Tensor Charge of the Nucleon
on a Lattice*

Tetsuo Hatsuda

Institute of Physics, University of Tsukuba,
Tsukuba, Ibaraki 305, Japan.

Abstract
The tensor charge of the nucleon, which will be measured in Drell–Yan processes in polarized
proton–proton collisions at the RHIC, is studied in a quenched lattice QCD simulation. On
the $16^3 \times 20$ lattice with $\beta = 5.7$, connected parts of the tensor charge are determined with
small statistical error, while the disconnected parts are found to be small with relatively large
error bars. The flavour-singlet tensor charge ($\delta \Sigma = \delta u + \delta d + \delta s$) is not suppressed, as opposed
to the flavour-singlet axial charge ($\Delta \Sigma = \Delta u + \Delta d + \Delta s$).

1. Introduction
The parton structure of the nucleon in the twist 2 level is known to be
characterized by three structure functions $f_1(x, \mu)$, $g_1(x, \mu)$ and $h_1(x, \mu)$ with $x$
being the Bjorken variable and $\mu$ being the renormalization scale (see e.g. [1]).
Here $f_1$ and $g_1$, which represent the quark-momentum distribution and quark-
spin distribution respectively, can be measured by deep inelastic lepton–hadron
scattering (DIS). On the other hand, $h_1$, which represents the quark-transversity
distribution, can only be measured in the polarized Drell–Yan processes, since it
is related to the matrix element of the chiral-odd quark operator. Although such
experiments are not yet available, they are planned for the Relativistic Heavy Ion
Collider at Brookhaven National Laboratory. Therefore, theoretical prediction of
$h_1$ has some importance. Also, whether there is a ‘transversity crisis’ in the first
moment of $h_1$, as in the case of ‘spin crisis’ in $g_1$, is an interesting question to
be examined. In this paper, we will concentrate on the first moment of $h_1(x)$
and report our recent studies using lattice QCD simulations [2].

2. Tensor Charge versus Axial Charge
The first moments of $g_1(x, \mu)$ and $h_1(x, \mu)$ are called $\Delta q(\mu)$ and $\delta q(\mu)$
respectively. They are related to the nucleon matrix elements of the axial current
and tensor current as follows:

* Refereed paper based on a contribution to the Japan–Australia Workshop on Quarks,
Hadrons and Nuclei held at the Institute for Theoretical Physics, University of Adelaide, in

10.1071/PH96030  0004-9506/97/010205$05.00
\[ \langle p s | \bar{q} \gamma_\mu \gamma_5 q | p s \rangle = 2 M s_\mu \Delta q, \quad (1) \]
\[ \langle p s | \bar{q} i \sigma_\mu \gamma_5 q | p s \rangle = 2 (s_\mu p_\nu - s_\nu p_\mu) \delta q, \quad (2) \]

where \( p_\mu \) is the nucleon’s four momentum, \( M \) is the nucleon’s rest mass, and \( s_\mu \) is the nucleon’s covariant spin-vector.

In the light cone frame, \( \Delta q \) is interpreted as the total quark-helicity in the nucleon, while \( \delta q \) is a total quark-transversity in the nucleon [1]:

\[ \Delta q(\mu) = \int_0^1 [g_1(x,\mu) + \bar{g}_1(x,\mu)] dx \]
\[ = \int_0^1 [N_+^+(x,\mu) - N_-(x,\mu) + \bar{N}_+^-(x,\mu) - \bar{N}_-(x,\mu)] dx, \quad (3) \]

and

\[ \delta q(\mu) = \int_0^1 [h_1(x,\mu) - h_1(x,\mu)] dx \]
\[ = \int_0^1 [N_+^- (x,\mu) - N_- (x,\mu) - \bar{N}_+^- (x,\mu) + \bar{N}_- (x,\mu)] dx. \quad (4) \]

Here \( N_+^+(x) \) [\( N_- (x) \)] denotes the momentum distribution of quarks having the same (opposite) helicity with the nucleon, while \( N_+^- (x) \) [\( \bar{N}_+^- (x) \)] denotes that having the same (opposite) transverse polarization with the nucleon. The quantity with bar is the distribution for anti-quarks.

On the other hand, in the rest frame of the nucleon, \( \Delta q \) (\( \delta q \)) denotes the quark-spin + anti-quark-spin (quark-spin–anti-quark-spin), which can be seen by taking \( \mu = i \) (\( \mu = 0, \nu = i \)) in equations (1, 2). In this frame, estimates by using hadron models are possible. For example, relativistic quark models which can reproduce \( g_A = 1 \cdot 25 \) correctly give simple inequalities:

\[ | \delta u | > | \Delta u |, \quad | \delta d | > | \Delta d |. \quad (5) \]

The lower component of the Dirac spinor of the confined quarks plays an essential role for the above inequalities.

Drawbacks of such model calculations are (i) the renormalization scale where the matrix elements are evaluated is not clear, and (ii) the strange quark contribution, which originates from the OZI violating processes, is hard to estimate in a reliable manner. Lattice QCD simulations can overcome these problems even within the quenched approximation. In particular, \( \Delta u, \Delta d \) and \( \Delta s \) have been studied by two groups [3, 4] and their results are consistent with the recent experimental data on the spin structure of the nucleon [5]. At the quenched level, two kinds of diagrams arise: one is the connected amplitude (Fig. 1a) in which the external operator is connected to one of the valence nucleon lines, and another is the disconnected amplitude (Fig. 1b) where the quark line coming from the external operator is closed by itself. The latter gives an OZI violating amplitude leading to the strangeness content in the nucleon.
3. Matrix Elements on the Lattice

(a) Mass and Matrix Elements

We use the standard Wilson action in our simulation in which two basic parameters read $\beta \equiv 6/g^2$ ($g$ being the bare gauge coupling) and the hopping parameter $K$. In the following, instead of $K$, we use the ‘quark-mass’ $m a \equiv (1/K - 1/K_c)/2$ where $a$ is the lattice spacing and $K_c$ is the critical hopping parameter at which the pion becomes massless.

Hadron masses are obtained by the correlation function of composite operators for large time $t$. For example, the nucleon mass is obtained from

$$\langle N(t)\overline{N}(0) \rangle \rightarrow \text{const.} \times e^{-m_N t}, \quad (6)$$

with $N(t)$ being the spatially integrated interpolating operator for the nucleon $N(t) = \int d^3x (q C^{-1} \gamma_5 q)$. On the other hand, the matrix element of the local operator is obtained as

$$R(t) \equiv \frac{\langle N(t) \sum_{t',x} O(t',x)\overline{N}(0) \rangle}{\langle N(t)\overline{N}(0) \rangle} \rightarrow \text{const.} + \langle N | O | N \rangle \ t. \quad (7)$$

Namely, the linear slope of $R(t \rightarrow \text{large})$ gives matrix elements defined on the lattice.

(b) Some Remarks

In our actual simulation, the following points have been considered [3]. (i) To get a large overlap of $N(0)$ with the real nucleon, we use a wall source at initial time slice $t = 0$. To do this, we made Coulomb gauge fixing only at $t = 0$. (ii) To avoid a mirror source at the final time slice $t = t_f$, we set the Dirichlet boundary condition at $t_f$. (iii) To compare the matrix element on the lattice with that in the $\overline{MS}$ scheme, we multiplied the data obtained by a renormalization factor $Z(\mu a)$. For the tensor charge, $Z$ calculated for $\mu = 1/a$ using tadpole improved perturbation theory [6] reads

$$Z = \left(1 - \frac{3K}{4K_c}\right)[1 - 0.44 \alpha_s(1/a)]. \quad (8)$$
4. Results

We have done simulations in three cases: \(12^3 \times 20\) (\(\beta = 5.7\)), \(16^3 \times 20\) (\(\beta = 5.7\)) and \(16^3 \times 20\) (\(\beta = 6.0\)). In this paper we show the results of the second simulation. We have used a Fujitsu VPP500 computer and analysed 550 gauge configurations. Each configuration is taken after every 1000 sweeps. Three different values of the quark masses \(K = 0.160, 0.164\) and \(0.1665\) are adopted to extract physical quantities in the chiral limit. The statistical errors are estimated by the jackknife procedure.

Hadron masses are extracted by the \(\chi^2\) fitting of the data in the interval \(5 \leq t \leq 10\). By using the physical hadron masses \(m_{\pi, \rho, K} = 135, 770\) and \(498\) MeV, one obtains \(a^{-1} = 1.42\) GeV (\(a = 0.14\) fm), \(m = 4.8\) MeV and \(m_s = 125\) MeV. This also predicts the nucleon mass as \(m_N = 1.13\) GeV. The physical volume of the lattice is \(V = (2.24)^3 \times 2.8\) fm\(^4\).

A \(\chi^2\) fitting in the interval \(5 \leq t \leq 10\) is also applied for the connected and disconnected part of the matrix elements. Shown in Fig. 2 are the data for the correlation function \(R(t)\) for \(K = 0.164\). For connected \(u,d\) contributions (\(\delta u_{\text{con.}}, \delta d_{\text{con.}}\)), one can see a clear non-vanishing linear slope in \(5 \leq t \leq 10\), while the disconnected \(u - d\) contribution (\(\delta u_{\text{dis.}}, \delta d_{\text{dis.}}\)) has a very small slope, if at all.

\[\text{Fig. 2. } R(t) \text{ as a function of } t \text{ for a medium-heavy quark mass } K = 0.164. \text{ The black (white) circles denote the connected amplitude } \delta u_{\text{con.}}, \delta d_{\text{con.}}, \text{ while the black triangles denote the disconnected amplitude } \delta u_{\text{dis.}}, \delta d_{\text{dis.}}.\]

Table 1 shows the results of the data extrapolated down to the chiral limit. The results compared with that for \(\Delta q\) in [3] with the same lattice size and \(\beta\).
Table 1. Comparison of the tensor and axial charges measured on the lattice

<table>
<thead>
<tr>
<th>Tensor charge $\delta q$ (this work)</th>
<th>Axial charge $\Delta q$ (ref. [3])</th>
</tr>
</thead>
<tbody>
<tr>
<td>16$^3 \times 20$, $\beta = 5.7$</td>
<td>16$^3 \times 20$, $\beta = 5.7$</td>
</tr>
<tr>
<td>550 gauge configurations</td>
<td>260 gauge configurations</td>
</tr>
<tr>
<td>$\mu^2 = 3$ GeV$^2$</td>
<td>$\mu^2 = 3$ GeV$^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta u_{\text{con.}}$</th>
<th>$\Delta u_{\text{con.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+0.89 \pm 0.02$</td>
<td>$+0.76 \pm 0.04$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta u_{\text{dis.}}$</th>
<th>$\Delta u_{\text{dis.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.05 \pm 0.02$</td>
<td>$-0.12 \pm 0.04$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta u$</th>
<th>$\Delta u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+0.84 \pm 0.02$</td>
<td>$+0.64 \pm 0.04$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta d_{\text{con.}}$</th>
<th>$\Delta d_{\text{con.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.18 \pm 0.02$</td>
<td>$-0.23 \pm 0.04$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta d_{\text{dis.}}$</th>
<th>$\Delta d_{\text{dis.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.05 \pm 0.02$</td>
<td>$-0.12 \pm 0.04$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta d$</th>
<th>$\Delta d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.23 \pm 0.02$</td>
<td>$-0.35 \pm 0.04$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta s$</th>
<th>$\Delta s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.05 \pm 0.02$</td>
<td>$-0.11 \pm 0.03$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta \Sigma$</th>
<th>$\Delta \Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+0.56 \pm 0.13$</td>
<td>$+0.18 \pm 0.10$</td>
</tr>
</tbody>
</table>

5. Summary and Discussion

In our simulation, we have found the following.

1. For the connected part of the tensor/axial charge, the following inequalities hold:
   \[ |\delta u| > |\Delta u|, \quad |\delta d| < |\Delta d|. \] (9)

   This is different from the prediction of relativistic quark models which have the universal inequality eq. (5).

2. The disconnected part still has large statistical error and one cannot make definite conclusions from our simulation. Nevertheless, there is an indication that (i) the disconnected part is flavour independent, i.e. $\delta u_{\text{dis.}} \sim \delta d_{\text{dis.}} \sim \delta s_{\text{dis.}}$, and (ii) they are small but slightly negative.

3. Flavour-singlet tensor charge $\delta \Sigma = \delta u + \delta d + \delta s$ is not suppressed as opposed to $\Delta \Sigma$:
   \[ \delta \Sigma(3\text{GeV}^2) = 0.56 \pm 0.13 \quad \leftrightarrow \quad \Delta \Sigma(3\text{GeV}^2) = 0.18(10), \] (10)

   which implies that there is no ‘transversity crisis’ for the tensor charge.

Now, what we need to understand is the origin of the smallness of the disconnected part of the tensor charge. Since the operator $\bar{q} \sigma_{\mu\nu} \gamma_5 q$ $(\bar{q} \gamma_{\mu} \gamma_5 q)$ is a charge conjugation odd (even) operator, $\delta q$ ($\Delta q$) can be interpreted as $q$-spin $-\bar{q}$-spin ($q$-spin + $\bar{q}$-spin). This indicates that there is a large cancellation of the quark-spin content and the anti-quark-spin content of the nucleon. Also, the disconnected tensor charge is zero in any order of perturbation theory in massless QCD, which might have some relation to its smallness in the non-perturbative regime. To clarify the above issue, we are currently collecting more data on $\delta q_{\text{dis.}}$ and $\Delta q_{\text{dis.}}$ simultaneously with a $16^3 \times 20$ lattice ($\beta = 5.7$).
Acknowledgments

This article is based on work in collaboration with Dr S. Aoki, Dr M. Doui and Dr Y. Kuramashi, to whom I am grateful. I thank Prof. A. W. Thomas, Dr. A. Williams and Prof. K. Saito for warm hospitality at Adelaide and for giving me an opportunity to present a talk at the Japan–Australia workshop on Quarks, Hadrons and Nuclei. This work was supported in part by the Grants-in-Aid of the Japanese Ministry of Education, Science and Culture (No. 06102004).

References