by drilling and mapping, especially if susceptibility is known (within bounds) and if remanence can be assumed uniform. Accordingly, inverting for the remanent magnetisation of individual homogeneous geological units of arbitrary 3D shape is the subject of this paper. Our remanent magnetisation inversion (RMI) approach can be regarded as a generalisation of parametric inversion of simple geometric bodies. If susceptibility is high, self-demagnetisation and interactions can be taken into account (Fullagar & Pears, 2013). If remanence varies, e.g. between fold limbs with pre-folding NRM, the complex body can be divided into uniform remanence sub-domains. In the absence of geological constraints, an initial source shape can be estimated after unconstrained inversion of a quantity such as total magnetic gradient (TMG).

The application of the RMI algorithm is illustrated in examples.

METHODOLOGY

The RMI algorithm has been implemented in VPmg (Fullagar Geophysics Pty Ltd), which can operate on geological models. When inverting for remanence it is more convenient to characterise the strength of remanent magnetisation in absolute terms, as the amplitude of a vector, rather than in relative terms, using the Koenigsberger Ratio (KR), since the KR becomes meaningless if the susceptibility approaches zero. Moreover, the magnetic (vector) response is linear with respect to the three components of magnetisation, whereas inversion with respect to inclination and declination is non-linear.

We define the amplitude of remanent magnetisation, $\vec{J}_R$, normalised with respect to the amplitude, F, of ambient field, $\vec{F}$, as the effective remanent susceptibility, k_{eff}, i.e.

$$k_{eff} = \frac{\left|\vec{J}_R\right|}{F}. \tag{1}$$

The resultant magnetisation is then expressed as

$$\vec{J} = k\vec{H} + \vec{J}_R = F\left(k\frac{\left|\vec{H}\right|}{F}\hat{h} + k_{eff}\,\hat{q}\right) = F\left(k'\,\hat{h} + \vec{q}\right) \tag{2}$$

where k is the susceptibility, k' is the susceptibility adjusted for demagnetisation, $\hat{h}$ is the unit vector in the direction of the inducing field, $\vec{H}$, and where $\hat{q}$ is the unit vector in the direction of remanent magnetisation, with $\vec{q}$ denoting the normalised remanent magnetisation vector, i.e.

$$\vec{q} = \frac{\vec{J}_R}{F}. \tag{3}$$

A different remanent magnetisation can be inferred for each geological unit in the model. For the *i*th geological unit involved in the inversion there are three parameters, $\left(q_x^i, q_y^i, q_z^i\right)$, denoting the components of $\vec{q}^{\,i}$. The remanent magnetisation inverse problem is solved by the method of steepest descent (Schaa & Fullagar, 2010). At each iteration a perturbation in the remanent magnetisation vector is sought which reduces the chi-squared misfit, χ^2, defined by

$$\chi^2 = \frac{1}{N}\sum_{n=1}^{N}\left(\frac{o_n - c_n}{\varepsilon_n}\right)^2, \tag{4}$$

where $\{o_n : n = 1, N\}$ are the observed data, $\{c_n\}$ are the calculated data, and $\{\varepsilon_n\}$ are the corresponding uncertainties. The susceptibility, k^i, is held fixed, and sensitivity of χ^2 to change in each of the magnetisation components is computed:

$$\frac{\partial\chi^2}{\partial q_m^i} = \frac{-2}{N}\sum_{n=1}^{N}\left(\frac{o_n - c_n}{\varepsilon_n^2}\right)\frac{\partial c_n}{\partial q_m^i}. \tag{5}$$

A perturbation anti-parallel to the gradient of χ^2 is then applied, scaled to halve the predicted misfit. When the susceptibilities are high, self-demagnetisation is taken into account.

Normally, unless the susceptibility is known, RMI is applied in conjunction with optimisation of susceptibility. During susceptibility optimisation, the *a priori* remanent magnetisation (if any) is fixed, and during RMI the susceptibility is fixed. The optimal susceptibility can be constrained to lie between upper and lower bounds appropriate for the unit, if available. Susceptibility optimisation is usually performed first, with the result that the induced contribution to net magnetisation parallel to inducing field is maximised. On the other hand if RMI were performed first, the remanent contribution to net magnetisation parallel to inducing field would be maximised. In general, in the absence of susceptibility or remanence measurements, it is not possible to unambiguously resolve the magnetisation component parallel to inducing field into induced and remanent contributions, especially since the remanent magnetisation component may oppose the induced magnetisation. Therefore, if RMI is performed on a zero susceptibility body, the result should be regarded as an estimate of undifferentiated *in situ* magnetisation.

The inverted magnetisation will be affected by the shape of the magnetic source and by its susceptibility. RMI will adjust the remanent magnetisation so that the resultant magnetisation is optimal in terms of data misfit. If J_h^{opt} denotes the component of optimal resultant magnetisation parallel to inducing field, then the corresponding component q_h of remanent magnetisation is traded off against the induced magnetisation according to

$$q_h = \frac{J_h^{opt}}{F} - k'\,. \tag{6}$$

An extension of the algorithm, for optimisation of remanent magnetisation for heterogeneous units, is under development.

EXAMPLES

Magnetised cube

The application of RMI is illustrated first on TMI data calculated at altitude 20m for a 40m cube, with zero susceptibility, buried to a depth of 20m. The cube has horizontal east remanent magnetisation, with effective susceptibility 50 x 10^{-3} SI, and the ambient field is vertical down, with intensity 24000 nT. This example has been discussed previously by Ellis et al (2012).

The TMI anomaly of the cube is dipolar in the vertical ambient field so, assuming a compact source, it is reasonable to infer remanence. The source shape and location is estimated via inversion of TMG data. The TMG anomaly is a simple apical high (Fig. 1). The ground was divided into 8 x 8 x 5m cells, and depth weighting was applied. An E-W section through the centre of the anomaly is shown in Fig. 1.

The iso-surface for 5 x 10^{-3} SI total magnetisation from the model in Fig. 1 was adopted as the exterior of the magnetic source, assumed homogeneous. RMI was then applied, assuming zero remanence initially, to determine the remanent magnetisation required for this body to reproduce the TMI data. The susceptibility was fixed as zero everywhere. An E-W section through the centre of the body is shown in Fig. 2. The inverted remanence had declination 89.7° and inclination 0.2°, virtually identical to the true values of 90° and 0° respectively. The inverted total magnetisation was 13.5 x 10^{-3} SI, much smaller than the true value of 50 x 10^{-3} SI, reflecting the larger volume of the adopted shape.

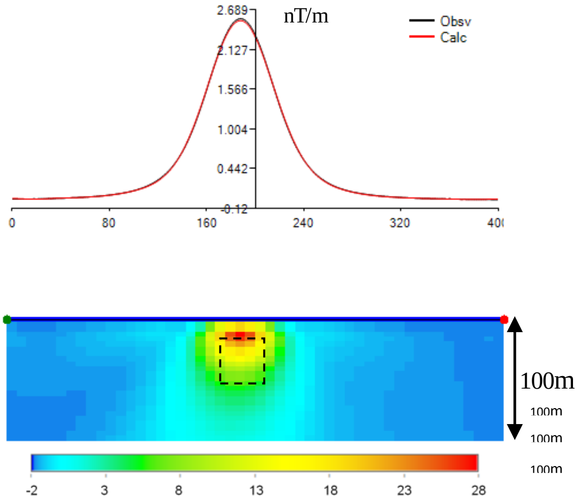

Figure 1. E-W section through total magnetisation model (mSI) after inversion of total magnetic gradient (TMG) data. Observed and calculated TMG plotted above. Location of true cubic source, with total magnetisation 50 x 10^{-3} SI, shown dashed.

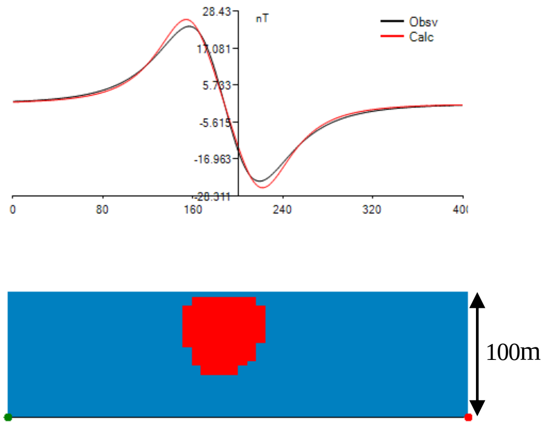

Figure 2. E-W section through homogeneous magnetic body enclosed by TMG inversion 5 x 10^{-3} SI iso-surface. Observed and calculated TMI after remanent magnetisation inversion plotted above. RMI recovered the true (horizontal east) remanent orientation. Susceptibility is zero everywhere.

RMI has successfully recovered the true remanent orientation in this case, since the true susceptibility of zero was assigned to the magnetic body. In general the remanent magnetisation direction cannot be uniquely determined unless the susceptibility is known.

Remanently magnetised intrusive

The second example involves RMI of synthetic data computed for a realistic geological model representing an igneous plug intruded into a mixed volcanic and sedimentary terrane. The susceptibility varies within the host rocks but is assumed uniform in the plug. For the purposes of illustration, the plug was assigned extreme magnetic properties: susceptibility of 2.50 SI and remanence with a KR of 2, declination 150^{o} and inclination 40^{o}. TMI was computed along aeromagnetic survey lines, assuming ambient field of 40000 nT with declination -20^{o} and inclination -30^{o}. Self-demagnetisation was taken into account. The synthetic data are presented in Fig. 3.

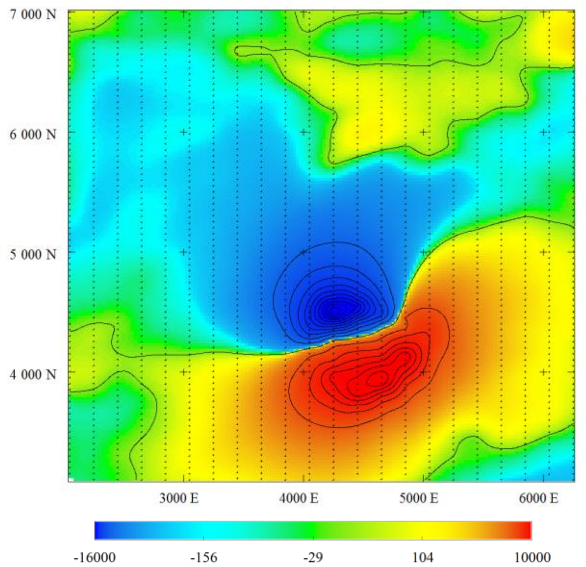

Figure 3. Synthetic TMI data (nT) for magnetic plug and host rocks. Histogram equalised image, with 2000 nT contours superimposed.

First RMI was applied to the synthetic data set with the actual susceptibility for the plug (2.50 SI) assumed known. The ground was divided into 100 x 50 x 50m cells and self-demagnetisation was taken into account. RMI recovered the correct remanent magnetisation, both direction and amplitude (k_{eff} = 5.0 SI).

Secondly, RMI was re-run, assuming zero susceptibility for the plug, in order to determine the resultant magnetisation. A north-south section through the plug, and the corresponding "observed" and calculated profiles (after the 2nd inversion), are shown in Fig. 4.

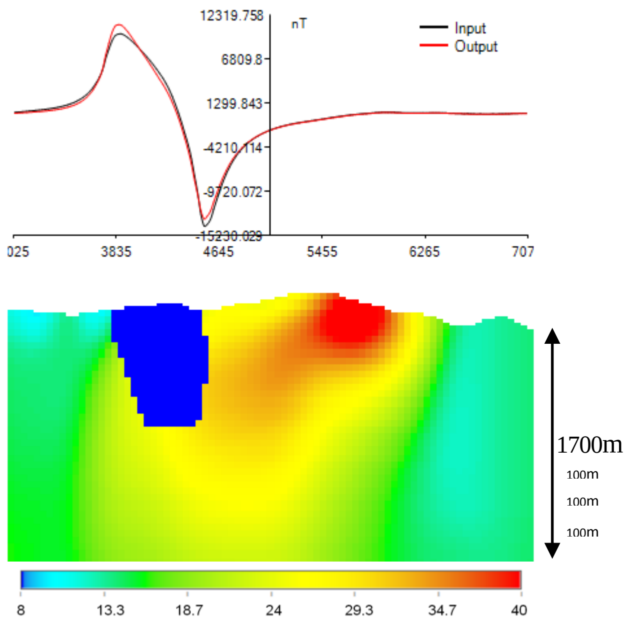

Figure 4. N-S section at 4450E through magnetic plug intruded into variably magnetic host (mSI). Observed and calculated TMI after remanent magnetisation inversion

(RMI) plotted above. RMI recovered an optimal uniform *in situ* magnetisation in the plug, with susceptibility fixed at zero.

RMI has recovered an optimal uniform *in situ* magnetisation vector for the plug, with declination 136.9^o, inclination 42.3^o, and effective susceptibility 1.41 SI. This is consistent with expectations: the combined (induced plus remanent) magnetisation in the plug can be estimated as (Clark et al, 1986)

$$\vec{J} \approx \frac{k\vec{F} + \vec{J}_R}{1 + k/3} \qquad (7)$$

since the plug is approximately equi-dimensional, hence warrants a demagnetisation factor of about 1/3. The *in situ* magnetisation estimated using this formula has declination 137.5^o, inclination 48.5^o, and effective susceptibility 1.43 SI. Unambiguous decomposition into induced and remanent components is not possible without additional information.

CONCLUSIONS

Remanent magnetisation is an important consideration in magnetic interpretation but remanence parameters are rarely known. Two main strategies have been pursued for "unconstrained" inversion of large data sets. One strategy is to invert quantities which are insensitive to magnetisation direction. The inverted property is then magnetisation amplitude. Another strategy is to invert for the magnetisation vector, allowing its three components to vary freely. These approaches are useful, but the resulting models are highly non-unique.

There is greater potential to infer remanence parameters if the shape and susceptibility of magnetic domains are constrained, especially if remanence can be assumed uniform. Accordingly, an algorithm for inverting for the remanent magnetisation of individual homogeneous geological units of arbitrary 3D shape has been developed. Our remanent magnetisation inversion (RMI) approach adjusts the remanent magnetisation vector within selected units, with the susceptibility held fixed, in order to minimise the data misfit. Sensitivity to change in magnetisation is determined in the x-, y-, and z-directions, and the perturbation vector is found via the method of steepest descent. RMI can be combined with susceptibility optimisation if susceptibility is unknown. The geological units can carry remanent magnetisation, but it is fixed during susceptibility optimisation. Self-demagnetisation and interactions are taken into account when susceptibilities are high.

The application of RMI has been described in two examples which illustrate the interplay between shape, susceptibility, and remanence of magnetic bodies. In the first example the shape of the magnetic source was estimated from a total magnetisation iso-surface after an unconstrained inversion of TMG data. The subsequent RMI inversion of TMI recovered the correct remanent magnetisation orientation, because the actual susceptibility of zero was assumed. However, the inferred amplitude of remanent magnetisation was reduced because the modelled source volume was greater than the true source volume.

In the second example, the remanent magnetisation of a highly magnetic plug was recovered accurately when its susceptibility and shape were known. Self-demagnetisation was taken into account. When susceptibility of the plug was assumed to be zero, RMI successfully determined the resultant magnetisation in the plug.

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