Inversion of magnetotelluric data with fuzzy cluster petrophysical and boundary constraints

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SUMMARY

Inverse magnetotelluric (MT) problems are naturally ill-posed and smoothing criteria are typically added to stabilize the process. Smoothing and geo-electrical equivalency tend to produce unrealistic geological models. In reality the subsurface geology is differentiated by distinct rock units that are often better defined by boundaries rather diffuse or smooth boundaries. We present the application of fuzzy clustering as an added constraint within the inversion process to guide model updates toward earth models that resemble geological units. Fuzzy clustering divides the simulated model into clusters based on the similarity of model features. Moreover, fuzzy clustering enables the inclusion of additional prior information in the inversion process such as structural and/or petrophysical information. The inclusion of this information produces geo-electrical distributions that more closely reflect the true rock units and unit boundaries. This is demonstrated through several synthetic examples. The simulations show that by including prior petrophysical and/or boundary location information within the inversion the original conductivity distribution is well resolved.

Key words: Magnetotellurics, inversion, fuzzy clustering, constraint.

INTRODUCTION

Conventionally, smoothness criteria is added to constrain the inversion of magnetotelluric (MT) data (deGroot-Hedlin and Constable 1990). This constraint may produce unrealistic geological models because both sharp and smooth geological boundaries exist in nature. Typical subsurface structure consists of geological units of nearly uniform conductivity. Thus the model construction is more realisable if the grouping criteria is added to constrain the inversion process. We propose to exploit the robustness of fuzzy c-mean (FCM) clustering techniques (Bezdek, Ehrlich, and Full 1984) to constrain the inversion process (Sun and Li 2011).

Another difficulty of MT inversion is equivalency issues. This occurs when multiple models conductivity distributions may generate the same electromagnetic signature. In order to reduce ambiguity, extra information from other sources is needed, such as boundary geometry derived from seismic and/or petrophysics from borehole data. In this work, we utilise FCM like an adapter to put the prior information in the inversion.

The MT method is based upon diffusive fields as it utilizes low frequency electromagnetic waves and is most sensitive to conductive environments. Therefore, the method looses resolution and 'sharpness' with depth and is relatively insensitive to resistive units. Resolution and boundary 'sharpness" can be improved with appropriate constraints by prior information. However, the prior information is usually localized; only partly available in the area of interest. This research demonstrates that this prior information can assist to enhance the accuracy of whole the model.

METHOD AND RESULTS

Our inversion algorithm is formulated with the minimization of the following objective function (Sun and Li 2011):

$$\Phi = \Phi_d + \beta\Phi_m + \gamma\Phi_{FCM}, \tag{1}$$

where Φ_d measures the difference between observed data and the synthetic data from the inverted models, Φ_m represents the smooth constraint and Φ_{FCM} is the FCM objective function (equation 2). This "model guider" term directs the updating model process. More specifically, it drives the incorporation of rock units within the inverted model. The regularization parameters β and γ balance between misfit, model structure and FCM constraint terms.

The prior petrophysicical representative values are included in the inversion routine via FCM (Kieu and Kepic 2015), which classifies N samples of a dataset $Z\{z_j\}$ into C subsets based on feature similarities, driving the groups central value $V\{v_k\}$ towards the prior representative conductivity $P\{p_k\}$

$$\Phi_{FCM} = (1-\eta)\sum_{j=1}^{N}\sum_{k=1}^{C} u_{jk}^{q}\left\|z_j - v_k\right\|_2^2 + \eta\sum_{k=1}^{C}\|p_k - v_k\|_2^2 \,, \tag{2}$$

where q is the fuzziness parameter, $q>1$, in this study q is set to be 2 (Sun and Li 2011), u_{jk} is the membership degree of sample *j*th belong to the *k*th cluster, with the constraint $\sum_{k=1}^{C} u_{jk} = 1$. η is the weighting value that represents the confidence level of the prior information.

In order to integrate boundary information within the inversion via FCM, the boundary information b is combined with the model parameter m to form the data input *Z=[m b]*.
We test our program with two synthetic model cases (Figure 1) and several prior initial condition scenarios (Table 1). Three different inversion scenarios are presented:

i. The typical petrophysical values of the media (i.e., the centre values (Equation 2) *P =[100; 30; 300; 10]* for model A and *P=[100; 30; 300; 1000]* for model B) are included
ii. Boundary information is available
iii. A combination of both the prior boundaries and petrophysical information.

The purpose of each test is to determine the importance of each prior initial condition within the inversion process. Additionally we attempt to determine an approach to improve the resulting conductivity distribution to better reflect the true geology. Particularly, differentiating between the basement rock units and the upper layer will be difficult in model B (Figure 1) due to low conductivity contrast and high resistivity values.

We modified the 2D MT inversion code from Lee et al. (2009) whilst retaining the original forward solution. An inversion is performed with synthetic data (Figure 2) for both models with the same regularization parameters β and γ. The Initial model is set homogeneously to 400 Ω·m. Figures 3 and 4 show the resulting inverted models after 10 iterations. Generally, the inversion results show that the boundaries B1, B2 and the objects O1, O2 are reasonably recovered. This is even the case for the basic scenario (case C1), when petrophysical and boundary information are unknown and only the number of units are available (Figure 3 and 4). This demonstrates the power of being able to direct the inversion algorithm to pick a limited number of petrophysical properties to construct a model. Such direction also leads to a model more representative and interpretable for mineral exploration.

For cases where some petrophysical information is known, but the boundary information is unknown (case C2), the resulting geo-electrical models, A and B are improved, particularly in the top two units when compared to C1 (Figure 3a and 4a). Significant improvements of the bottom two units are not encountered, especially in model B where the basement is resistive and the resistivity contrast with the above unit is low. Improvements in resolving the lower units are made by the inclusion of any unit boundary within the inversion as prior information (Figure 3b and 4b). Case C3 illustrates this. In model A, when the shallowest boundary, B1, is included as an inversion constraint, the resistive artefacts seen in C1's basement (i.e., the same model without the boundary constraint) are entirely removed. Similarly, the basement of model B is better recovered with the inclusion of the boundary constraint. Similar results are encountered for scenarios C4 and C5 when the boundary B2 and B3 are utilized respectively.

If more layer boundary information constrains the inversion (Figures 3b, 3c and 3d, and Figures 4b, 4c and 4d), both the boundaries and conductivity distributions more closely resemble the true model. Further improvements are made by constraining the inversion with a combination of geometry and petrophysical information (i.e., case C10). In case C10 (Figure 3d and 4d), the inversion almost recovers the true geo-electrical distribution in both the objects and units. Note that knowing the location of the uppermost boundary offers the greatest improvement. Thus, if we were combining seismic reflection data to guide MT inversion it is perhaps more important to gather information about the shallow boundaries rather than using seismic data to constrain deep boundaries alone.

CONCLUSIONS

Constraining the magnetotelluric inversion via fuzzy clustering provides a powerful tool to include prior petrophysical and geometrical information in model construction. The synthetic examples show that fuzzy clustering based inversion with prior information yields considerably better inversion results, particularly the capability of better resolving deep, low sensitivity, resistive units. We find that 2D inversion of magnetotelluric data is influenced more by geometrical information than petrophysical information. Thus an additional geophysical method such as seismic reflection or refraction can assist considerably in MT inversion in a co-operative manner. An important outcome of this research is the significantly improved resolution of the deeper conductivity distribution resulting from the subsequent near surface boundary constraints.

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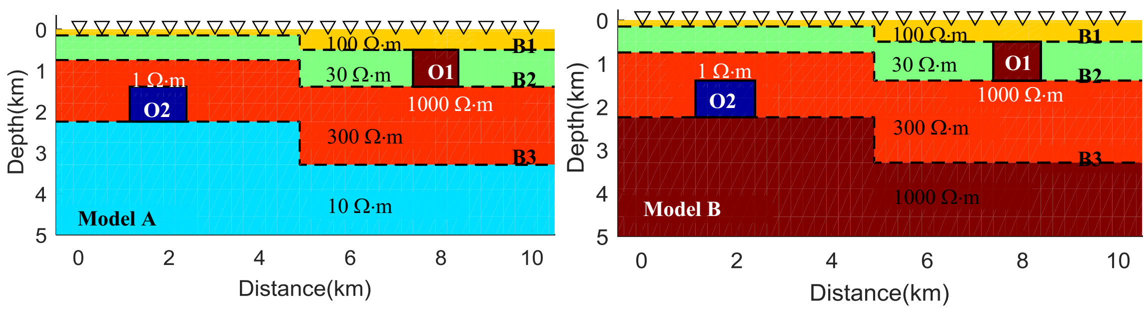

Figure 1. 2-D geo-electrical models, model A is the same as model B except the resistivity of the basement is changed. The three boundaries B1, B2 and B3 (dashed lines) divides the section in four layers. The two objects O1 and O2 are superimposed in the second and the third layer respectively. The triangles on the top profile mark the position of MT stations

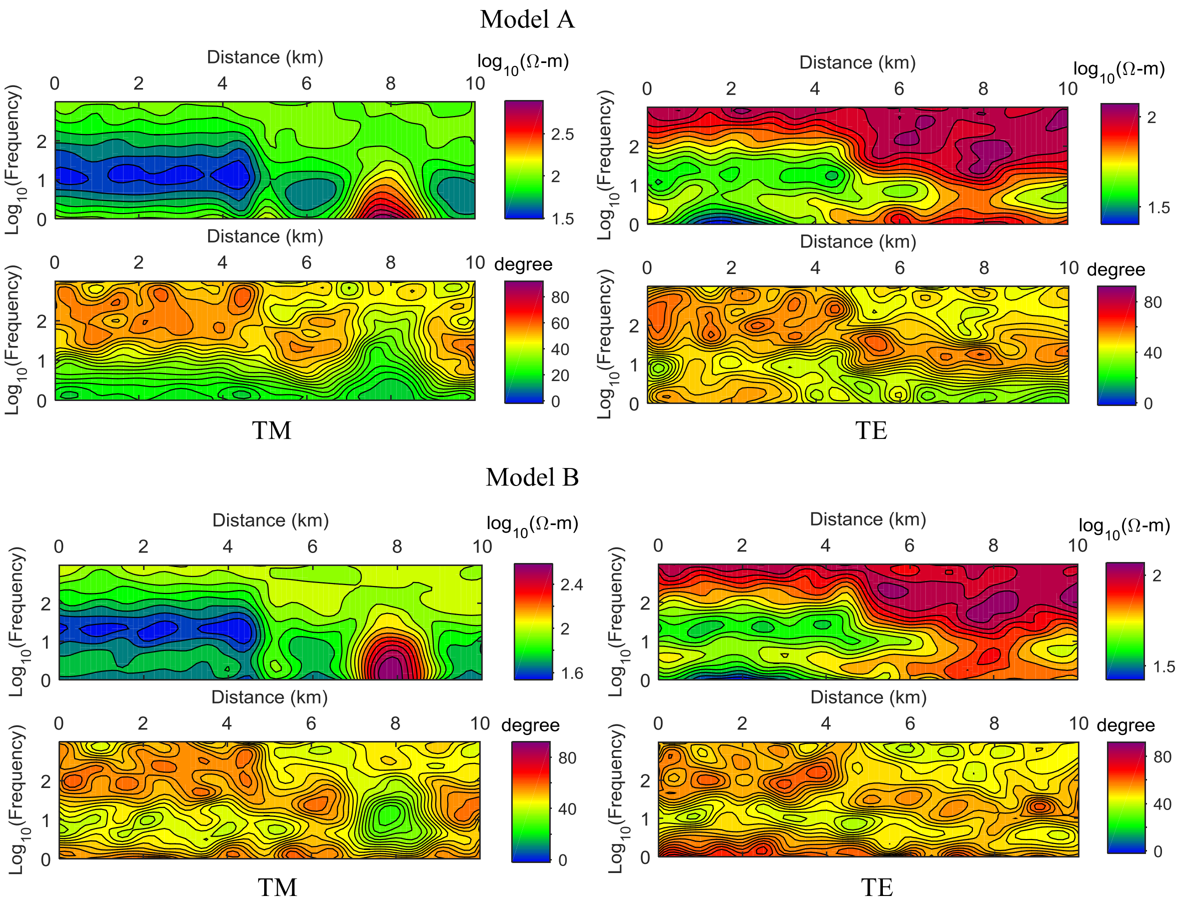

Figure 2. Synthetic data is generated from the models (Figure 1) and added 5% Gaussian random noise.

Cases	Petrophysics	B1	B2	B3
C1	N	N	N	N
C2	Y	N	N	N
C3	N	Y	N	N
C4	N	N	Y	N
C5	N	N	N	Y
C6	N	Y	Y	N
C7	N	Y	N	Y
C8	N	N	Y	Y
C9	N	Y	Y	Y
C10	Y	Y	Y	Y

Table 1. Prior information relating to the four layers (Figure 1) includes in the inversion via FCM. 'Y' and 'N' mean that the information is included and excluded in the inversion respectively. The cases are separated in groups: C1÷C2 : No boundary ; C3÷C5 : One boundary ; C6÷C8 : Two boundaries ; C9÷C10 : Three boundaries.

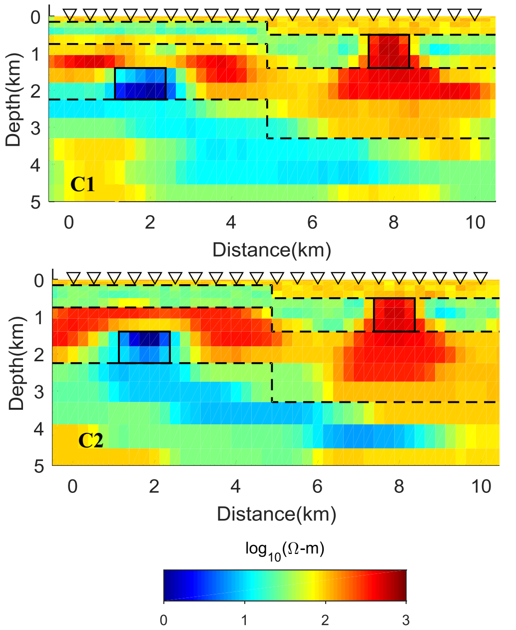

Figure 3a. Inversion results of model A without (C1) and with (C2) the prior petrophysics and no information about boundaries. Both are better than smooth inversions. Just the knowledge that the earth is made of discrete units assists inversion greatly.

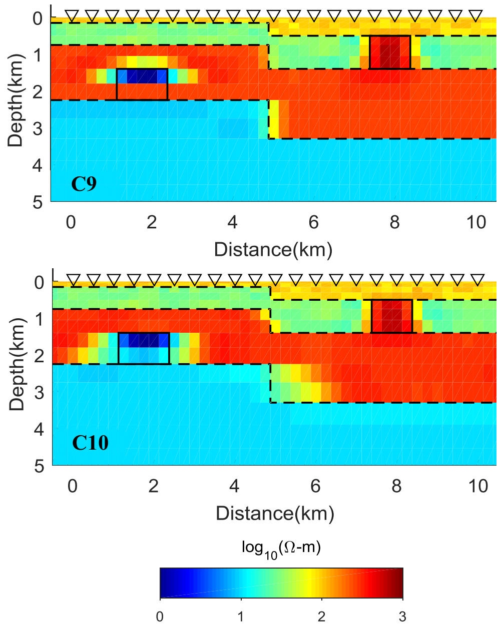

Figure 3d. Inversion results of model A using the information about the three boundaries and without (C9) and with (C10) the petrophysics. The inclusion of some boundary location information and that the earth is composed of discrete units allows very good recovery of the "true" model.

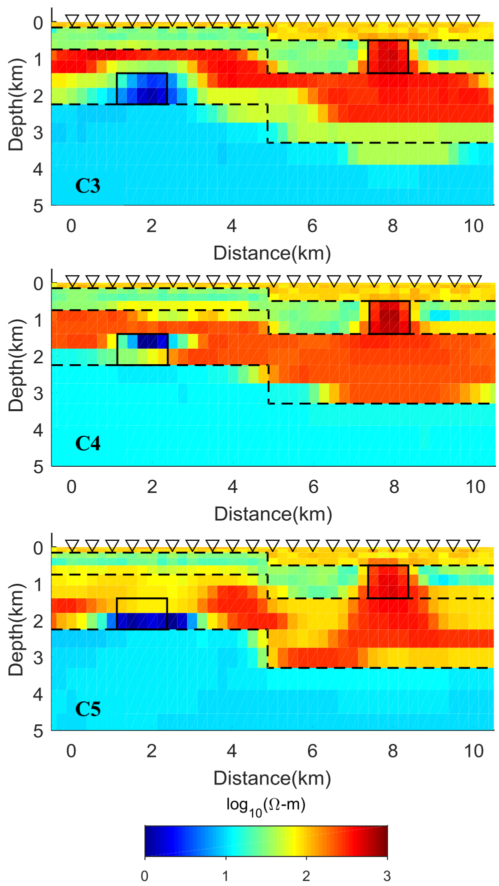

Figure 3b. Inversion results of model A using the information of one boundary: B1 (C3); B2 (C4); and B3 (C5) and no the petrophysical information. In this case the inclusion of the uppermost boundary information makes the greatest difference.

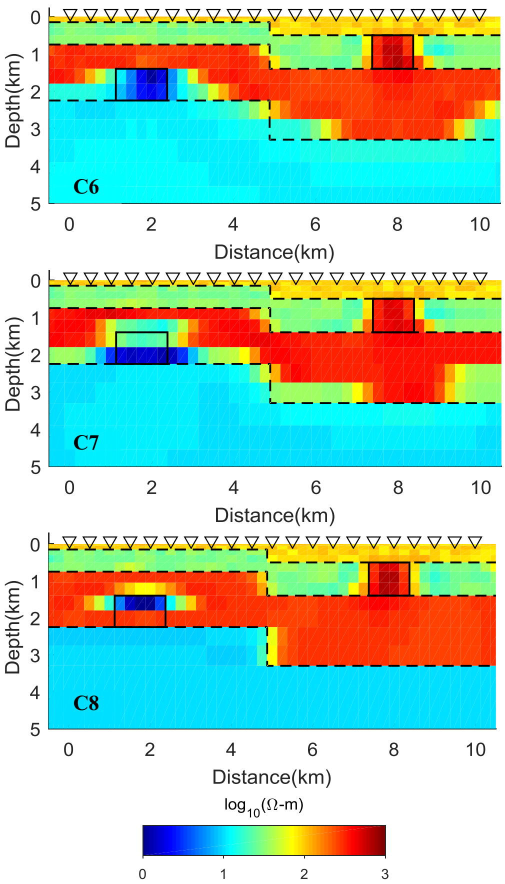

Figure 3c. Inversion results of model A using the information of two boundaries: B1 and B2 (C6); B1 and B3 (C7); and B2 and B3 (C8) and no the petrophysical information. Even with less boundary information at depth the revered models are very good.

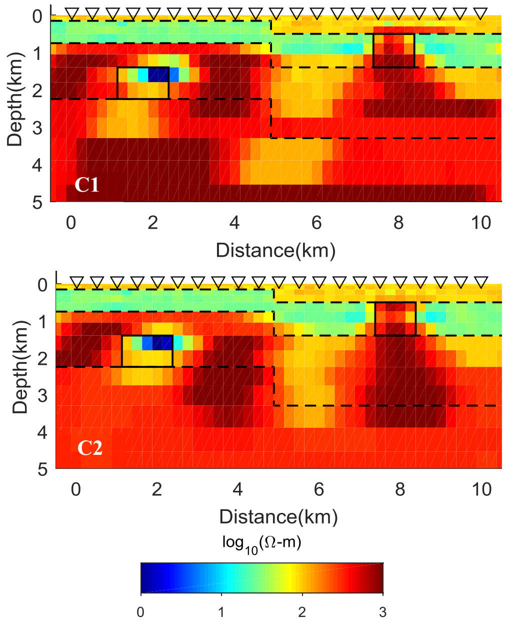

Figure 4a. Inversion results of model B without (C1) and with (C2) the prior petrophysics and no information about boundaries. The first, the second layers and the two objects are well defined, but the boundary between the basement and above layer is unrecovered because it is resistive media and the contrast of conductivity is small.

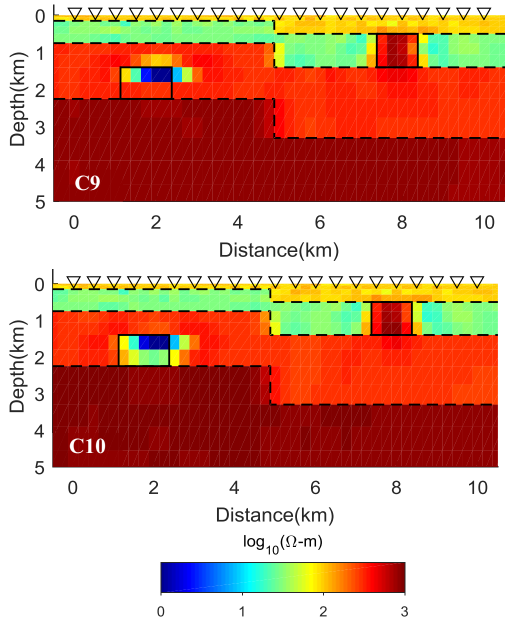

Figure 4d. Inversion results of model B using the information of the three boundaries and without (C9) and with (C10) the prior petrophysics. The inversed results are almost 'true' model. It is worth noting that the information of the two objects is initially unknown.

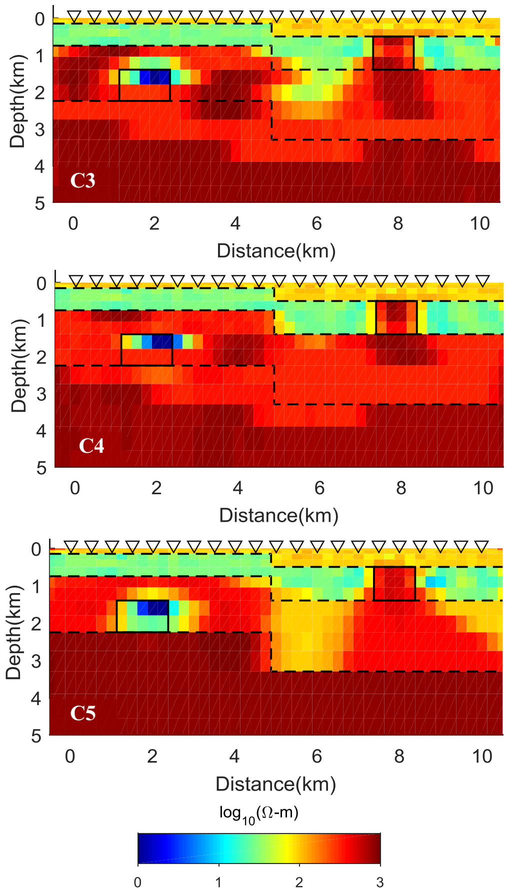

Figure 4b. Inversion results of model B using the information of one boundary*:* B1 (C3); B2 (C4); and B3 (C5) and without the pior petrophysics. The inclusion of one boundary information makes a significant improvement in comparison with the cases without boundary information.

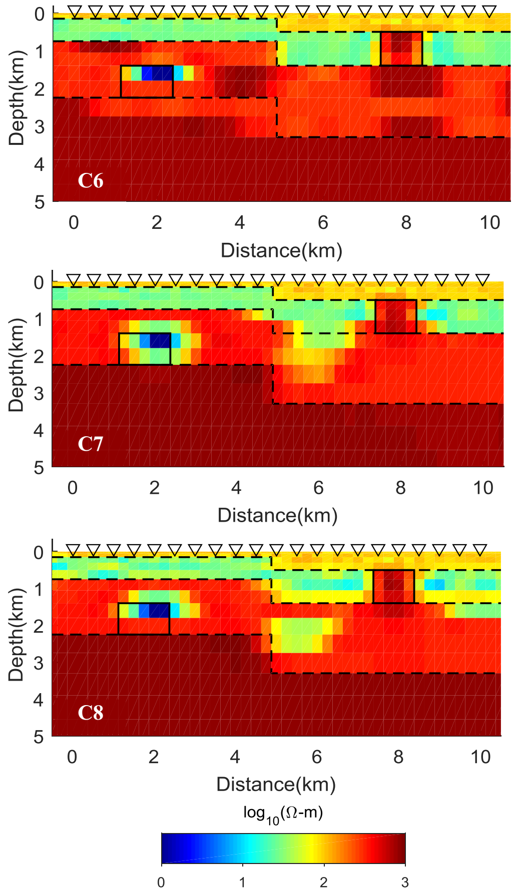

Figure 4c. The resulting inverted electrical resistivity distribution of model B using the information of two boundaries: B1 and B2 (C6); B1 and B3 (C7); and B2 and B3 (C8) and without the prior petrophysics. The inversion shows better result than the cases with only one boundary is added as the constraint.

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