

# Aeromagnetic compensation with partial least square regression

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## SUMMARY

Magnetic exploration plays a significant role in regional geological investigation and detection of underground geological bodies with magnetic anomaly. At the moment, aeromagnetic survey is widely applied for its high efficiency, low cost and less subject to terrain restrict. Magnetic compensation is a key step in pre-processing survey data and several methods have been used. In this paper, we would use partial least square method to complete aeromagnetic compensation. Partial least square regression is frequently used to find the fundamental relations between two matrices. It combines linear regression analysis, canonical correlation analysis and principal components analysis. It can be applied into data with multicollinearity among independent variables and the number of variables is larger than that of observations. Before compensation, we should have several pre-processing steps such as parallax correction, diurnal variations correction, geomagnetic field correction and high-frequency noise removal. This will provide us magnetic data with higher quality and make compensation process more accurate. We set synthetic aeromagnetic data with interference of aircraft's maneuvers and used partial least square method to do compensation. From the results of simulation, we can see that the interference signal is reduced to a low degree and satisfied compensation effect is obtained. Partial least square regression is a stable and effective method in the application of aeromagnetic compensation.

**Key words:** aeromagnetic compensation, partial least square, multicollinearity.

## INTRODUCTION

Aeromagnetic survey has been a main method to improve geo-exploration efficiency effectively. It can be applied in both large scale general survey and small scale detailed information investigation. In the field survey, aerial magnetometer is mounted on the aircraft and the aircraft will fly along the pre-set survey lines to let magnetometer acquire magnetic data. However, the acquired data also includes interference produced by magnetic objects on the aircraft and induced field from metal cutting geomagnetic field. These interference added to the survey data will affect the quality of survey result. To avoid this, magnetic compensation must be done to remove these interference.

On the issue of aeromagnetic compensation, a magnetic interference model was built by Tolles and Lawson (1950). This model classified interference into three categories based on their formation mechanism and basic properties, permanent field, induced field and eddy current field. On this basis, Leliak (1961) designed an experiment to estimate the real magnetic interference. In his paper, the nature of magnetic interference was regarded as a set of linear equations and it depends on maneuvers of the aircraft. With the testing magnetic interference data and the attitude data, 16 coefficients can be obtained and then used to remove interference in field survey. So aeromagnetic compensation is a linear regression problem and several methods were used to solve these linear equations. Least square method (Zhou et al., 2015) could be used to solve this overdetermined equation, but it doesn't perform well in the fitting of pathological data. Mean square error is large using least square method as the singularity of matrix  $X^T X$ . The source of this singularity is that the amplitude of maneuvers is small to keep the aircraft stable. This will lead to close values of the components along each row of  $X^T X$ , and strong relevance exists among the rows. Ridge regression (Zheng et al., 2014) solved the problem of multicollinearity among attitude data, however, it abandons the unbiasedness of least square and there is not good method to determine the ridge parameter. Nonlinear methods such as neural networks (Williams, 1993) and genetic algorithm (Wu et al., 2012) were applied to calculate compensation coefficients, but several parameters should be set before calculation and this would affect the accuracy obviously. So in this paper, we consider another linear regression method—partial least square regression (PLS), which can effectively overcome multicollinearity of the independent variables.

For matrices X and Y, PLS will try to find the multidimensional direction in X space that explains the maximum multidimensional variance direction in Y space. It builds the linear regression model with respect to latent variables of respective independent variables and dependent variables and this model can reflect their relations. We extract the latent variables from observed and manifest variables iteratively and set a limit for the prediction standard deviation. With this method, we designed a unidirectional flight of the aircraft to simulate the process of compensation experiment, calculated the 16 coefficients and achieved a satisfied result.

## METHOD AND RESULTS

### Aeromagnetic compensation model

Aeromagnetic compensation model is a set of linear equations, which has 16 independent unknown coefficients, attitude data of aircraft as independent variables and the magnitude of interference field as dependent variables. It can be expressed as follows:

$$\begin{aligned}
 H_T = & c_1 \cos X + c_2 \cos Y + c_3 \cos Z + \\
 & H_e \{ c_4 \cos^2 X + c_5 \cos X \cos Y + c_6 \cos X \cos Z + \\
 & c_7 \cos^2 Y + c_8 \cos Y \cos Z + c_9 \cos^2 Z \} + \\
 & H_e \{ c_{10} \cos X (\cos X)' + c_{11} \cos X (\cos Y)' + c_{12} \cos X (\cos Z)' + \\
 & c_{13} \cos Y (\cos X)' + c_{14} \cos Y (\cos Y)' + c_{15} \cos Y (\cos Z)' + \\
 & c_{16} \cos Z (\cos X)' + c_{17} \cos Z (\cos Y)' + c_{18} \cos Z (\cos Z)' \}
 \end{aligned} \tag{1}$$

in which  $H_T$  and  $H_e$  denote intensity of interference field and geo-magnetic field, respectively.  $\{\cos X, \cos Y, \cos Z\}$  is the directional cosine of geo-magnetic field vectors in regard to the transverse, longitude and vertical axes of aircraft,  $()'$  is the rate of variation with time. In the equation, the first three terms represent permanent field, the next six terms represent induced field and the last nine terms denote eddy current interference. Simplifying the equation with the following two identities

$$\begin{aligned}
 \cos^2 X + \cos^2 Y + \cos^2 Z &= 1 \\
 \cos X (\cos X)' + \cos Y (\cos Y)' + \cos Z (\cos Z)' &= 0
 \end{aligned} \tag{2}$$

Then the model equation has only 16 coefficients. If  $n$  sets of samples are selected, (1) can be written in the form of matrix:

$$\begin{aligned}
 & Y = XC \\
 \text{where } Y = & \begin{pmatrix} H_T(1) \\ H_T(2) \\ \vdots \\ H_T(n) \end{pmatrix}, X = \begin{pmatrix} x_1(1) & x_2(1) & \cdots & x_{16}(1) \\ x_1(2) & x_2(2) & \cdots & x_{16}(2) \\ \vdots & \vdots & & \vdots \\ x_1(n) & x_2(n) & \cdots & x_{16}(n) \end{pmatrix}, C = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_{16} \end{pmatrix}.
 \end{aligned} \tag{3}$$

### Partial least square and simulation

Given independent variables  $X = \{x_1, \dots, x_p\}$  and dependent variables  $Y = \{y_1, \dots, y_q\}$  and  $n$  sample points, we can extract latent variables  $t_1$  and  $u_1$  from them,  $t_1$  and  $u_1$  are respectively the linear combination of  $X$  and  $Y$ . To satisfy the need of regression analysis,  $t_1$  and  $u_1$  should contain as much variation information in their own data as possible and have a maximum correlation. According to principal components analysis, they should satisfy the following terms:

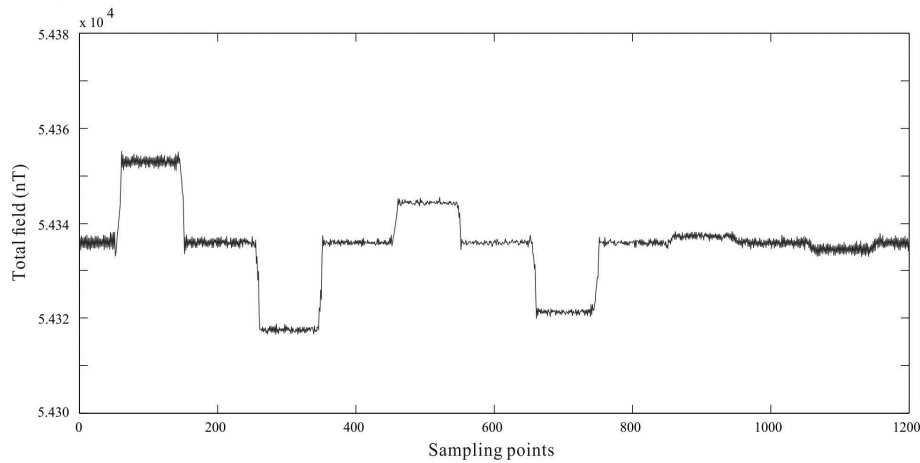
$$\text{Var}(t_1) \rightarrow \max, \text{Var}(u_1) \rightarrow \max; R(t_1, u_1) \rightarrow \max \tag{4}$$

where  $R$  calculates the relevancy between the two variables. To combine the three terms, we need to maximum the covariance of  $t_1$  and  $u_1$  in PLS:

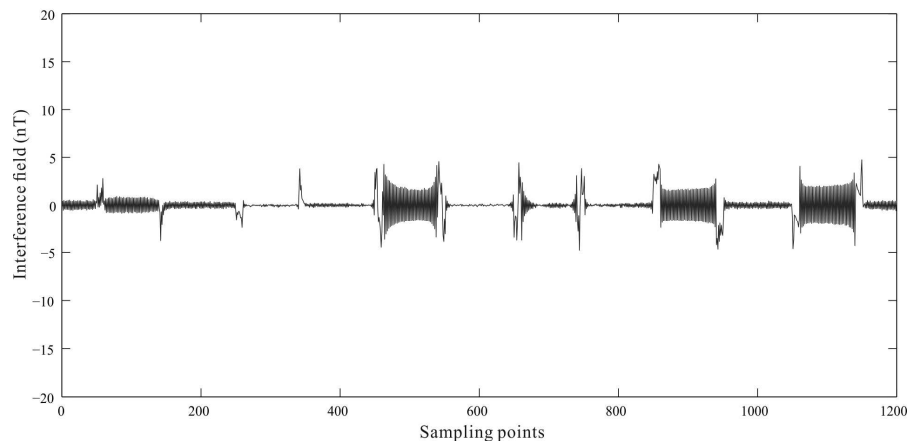
$$\text{Cov}(t_1, u_1) = \sqrt{\text{Var}(t_1)\text{Var}(u_1)} R(t_1, u_1) \rightarrow \max \tag{5}$$

With  $t_1$  and  $u_1$ , we can build the regression relation between  $y_1, \dots, y_q$  and  $t_1$ . After extracting the first components, regression analysis will be conducted on  $X \sim t_1$  and  $Y \sim u_1$ . If the accuracy of regression equations satisfies the pre-set value, the calculation is over, or the second components will be extracted from residual information of  $X$  and  $Y$ . If the number of extracted components is  $r$  finally, we will build the regression equation between  $y_1, \dots, y_q$  and  $t_1, \dots, t_r$ , and then between  $y_1, \dots, y_q$  and  $x_1, \dots, x_p$ , that is the PLS regression equation.

To verify the effectiveness of PLS in aeromagnetic compensation, we designed a straight line flight with magnetometer. The aircraft has maneuvers of yaw, roll and pitch during the flight, their maximum limits are  $5^\circ, 10^\circ$  and  $5^\circ$  in two opposite directions with frequency of 0.1Hz. The inclination and declination are  $45^\circ$  and  $-10^\circ$ ,  $H_e = 54000 \text{ nT}$ . Gaussian white noise with standard deviation of 0.2 was added to the simulation data. The results of compensation are shown in Figure 1 and Figure 2.



**Figure 1: Total field magnitude in the simulation flight.**



**Figure 2: Interference field magnitude after PLS compensation.**

## CONCLUSIONS

Partial least square is effective in solving linear regression problems with multicollinearity. In this paper we used PLS in aeromagnetic compensation and obtained satisfied results in simulation test. From the simulation results, we can see that PLS can effectively suppress the interference field to a low level. PLS demonstrated a preferable result in the compensation of yaw maneuvers, but less satisfied in the maneuvers of roll and pitch. That is also where we should do more research to investigate the reason and furthermore, to figure out a solution in order to get a better compensation effect.

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