Performance of Hankel transform filters for marine controlled-source electromagnetic surveys: a comparative study

Hangilro Jang*
Sejong University
Seoul, Korea
question5150@gmail.com

Hee Joon Kim
Pukyong National University
Busan, Korea
hejkim@pknu.ac.kr

Myung Jin Nam
Sejong University
Seoul, Korea
nmj1203@gmail.com

SUMMARY

For the numerical calculation of electromagnetic responses by an arbitrary source in a one-dimensional model, integral-equation based approaches can be the best method since they are semi-analytic and thus very fast and accurate. In the numerical computation of the integral-based approaches, digital linear filters (e.g., Anderson's filter, Kong's filter and Mizunaga's filter) play a key role. Using a closed-form solution of the Hankel transform in transverse magnetic mode for a homogeneous half-space model, we can assess the accuracy of digital linear filters for evaluating the Hankel transform. In this paper, we conduct comparative performance tests on the linear filter with known integral transforms. We examine three kinds of filters developed by Anderson, Kong and Mizunaga, which are known to be suitable for marine controlled-source electromagnetic (CSEM) applications. Kong's filters perform best in the three kinds of filters over a practical range of offset distances in marine CSEM surveys. While the relative error versus distance appears as a V-shaped curve in semi-log scale, Mizunaga's filters are the shortest in length and have a performance comparable to Kong's filters. Anderson's filters have a quite similar performance between the J_0 and J_1 filters, although these are somewhat inferior to Kong's and Mizunaga's filters when computing marine CSEM fields.

Key words: closed-form solution, CSEM, digital linear filter, Hankel transform

INTRODUCTION

Digital linear filters have proven to be a fast and accurate method of computing Hankel transforms. In general, the accuracy of the digital linear filter increases with filter length and sampling density (Guptasarma and Singh, 1997). Longer filters are expected to produce smaller errors and to be effective for a larger range of offset distances. It is known that the determination of the digital linear filter is not unique (Kong, 2007). Hence, a filter, which is optimum for one application, may not be optimum for another application. In this paper, we examine digital linear filters for evaluating the Hankel transform related to EM fields generated by a source embedded in a very conductive medium, such as those encountered in a marine CSEM problem (Eidesmo et al., 2002; Ellingsrud et al., 2002). Since a closed-form solution of the Hankel transform exists in transverse magnetic mode for a homogeneous half-space, we can assess the accuracy of digital linear filters for evaluating the Hankel transform. This paper presents some results of comparative performance tests on the digital linear filter with known integral transforms that are directly applicable to a practical marine CSEM problem. We compare three kinds of J_0 and J_1 filters introduced by Anderson (1982), Kong (2007) and Mizunaga (2015), which are known to be suitable for marine CSEM applications.

FILTER DESIGN

The Hankel transform of $h(\lambda)$ of integer order n is defined as

$$H(r) = \int_0^\infty h(\lambda) J_n(\lambda r) d\lambda \, . \tag{1}$$

To design a digital linear filter, assume $r = e^x$ and $\lambda = e^{-y}$, then one has

$$e^x H(e^x) = \int_{-\infty}^{\infty} h(e^{-y}) e^{x-y} J_n(e^{x-y}) dy \, . \tag{2}$$

This equation can be regarded as a convolution equation, in which $e^x H(e^x)$ is the known output function, $h(e^{-y})$ is the known input function, and $e^{x-y} J_n(e^{x-y})$ is the kernel response to be determined. Guptasarma and Singh (1997) used the Wiener–Hopf minimization method to solve the convolution equation, while Kong (2007) performed the deconvolution after constructing the convolution equation as a matrix equation. In contrast, Mizunaga (2015) developed a digital linear filter based on the continuous Euler transformation that can accelerate the convergence of an alternating series.

FILTER PERFORMANCE TEST

Of the total 36 source-receiver configurations (three components of electric and magnetic fields due to three orientations of electric and magnetic dipoles), only nine pairs produce EM fields decreasing almost linearly in semi-log scale with offset distance (Kim, 2011). All of the specific pairs have closed-form solutions for a homogeneous half-space model. For example, a vertical electric field (E_z) due to a horizontal electric dipole (J_x) and horizontal magnetic (H_y) fields due to a vertical electric dipole (J_z) in a homogeneous half-space are written as

$$E_z^{J_x} = \frac{x-x'}{4\pi\sigma}\frac{1}{\rho}\int_0^\infty \lambda^2\left[e^{-u|z-z'|}+e^{-u(z+z')}\right]J_0(\lambda\rho)d\lambda = \frac{x-x'}{4\pi\sigma}\left[\frac{z-z'}{R^5}(3+3ikR-k^2R^2)e^{-ikR}+\frac{z+z'}{R_s^5}(3+3ikR_s-k^2R_s^2)e^{-ikR_s}\right], \quad (3)$$

and

$$H_y^{J_z} = \frac{x-x'}{4\pi}\frac{1}{\rho}\int_0^\infty \frac{\lambda^2}{u}\left[e^{-u|z-z'|}-e^{-u(z+z')}\right]J_1(\lambda\rho)d\lambda = \frac{x-x'}{4\pi}\left[\frac{e^{-ikR}}{R^3}(1+ikR)-\frac{e^{-ikR_s}}{R_s^3}(1+ikR_s)\right], \quad (4)$$

where the source and receiver are located at (x', y', z') and (x, y, z), respectively,

$$R=\sqrt{\rho^2+(z-z')^2}\ ,$$

$$R_s=\sqrt{\rho^2+(z+z')^2}\ ,$$

$$\rho=\sqrt{(x-x')^2+(y-y')^2}\ ,$$

$$u=\sqrt{\lambda^2-k^2}\ ,$$

$$k=\sqrt{-i\omega\mu_0\sigma}\ ,$$

under quasi-static approximation, $\omega = 2\pi f$, f is the frequency, μ_0 is the magnetic permeability in the air, and σ is the electrical conductivity. Each EM field is composed of two terms: the first is a whole-space solution and the second is a field due to the image source with respect to the surface of the half-space. Using equations (3) and (4), we can check the accuracy of published J_0 and J_1 filters for marine CSEM applications.

In general, the numerical result fits the analytical one very well at a shorter distance and flattens out from a certain distance. Because of this flattening, the smallest signal evaluated over the total distance becomes equivalent to the field at the turning point where the fit starts to deteriorate. Table 1 compares vertical electric field magnitudes and distances at the turning points from the J_0 filters developed by Anderson (1982), Kong (2007) and Mizunaga (2015) for a homogeneous half-space model (Equation 3). In the evaluation, the parameters are set to σ = 3.2 S/m, z = 1 km, z' = 950 m, and three frequencies are considered: f = 0.2, 1 and 5 Hz. The 241-point filter of Kong (2007) has the best performance at frequencies of 1 Hz and 5 Hz. The 90-point filter of Mizunaga (2015) can evaluate weaker fields at every frequency as opposed to the 801-point filter of Anderson (1982). In Anderson's filter, the field magnitude at the turning point decreases with decreasing frequency unlike those in Kong's and Mizunaga's filters. However, in the frequency range tested, Anderson's filter cannot calculate smaller fields (by a factor of $10^2 - 10^4$) than the other filters.

Table 1 Vertical electric field magnitudes and distances at the turning points from the J_0 filters.

Frequency	Anderson (1982)		Kong (2007)		Mizunaga (2015)	
(Hz)	R (km)	$\lvert E_z\rvert$ (V/m)	R (km)	$\lvert E_z\rvert$ (V/m)	R (km)	$\lvert E_z\rvert$ (V/m)
0.2	12.4	3.9×10^{-21}	18.0	2.2×10^{-25}	18.0	2.8×10^{-25}
1.0	5.7	1.6×10^{-20}	9.0	3.2×10^{-25}	8.0	3.8×10^{-24}
5.0	2.4	5.8×10^{-20}	3.7	1.4×10^{-24}	3.4	1.8×10^{-23}

Table 2 shows horizontal magnetic field magnitudes and distances at the turning points from the J_1 filters developed by Anderson (1982), Kong (2007) and Mizunaga (2015) for a homogeneous half-space model (Equation 4). Comments on Table 2 are similar to those made in Table 1, while Kong's J_1 filter shows better performance than his J_0 filter. In Anderson's filters, the turning point in the magnetic field appears at almost the same distance as that in the electric field in Table 1. This means that Anderson's filters have a nearly identical performance between the J_0 and J_1 transforms, although these are somewhat inferior to Kong's and Mizunaga's filters for marine CSEM applications.

Table 2 Horizontal magnetic field magnitudes and distances at the turning points from the J_1 filters.

Frequency	Anderson (1982)		Kong (2007)		Mizunaga (2015)	
(Hz)	R (km)	$\lvert H_y\rvert$ (A/m)	R (km)	$\lvert H_y\rvert$ (A/m)	R (km)	$\lvert H_y\rvert$ (A/m)
0.2	12.4	1.1×10^{-17}	22.0	5.5×10^{-24}	18.5	4.1×10^{-22}
1.0	5.6	1.4×10^{-16}	9.8	2.4×10^{-23}	9.0	4.0×10^{-22}
5.0	2.5	7.6×10^{-16}	4.4	1.5×10^{-22}	4.0	3.3×10^{-21}

CONCLUSIONS

In this paper, we conducted comparative performance tests on the digital linear filter with known integral transforms that are directly applicable to a practical marine CSEM problem. We examined three kinds of J_0 and J_1 filters introduced by Anderson (1982), Kong (2007) and Mizunaga (2015), which are known to be suitable for solving a marine CSEM problem. To examine the performance of these filters, we used the analytic solutions of the vertical electric field due to a horizontal vertical electric dipole and horizontal magnetic field due to a vertical electric dipole in a homogeneous half-space.

The 241-point filters of Kong (2007) perform best in the three kinds of filters tested over ranges shorter than about 18 km and 27 km for the J_0 and J_1 transforms, respectively. Since the distances are long enough for marine CSEM surveys, we can say that Kong's filters are quite useful in a practical point of view. Use of Mizunaga's filters will save calculation time due to the short length, giving a performance comparable to Kong's filters. The 801-point filters of Anderson (1982) have a nearly identical performance between the J_0 and J_1 filters, although these are somewhat inferior to Kong's and Mizunaga's filters for evaluating marine CSEM fields.

ACKNOWLEDGMENT

This research was supported by the Korea Agency for Infrastructure Technology Advancement under the Ministry of Land, Infrastructure and Transport of the Korean government. (Project Number: 15SCIP-B108153-01)

REFERENCES

Anderson, W.L., 1982, Fast Hankel transforms using related and lagged convolutions: ACM Transactions on Mathematical Software, 8, 344-68.

Guptasarma, D., and Singh, B., 1997, New digital linear filters for Hankel J0 and J1 transforms: Geophysical Prospecting, 45, 745-762.

Eidesmo, T., Ellingsrud, S., MacGregor, L.M., Constable, S., Sinha, M.C., Johansen, S., Kong, F.N., and Westerdahl, H., 2002, Sea Bed Logging (SBL), a new method for remote and direct identification of hydrocarbon filled layers in deepwater areas: First Break, 20, 144-152.

Ellingsrud, S., Eidesmo, T., Johansen, S., Sinha, M.C., MacGregor, L.M., and Constable, S., 2002, Remote sensing of hydrocarbon layers by seabed logging (SBL): Results from a cruise offshore Angola: Leading Edge, 21, 972-982.

Kim, H.J., 2011, A scheme for computing primary fields in modeling of marine controlled-source electromagnetic surveys: Jigu-Mulli-wa-Mulli-Tamsa, 14, 185-190. (in Korean with English abstract)

Kong, F.N., 2007, Hankel transform filters for dipole antenna radiation in a conductive medium: Geophysical Prospecting, 55, 83-89.

Mizunaga, H., 2015, High-precision linear filter using continuous Euler transformation: Butsuri-Tansa, 68, 1-12. (in Japanese with English abstract)