



## Joint inversion of P-, and S-wave travel times for characterisation of anisotropic materials using laser Doppler interferometry measurements

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### SUMMARY

We used laser Doppler interferometer for measuring the displacement on the sample surface. These measurements allow us to clearly separate different wave types, whose picked travel times are used for estimation of VTI anisotropy parameters. One of the observations in this study is the very strong amplitude of critically refracted SP wave at the measurement surface. We confirmed the characteristics of this wave by numerical modelling. We used this wave to improve the estimates of the anisotropy. The observed strong amplitude of this wave can have strong implications for the interpretation of ultrasonic measurements.

**Key words:** anisotropy, inversion, converted waves, laboratory experiments, rock characterisation.

### INTRODUCTION

Laboratory measurements of elastic properties of rocks are important for calibration of seismic data and for corroboration of theoretical models of rocks. The most common way of determining the elastic properties of rock samples in laboratory settings is to estimate the velocities of the ultrasonic waves propagating in different directions. The wave velocities are established from the travel times of waves generated and recorded by ultrasonic piezoelectric transducers (e.g., Pros and Babuska, 1967; Jech, 1991; Rasolofosaon and Zinszner, 2002). This approach has a large uncertainty associated with S-wave travel time estimation and separation of differently polarised S-waves, and uncertainty as to whether phase or group velocity is measured. One way to address some of these issues is by using laser Doppler interferometer, which allows to record a particle like movement that can serve to separate the waves and to pick the travel times from which the ray velocities can be estimated (Guilbaud and Audoin, 1999; Rasolofosaon et al., 1994; Lebedev et al., 2011)).

Even for the most simple nontrivial anisotropy type, the polar anisotropy (VTI media), there are no exact analytic expressions for the ray velocities. Most of the available approximate expressions for VTI media are summarised by Golikov and Stovas (2012) and Asgharzadeh et al. (2014), who study their accuracy and ranges of validity. At large

offsets, the approximations might not be valid and can cause large errors in the anisotropy parameter estimation (Tsvankin, 2001). To address this potential issue, the exact ray velocities for a given measurement direction can be computed numerically and then be used for inversion of the elasticity parameters (e.g. Bóna et al. (2012), Nadri et al. (2012) and Lebedev et al. (2011)).

Lebedev et al. (2011) used the above approach to estimate elastic properties of an anisotropic material from travel times and polarisations of a quasi-P-wave. In this paper we aim to build up on that approach, by using travel times of all recorded transmitted waves (quasi-P, SV and SH), and using polarisations for the separation of the different types of waves.

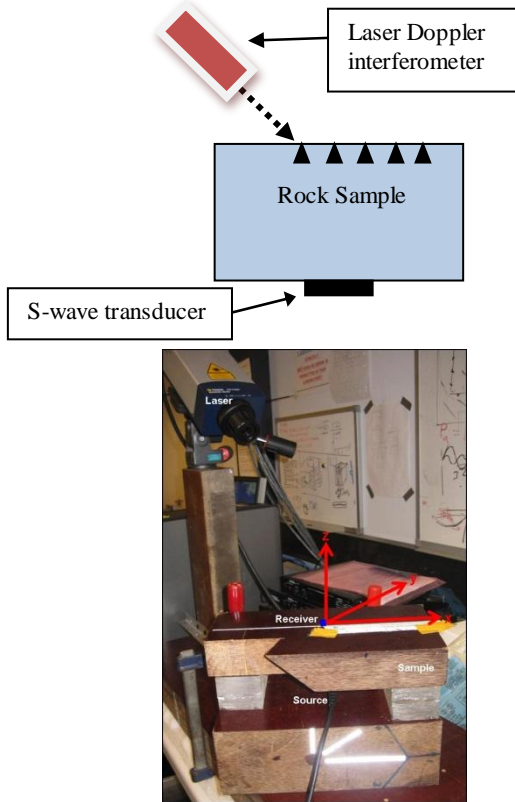
An unexpected observation from this study is the presence of a strong critically refracted SP wave at the measurement surface. We confirmed the presence of this wave by numerical simulations and used this wave to improve our inversion.

We demonstrate the effectiveness of the proposed inversion using measurements on paper-reinforced phenolic. Based on the observed equal travel times of the two S-waves along the presumed symmetry axis, we assume that the material has VTI symmetry. We invert for the VTI anisotropy parameters without relying on any approximations to the ray velocities.

### EXPERIMENTAL SETUP

Unlike in the typical laboratory ultrasonic experiments, in seismic field measurements, the size of the receivers is much smaller than the wavelength of the measured waves. This simplifies the determination of the time of arrival of the wavefronts and estimation of the polarization and, thus, determination of the wave type. Adoption of a similar approach in the laboratory requires receivers whose effective measurement area is small compared to the wavelength. To achieve measurements across such a small surface area, we use a set up similar to that described in details by Lebedev et al. (2011) (Figure 1). To study anisotropic wave propagation, we use a block (300 mm x 200 mm x 50 mm, L x W x H) of a paper-reinforced phenolic as a physical model of the subsurface. The ultrasonic waves (both P and S) were generated by a S-wave type transducer (V153-RB, Olympus Ltd) glued to the bottom of the sample. The excitation

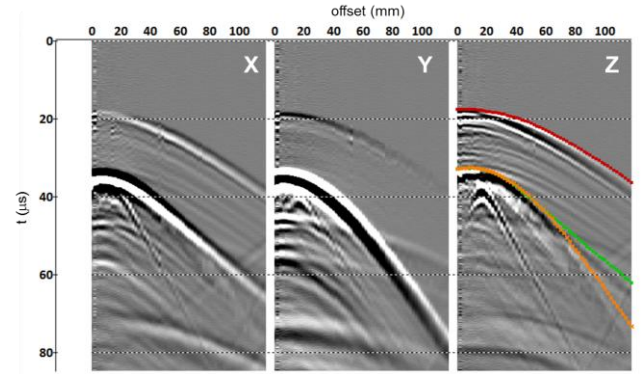
frequency was 1 MHz. A vibrometer OFV-5000 with the sensor head OFV-503 (Politec Ltd) was used to measure displacement of the surface upon arrival of ultrasonic waves. Displacement measurements were performed along a straight line on the surface in three spatial directions (3C). The distance between adjacent measurement points was 2 mm, with the uncertainties in positioning of measurement points less than 0.3 mm. We recorded the waveforms by a digital oscilloscope (TDS3001, Tektronix), with a time sampling rate of 20 ns.



**Figure 1** A schematic and a photo of laser Doppler interferometry experiments. The red axes correspond to the displacement directions used in the subsequent figures.

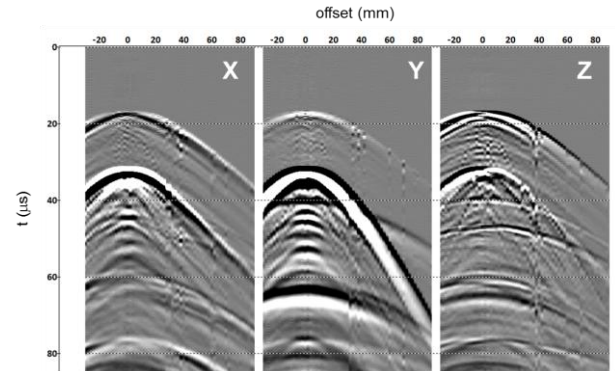
## DATA ANALYSIS

The recorded data along the line of measurement points is shown in Figure 2, in which we use the three-component measurements to separate the wave field into the three components: X – displacement along the measurement line, Z – displacement in the normal direction to the surface, and Y – displacement perpendicular to directions X and Z. The picked travel time curves are overlaid on the panel showing the Z component of the wave field. Based on the polarisation of the recorded wave, we initially interpreted the picked travel times as those corresponding to P-wave (red), SH-wave (orange), and SV-wave (green). The nomenclature for SH and SV waves is relative to the measurement surface (X-Y plane). In anisotropic media these should be called quasi-P, quasi-SH and quasi-SV waves but we drop the word ‘quasi’ for brevity.



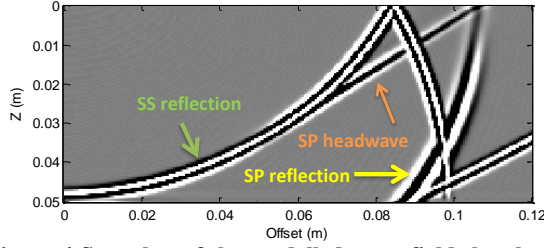
**Figure 2** Measured displacements in X, Y, and Z directions for anisotropic paper-reinforced phenolic. The red, green, and orange curves correspond to the picked travel times of the dominant waves with displacement along Z, X, and Y directions, respectively.

We initially inverted the picked travel times corresponding to the three identified waves using an algorithm discussed in the following section. However, the travel times of what we interpreted as the SV-wave could not be reconciled with the P-wave travel times. To understand the cause of this discrepancy, we performed the same experiment on a PMMA (acrylic) sample, which was expected to be isotropic. The resulting wave field components exhibit very similar behaviour to that of the original experiment, as shown in Figure 3.

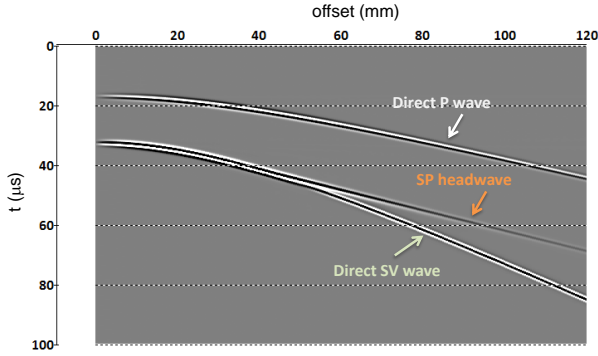


**Figure 3** Measured displacements in X, Y, and Z directions for PMMA sample. Even though the sample is isotropic, there is clear difference between the X and Y components of the wavefield.

In PMMA there is also a difference between what appeared to be the SV and SH waves. Since in an isotropic medium velocities of SH and SV waves should be identical, a suspicion arose that we did not identify the waves correctly. To confirm this and to properly identify the wave fields, we performed numerical simulations of the PMMA experiment using OASES software based on the Global Matrix Algorithm (Schmidt and Jensen, 1985). The results confirmed the misidentification of the waves and convinced us that part of the observed X – component is SV-P wave critically refracted from the free surface (for brevity also denoted as SP headwave). This can be seen on a snapshot of the wave field shown in Figure 4 and the corresponding seismogram recorded at the modelled receivers on the surface shown in Figure 5.



**Figure 4** Snapshot of the modelled wave field shortly after the SV wave arrives to the surface at the critical angle for the SV-P reflection.



**Figure 5** Modelled data showing the critically refracted SV-P wave. The model does not show why this wave is as strong as observed in real data.

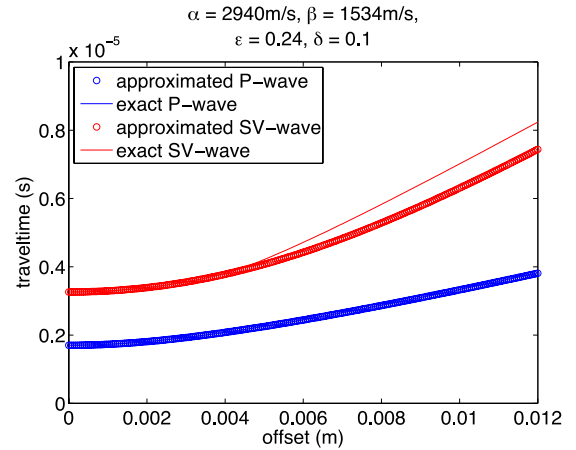
Once we knew what waves we observed in the data, we modified the inversion accordingly, which produced a good match with the data. In particular, applying the inversion algorithm on the modelled data resulted in the error in the anisotropy parameter estimation of less than 10%.

### INVERSION

To characterise the subsurface or a physical model we would like to invert the picked travel times for the anisotropy parameters. Instead of inverting for a generally anisotropic material, we can make some assumptions about the symmetries of the material. From the construction of the paper-reinforced phenolic, we can assume that the material should have at least orthotropic symmetry. Furthermore, the picked travel times show that the SV and SH velocities along the symmetry axis normal to the measurement/layering surface are the same. This suggests that the material has polar anisotropy with the vertical symmetry axis (VTI). Even though we cannot exclude the possibility that the material is very special orthorhombic material with the same S velocities along one of the symmetry axes, we believe that this is unlikely. For these reasons, we decided to invert the measured travel times for Thomsen's VTI anisotropy parameters.

To invert the travel times, we effectively convert them to velocities. However, in anisotropic media there are two types of velocities: phase and ray velocities. Phase velocity is the velocity of the wavefront and would correspond to the distance between the source and receiver divided by the travel time for a plane wave, such as the wave traveling normal to a large planar source. Ray velocity is the velocity of propagation of energy and can be computed by dividing the ray path length by the travel time. Thus, to invert the travel times we need to first establish what velocities we are computing.

Since the transducer is emitting waves in the direction of the symmetry axis, we can exclude the plane waves from our consideration by using only the receivers offset from the centre of the transducer by the radius of the transducer. By doing this, all the considered velocities are the ray velocities. There are no exact analytic expressions for anisotropic ray velocities. We could use some of the available approximations, such as the one suggested by Tsvankin and Thomsen (1994). However, as demonstrated in Figure 6, such an approximation at large offsets could cause large errors in the anisotropy parameter estimation (Tsvankin, 2001). These errors may be particularly problematic for inversion, since it is a priori unknown whether anisotropy is weak, and since the errors can affect the very behaviour of the misfit function.

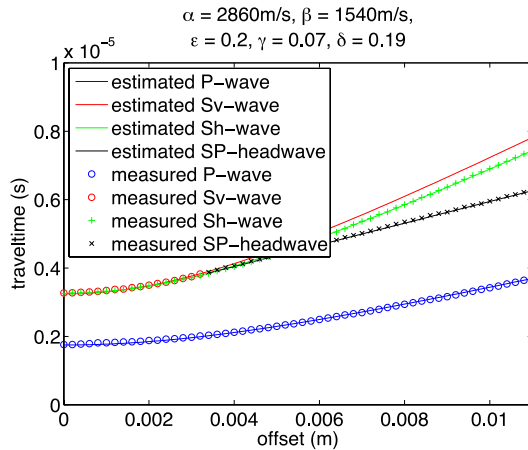


**Figure 6** Comparison between exact and approximated travel times, using travel time approximation for VTI media by Tsvankin and Thomsen (1994). Note that the SV approximation exhibits roughly similar shape as the observed SV/SP-headwave from the experiments; the approximation could fit the observed data better than the exact travel times while misrepresenting the SP-headwave as SV wave.

To invert for Thomsen's anisotropy parameters we perform a global search of the parameters. For a given set of parameters, we compute the exact travel time curves by parameterising the curves by the phase directions. First, for each phase direction  $\mathbf{n}$  we compute the three (P, SV, and SH) phase slownesses  $\mathbf{p}$  and the corresponding polarisations  $\mathbf{A}$ . Then we use them to compute the three ray velocities  $\mathbf{V}$  using relation  $\mathbf{V} = \Gamma(\mathbf{A})\mathbf{p}$ , where  $\Gamma(\mathbf{n})$  is the Christoffel matrix in direction  $\mathbf{n}$  (Bóna et al., 2008). Once we know the parametric curves corresponding to all the observed travel times, we find the distance to the relevant curve from each measurement point. The distance is defined as the square root of the sum of the offset differences squared and velocity-scaled travel time differences squared. The travel time scaling velocity is the vertical velocity of the corresponding wave:  $\alpha$  for P-wave and  $\beta$  for S-waves. The global search minimises the sum of the distances from the picked travel times for all waves to the corresponding travel time curves.

The joint inversion algorithm gives Thomsen's anisotropy parameters as:  $\alpha = 2860$  m/s,  $\beta = 1540$  m/s,  $\epsilon = 0.20$ ,  $\gamma = 0.07$ , and  $\delta = 0.19$ . Figure 7 shows a good match between the experimental travel time data and the corresponding estimated travel time curves.





**Figure 7** The picked travel times for the paper-reinforced phenolic are shown as discrete symbols. The fitted travel times are shown in the solid lines.

## CONCLUSIONS AND DISCUSSION

We have inverted the picked travel times of P, SV, SH, and critically refracted SP waves in paper-reinforced phenolic and obtained the following parameter estimates:  $\alpha = 2860$  m/s,  $\beta = 1540$  m/s,  $\varepsilon = 0.2$ ,  $\gamma = 0.07$ , and  $\delta = 0.19$ . These values can be compared with the estimates from joint inversion of travel times and polarisation angles for the same material:  $\alpha = 2887$  m/s,  $\beta = 1548$  m/s,  $\varepsilon = 0.23$ ,  $\gamma = 0.08$ , and  $\delta = 0.25$  (Lebedev et al., 2011). These sets of parameters have similar values but with some difference, which can probably be attributed to the miss-identification of the critically refracted SP wave as SV wave in Lebedev et al., (2011). The reason that such miss-identification did not cause bigger differences is the limited offset range (0 to 6cm) used in the previous work.

A key observation from this study is the very strong presence of critically refracted SP wave. Although we confirmed the presence of this wave by numerical modelling, its amplitude is much stronger in the laboratory-measured data. This inconsistency is part of the future research, but perhaps one possible explanation is a complex radiation pattern of the source. Such a non-trivial radiation pattern is probably also the reason for the different frequency content in the SH and P-waves compared to the SV and the critically refracted SP wave, as can be seen by the naked eye for both isotropic and anisotropic samples (Figure 2 and Figure 3). Once we know the radiation pattern of the source, we could try to deconvolve this from the observed data. Based on our preliminary results, such a deconvolution would also allow us to use semblance as a fitting algorithm, which would eliminate the need to pick the travel times (see e.g. Goldin, 1979, Grechka et al., 1999).

Another potential source of errors in the presented work is our assumption of polar anisotropy. We justified this assumption by the observation that along a symmetry axis both S-waves have the same velocity and an orthorhombic material with such property would be unlikely. To fully justify this assumption, we plan to look at the travel times measured across the surface of the material, not only along one receiver line.

Our observations should influence the quality control of using apparent SV-waves at nonzero offsets, as there is a potential danger of miss-identifying the critically refracted SP wave as SV-wave. Such situations are potentially not restricted to

laboratory ultrasonic measurements, but also to seismic field experiments.

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