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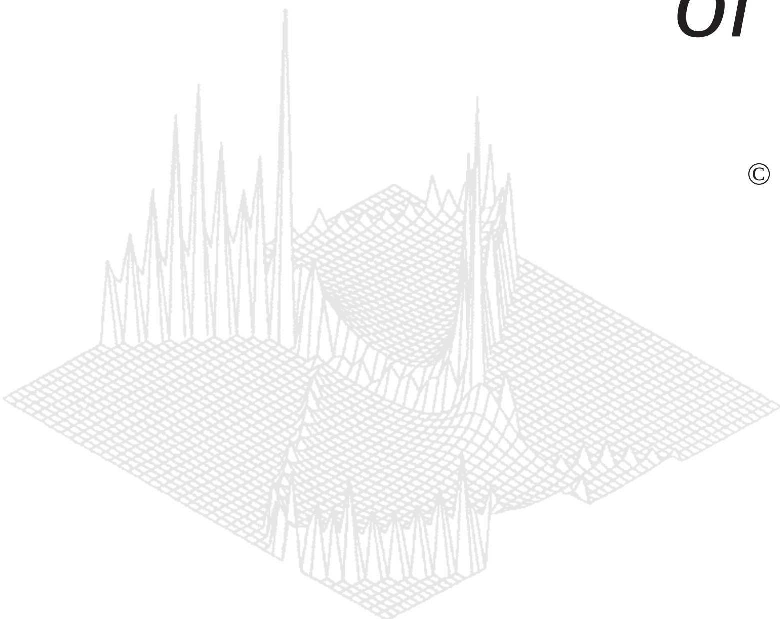

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Nuclear Transparency in a Relativistic Quark Model*

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Abstract

We examine the nuclear transparency for the $(e, e'p)$ reaction, applying a relativistic quark model to incorporate the internal dynamics of the struck proton. We find that the relativistic covariance plays an important role for so-called 'colour transparency'.

1. Introduction

Colour transparency reflects internal dynamics of hadrons, as was proposed by Brodsky and Mueller on the basis of perturbative quantum chromodynamics (QCD) [1]. They suggested that in the high-energy quasi-elastic process, such as $(e, e'p)$ or $(p, 2p)$, there is a possibility that the target nucleus becomes transparent. That the nucleus becomes transparent implies that the final-state interactions (FSI) of the struck proton become much weaker than those obtained from the conventional multiple-scattering theory. When this happens, we say 'colour transparency' occurs.

Experimental results on the nuclear transparency in $(e, e'p)$ have been reported recently [2, 3]. The results of Ref. [2] indicate a weak energy dependence and a slow onset of colour transparency. Most recently the occurrence of colour transparency has been reported in ρ-meson production on the nucleus [4].

In order to study how the nucleus becomes transparent, we should construct a quantum mechanical model for the struck proton. In our previous work [1] we formulated a model for the breathing mode of the proton (we call it the 'b-model'). The b-model gave a good description for the internal dynamics of the proton and brought about energy-dependent transparencies in the intermediate energy region.

Recently, however, we have realized that the struck proton in our b-model appeared to be too stiff. That is, the nucleus does not become transparent completely for the struck proton in the model even if the momentum transfer is very large. We believe that this is because the transverse motion breaks the causality. That is, since the b-model is based on the non-relativistic quark model (NRQM), i.e. the quark cluster model, it does not preserve the causality. One

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can easily see that the transverse velocity of the harmonic oscillator increases to infinity for the very highly excited states. The description in terms of NRQM will break down at least for the higher excited states.

In this work, we incorporate the Lorentz covariance into our formulation for the transparency. In the rest of this paper we explain how to describe the nuclear transparency in terms of the relativistic harmonic oscillator model. First we explain our basic formulation in Section 2, and second introduce the model in Section 3. In Section 4 the survival amplitude is given, and the numerical results are presented. Finally we summarize the results in Section 5.

2. Formulation

We confine ourselves to the $(e, e'p)$ reaction in this work, while we emphasize that our considerations are equally applicable to other more complicated, semi-exclusive reactions, such as $(p, 2p)$. Following the terms used in Ref. [1], we call the proton dynamical (inert) when its internal structure is (not) taken into account.

One should be extremely careful to choose the quantity that would serve as a signature of the colour transparency. As described with great care in our previous work [5], we define the nuclear transparency $T(\mathbf{q})$ as a function of the momentum transfer $\mathbf{q}$ as

$$T(\mathbf{q}) = \frac{1}{Z}\,\frac{1}{A(\mathbf{q})}\,\frac{d\sigma_{eA}}{d\Omega_{k'}}\Big/\frac{d\sigma_{\mathrm{ep}}}{d\Omega_{k'}}\,, \qquad (2\cdot1)$$

where Z is the atomic number of the target nucleus, and $A(\mathbf{q}) \simeq 1\cdot 05$ is the Fermi-motion averaging factor, almost independent of $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ [5]. Here $\mathbf{k}$ is the momentum of the incident electron, and $\mathbf{k}'$ is that of the outgoing electron. Note that the quasi-elastic cross sections in eq. $(2\cdot1)$ are results of integration over the struck-proton momentum $\mathbf{p}'$, and over the magnitude of the momentum of the outgoing electron $\mathbf{k}'$.

We refer the reader to Refs [5] for detailed discussions leading to the preceding definition of the nuclear transparency as well as the kinematical considerations involved. Note that we use relativistic kinematics for the electrons and struck proton throughout.

To set the reference frame for the colour transparency, we carried out the calculations of the nuclear transparency in the framework of the Glauber multiple scattering theory, the Glauber impulse approximation (GIA) [5]. The GIA is applied under the zero-range-no-recoil (ZRNR) approximation, which neglects the target nuclear recoil, and the finite NN-interaction range in the final-state interaction. In this framework, the nuclear transparency is expressed as

$$T(\mathbf{q}) = \int d\mathbf{r}\rho(\mathbf{r})P^{(-)}(\mathbf{r}),\;\; P^{(-)}(\mathbf{r}) = \exp\{-(A-1)\sigma^r_{\mathrm{NN}}\int_z^\infty dz'\rho(\mathbf{b}, z')\}\,, \qquad (2\cdot2)$$

if nuclear correlations are neglected. Here $\mathbf{r} = (\mathbf{b}, z)$, $\rho(\mathbf{r})$ is the nuclear density, normalized to unity, and the path of the integral in eq. $(2\cdot2)$ is taken to be along the classical path of the struck proton. Also σ^r_{NN} is the proton–nucleon reaction cross section, $\sigma^r_{\mathrm{NN}} = \sigma^{total}_{\mathrm{NN}} - \sigma^{elastic}_{\mathrm{NN}}$, and is about 70% of the total cross section if the struck proton momentum is in the region $2 - 15$ GeV$/c$ where the reaction cross section is practically constant. In Ref. [5], we proposed to use the mean-free path determined by the proton–nucleon *reaction* cross section, not by the total cross section in this energy region.

We then proceed with a QCD-motivated calculation using a dynamical model associated with the internal structure of the proton [1]. A general description of the following discussion can be found in Ref. [6].

First we introduce the electromagnetic form factor of the proton, which is a key quantity, and is written as

$$F_{ep}(q^2) = \langle N; \mathbf{P}_f | \hat{O}(q) | N; \mathbf{P}_i \rangle \,, \tag{2·3}$$

where $|N; \mathbf{P}\rangle$ is the lowest eigenfunction of $\hat{M}^2$ of eq. (3·1) below, corresponding to a nucleon state with a 3-momentum $\mathbf{P}$, while $\hat{O}(q)$ is an operator which causes a hard scattering and brings a 4-momentum, $q_\mu = P_{f,\mu} - P_{i,\mu}$, into the system.

In order to consider the transparency, we are interested in the amplitude for the propagation through the nuclear medium for a hard struck proton, which is observed as a proton after a certain time t. The time evolution operator is $e^{-i\hat{H}t}$. Here the full Hamiltonian, $\hat{H}$ is assumed to consist of two parts, $\hat{H} = \hat{H}_0 + \hat{H}_I$, where $\hat{H}_0$ describes the internal dynamics of a free proton and H_I describes an interaction with the nuclear medium. We introduce a proton survival amplitude defined by

$$M_{eA}^{(D)}(q^2; t) = \langle N; \mathbf{P}_f | e^{-i\hat{H}t} \, \hat{O}(q) | N; \mathbf{P}_i \rangle \,. \tag{2·4}$$

Here our initial condition at $t = 0$, when the proton is hit by a photon, is that the proton wave function is $\hat{O}(q)|N; \mathbf{P}_i\rangle$. In (2·4) t is the time taken by the proton to exit the nucleus. At $t = 0$, $M_{eA}^{(D)}(q^2; t)$ corresponds to the form factor (2·3).

To obtain the nuclear transparency including the internal dynamics of the proton, we substitute

$$P^{(-)}(\mathbf{r}) = |M_{eA}(q^2; t(\mathbf{r}))|^2 / |F_{ep}(q^2)|^2 \,, \tag{2·5}$$

in eq. (2·1). This gives a measure of FSI. How to convert the t-dependence to the $\mathbf{r}$-dependence is explained in detail in Ref. [1].

3. Relativistic Quark Model

We use a relativistic quark model to preserve the Lorentz covariance for the internal dynamics of the struck proton. We follow the formulation of Fujimura [7] and briefly explain it here.

In this formulation the internal dynamics of the quarks is included in a mass operator $\hat{M}^2$, which has a 4-dimensional harmonic oscillator form, and the proton is governed by the wave equation

$$\hat{M}^2 |\phi_n; \mathbf{P}\rangle = M_n^2 |\phi_n; \mathbf{P}\rangle, \text{ where } -\hat{M}^2 = \hat{p}_r^2 + \hat{p}_s^2 + \alpha^2(\hat{r}^2 + \hat{s}^2) + C \,, \tag{3·1}$$

and where α and C are parameters. C is determined in order to reproduce the observed proton mass. $\hat{r}_\mu$ and $\hat{s}_\mu$ in eq. (3·1) are relative coordinate 4-vectors defined by $\hat{r}_\mu = (\hat{x}_{2,\mu} - \hat{x}_{3,\mu})/\sqrt{6}$ and $\hat{s}_\mu = (-2\hat{x}_{1,\mu} + \hat{x}_{2,\mu} + \hat{x}_{3,\mu})/(3\sqrt{2})$, where $x_{i,\mu}$ $(i = 1, 2, 3)$ are coordinates of each quarks. The $\hat{p}_{r,\mu}$ and $\hat{p}_{s,\mu}$ are the corresponding relative momentum 4-vectors.

The excitations in the time-direction are prohibited due to the following physical constraints, as discussed by Takabayashi [8];

$$P\cdot(-i\hat{p}_{r,\mu}+\alpha\hat{r}_\mu)|\phi_n;\mathbf{P}\rangle = P\cdot(-i\hat{p}_{s,\mu}+\alpha\hat{s}_\mu)|\phi_n;\mathbf{P}\rangle = 0,\ \ P_\mu=(E_n,\mathbf{P})\,. \qquad (3\cdot 2)$$

The wave function of eq. (3·1) is nothing but a harmonic oscillator. The ground state, for example, is easily obtained as

$$\phi_0(r,s;\mathbf{P}) = \left(\frac{\alpha}{\pi}\right)^2\ \exp\left\{\frac{\alpha}{2}\left(r^2+s^2-\frac{2}{M_0^2}(P\cdot r)^2-\frac{2}{M_0^2}(P\cdot s)^2\right)\right\}, \qquad (3\cdot 3)$$

where $P_\mu=(M_0,\mathbf{P})$. The mass eigenvalues are

$$\begin{aligned} M_n^2 = &-(2n_{r,0}+1)\alpha+(2(n_1+n_2+n_3)+3)\alpha \\ &-(2n_{s,0}+1)\alpha+(2(n_4+n_5+n_6)+3)\alpha+C\,, \end{aligned} \qquad (3\cdot 4)$$

where n_i $(i=1,\cdots,6)$ are non-negative integers. Here M_0 is the proton mass. Due to the physical constraints mentioned above, in eq. (3·2), we put $n_{r,0}=n_{s,0}=0$. The normalization is $\int d^4r d^4s\,|\phi_n(r,s;P_\mu)|^2=1$. We can easily calculate the transverse velocity in this model. It has an upper limit even if we consider the highly excited states. The transverse motion preserves the causality.

The electromagnetic form factor of this model is

$$\begin{aligned} F_{ep}(q^2) &\equiv \int d^4r d^4s\ \phi_0^*(r,s;\mathbf{P}_f)\ e^{iq\cdot(ar+bs)}\ \phi_0(r,s;\mathbf{P}_i) & (3\cdot 5)\\ &= \left(1+\frac{(-q^2)}{2M_0^2}\right)^{-2}\times\exp\left\{-\frac{(-q^2)}{2\alpha}\Big/\left(1+\frac{(-q^2)}{2M_0^2}\right)\right\}, & (3\cdot 6) \end{aligned}$$

where $q_\mu=P_{f,\mu}-P_{i,\mu}$. We use a combination, $(a,b)=(0,-\sqrt{2})$, which corresponds to the case where a photon couples to the quark-1, $\Leftrightarrow -\sqrt{2}q\cdot s=(x_1-X)\cdot q$. This implies that we take $\hat{O}(q^2)=e^{-i\sqrt{2}q\cdot s}$. As one can see from eq. (3·6), the form factor reproduces the well-known power-law behaviour for $-q^2$ large. This is an advantage of this model. We determine α to reproduce the behaviour of the observed form factor (see Fig. 1).

Since we have prepared the proton in free space, we now put the proton into the nuclear medium and let it run. We assume that the struck proton feels a size-dependent interaction, and use the Low–Nussinov model [9]. The interaction is purely absorptive, and depends only on the transverse size of the proton. Then H_I becomes $\hat{H}_I=-ic_0\,(\hat{\mathbf{r}}_\perp^2+\hat{\mathbf{s}}_\perp^2)$, where c_0 is a parameter, and expresses a strength of the absorption, which is determined by the mean-free path of the proton in the nuclear medium.

The Hamiltonian of the proton in free space H_0 satisfies the usual relativistic energy–momentum relation, $\hat{H}_0=\sqrt{|\mathbf{P}|^2+\hat{M}^2}$. If we take the ultra-relativistic limit for it, then $\hat{H}_0$ becomes $\hat{H}_0=|\mathbf{P}|+(\hat{M}^2/2|\mathbf{P}|)$. Since the full Hamiltonian has the form $\hat{H}=\hat{H}_0+\hat{H}_I$, the absorption effect is absorbed into a new mass operator

$\hat{M}_I$ under the ultra-relativistic approximation, such as $\hat{H} = |\mathbf{P}| + (\hat{M}_I^2/2|\mathbf{P}|)$, where $-\hat{M}_I^2 = \hat{p}_r^2 + \hat{p}_s^2 + \alpha^2(\hat{r}_0^2 - \hat{r}_3^2 + \hat{s}_0^2 - \hat{s}_3^2) - \alpha_\perp^2(\hat{\mathbf{r}}_\perp^2 + \hat{\mathbf{s}}_\perp^2) + C$. Here $\alpha_\perp$ is the transverse frequency which is a measure of an absorption effect. It is defined by $\alpha_\perp^2 \equiv \alpha^2\{1 - i(2|\mathbf{P}|c_0/\alpha^2)\}$. Here we assume that the proton is moving in the z-direction.

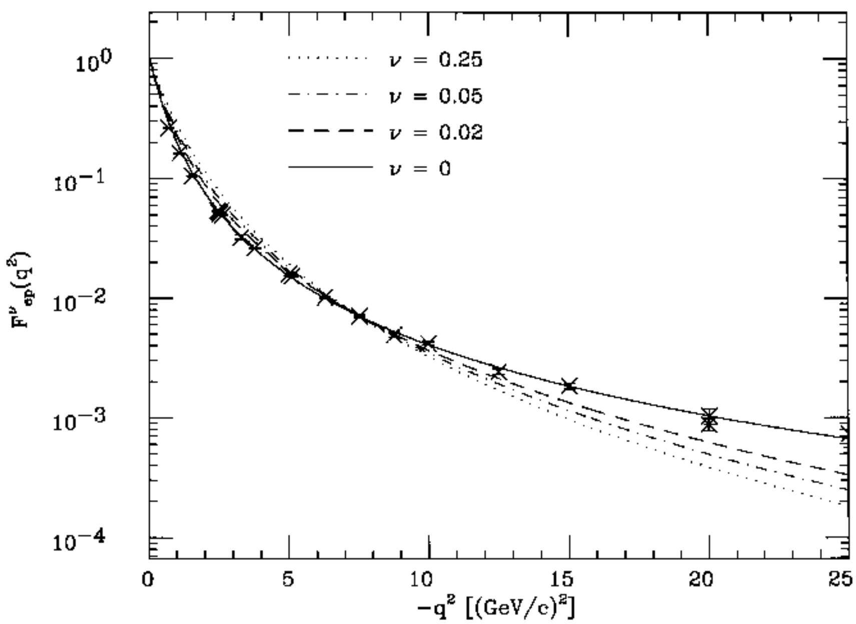

Fig. 1. A comparison of the electromagnetic form factors. The solid curve is the form factor for the original harmonic oscillator model ($\nu = 0$). The dotted curve, the dot–dash curve and the dashed curve are those of the modified harmonic oscillator model with $\nu = 0{\cdot}25$, $0{\cdot}05$ $0{\cdot}02$ respectively. The crosses indicate the experimental data from Ref. [11].

The wave function for the full Hamiltonian $\hat{H}$ satisfies the same form of the wave equation as that for the non-interacting case, eq. (3·1); $\hat{M}_I^2\ |\phi_{I,n};\mathbf{P}\rangle = M_{I,n}^2\ |\phi_{I,n};\mathbf{P}\rangle$. The wave function and the mass eigenvalues are

$$\begin{aligned}\phi_{I,n}(r,s;\mathbf{P}) = N_0 N_{n_1}{}' N_{n_2}{}' \cdots N_{n_6} \\ \times \exp\left\{\frac{\alpha}{2}\left(r_0^2 - r_3^2 + s_0^2 - s_3^2 - \frac{2}{M_n^2}(P\cdot r)^2 - \frac{2}{M_n^2}(P\cdot s)^2\right)\right\} \\ \times \exp\left\{-\frac{\alpha_\perp}{2}(\mathbf{r}_\perp^2 + \mathbf{s}_\perp^2)\right\} \\ \times H_{n_1}(\sqrt{2\alpha_\perp}\ r_1)\ H_{n_2}(\sqrt{2\alpha_\perp}\ r_2)\ H_{n_3} \\ \times \left(\sqrt{2\alpha}\sqrt{\frac{1}{M_n^2}(P\cdot r)^2 - (r_0^2 - r_3^2)}\right)\end{aligned}$$

$$\times H_{n_4}(\sqrt{2\alpha_\perp}\ s_1)\ H_{n_5}(\sqrt{2\alpha_\perp}\ s_2)\ H_{n_6}$$
$$\times \left(\sqrt{2\alpha}\sqrt{\frac{1}{M_n^2}(P\cdot s)^2-(s_0^2-s_3^2)}\right),$$

where $N_0=\sqrt{\alpha}/\sqrt{\pi},\ N_n^2=\sqrt{\alpha}/(\sqrt{\pi}n!),\ N_n'^2=\sqrt{\alpha_\perp}/(\sqrt{\pi}n!)$, and

$$\begin{aligned}M_{I,n}^2 = &-(2n_{r,0}+1)\alpha+(2(n_1+n_2)+2)\alpha_\perp+(2n_3+1)\alpha \\ &-(2n_{s,0}+1)\alpha+(2(n_4+n_5)+2)\alpha_\perp+(2n_6+1)\alpha+C\,.\end{aligned} \tag{3·8}$$

Here n_i $(i=1,\cdots,6)$ are non-negative integers, and $n_{r,0}=n_{s,0}=0$. Here also the proton is assumed to move in the z-direction.

4. Proton Survival Amplitude

Now we are in a position to calculate the survival amplitude of the struck proton in nuclear matter. For the dynamical proton it is written as

$$\begin{aligned}M_{eA}^{(D)}(q^2,t) &= \langle\phi_0;\mathbf{P}_f|e^{-i\hat{H}t}\ \hat{O}^\nu(q^2)\ |\phi_0;\mathbf{P}_i\rangle \\ &= \sum_{n=0}^{\infty}\langle\phi_0;\mathbf{P}_f|e^{-i\hat{H}t}|\phi_{I,n};\mathbf{P}_f\rangle\times\langle\phi_{I,n};\mathbf{P}_f|\ \hat{O}^\nu(q^2)\ |\phi_0;\mathbf{P}_i\rangle\,,\end{aligned} \tag{4·1}$$

where we introduce an operator $\hat{O}^\nu(q^2)\equiv e^{-i\sqrt{2}q\cdot s}\ e^{\nu q^2\ (\mathbf{r}_\perp^2+\mathbf{s}_\perp^2)}$, instead of the hard scattering operator $\hat{O}(q)$ in eq. (2·3). Here $q_\mu=P_{f,\mu}-P_{i,\mu}$. The new operator $\hat{O}^\nu(q^2)$ with finite ν introduces a longitudinal–transverse correlation into the system. The internal motion in the direction of the 3-momentum transfer $\mathbf{q}$ and that perpendicular to it decouple each other in the harmonic oscillator model. The new hard scattering operator couples both directions, and suppresses the components of the large transverse size when $-q^2$ is large. The strong ground state correlation was suggested in Ref. [10].

Since the hard scattering operator is changed, the form factor should be modified. The modified form factor becomes

$$F_{ep}^\nu(q^2)=\left(1+\frac{\nu(-q^2)}{\alpha}\right)^{-2}\ \left(1+\frac{(-q^2)}{2M_0^2}\right)^{-2}\ \exp\left\{-\frac{(-q^2)}{2\alpha}\Big/\left(1+\frac{(-q^2)}{2M_0^2}\right)\right\}. \tag{4·2}$$

We take $P_{i,\mu}=(M_0,\mathbf{P}_i)$, $\mathbf{P}_i=\mathbf{0}$ and $q_\mu=(q_0,0_\perp,q_3)$. The parameter α is determined in order to reproduce the experimental data for each fixed ν.

For the inert proton, the survival amplitude is written as

$$M_{eA}^{(I)}(q^2,t)=\langle\phi_0;\mathbf{P}_f|e^{-i(\hat{H}_0+\langle\phi_0;\mathbf{P}_f|\hat{H}_I|\phi_0;\mathbf{P}_f\rangle)t}\ \hat{O}^\nu(q^2)\ |\phi_0;\mathbf{P}_i\rangle\,. \tag{4·3}$$

Here $\hat{H}_I$ is replaced by its ground state expectation value, because the inert proton always stays in its ground state during propagation. The absorption

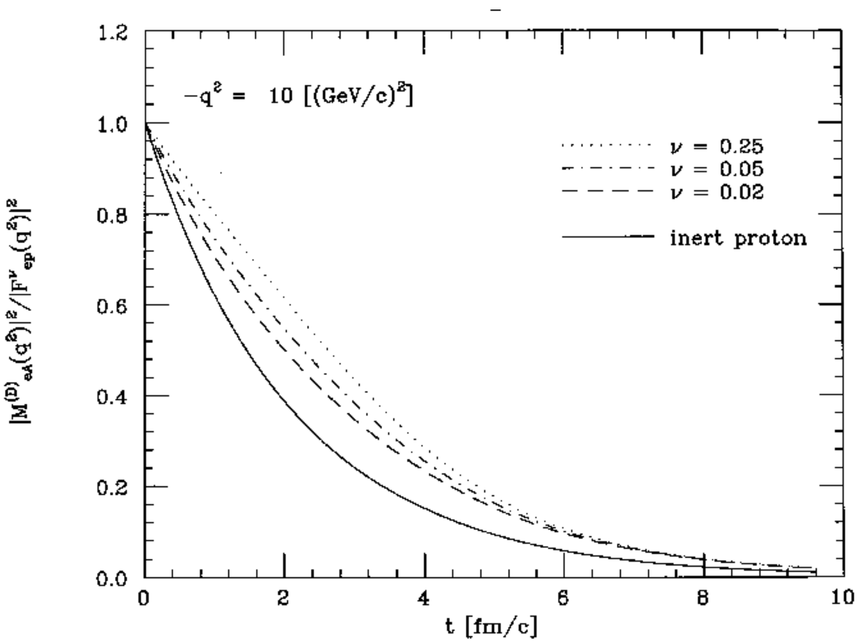

Fig. 2. Time development of the survival amplitude squared divided by the form factor squared, eq. (2·5), for $-q^2 = 10$ (GeV/c)^2. The solid curve is the case of the inert proton. The dotted curve, the dot–dash curve and the dashed curve are for the dynamical proton with $\nu = 0{\cdot}25$, $0{\cdot}05$ and $0{\cdot}02$ respectively.

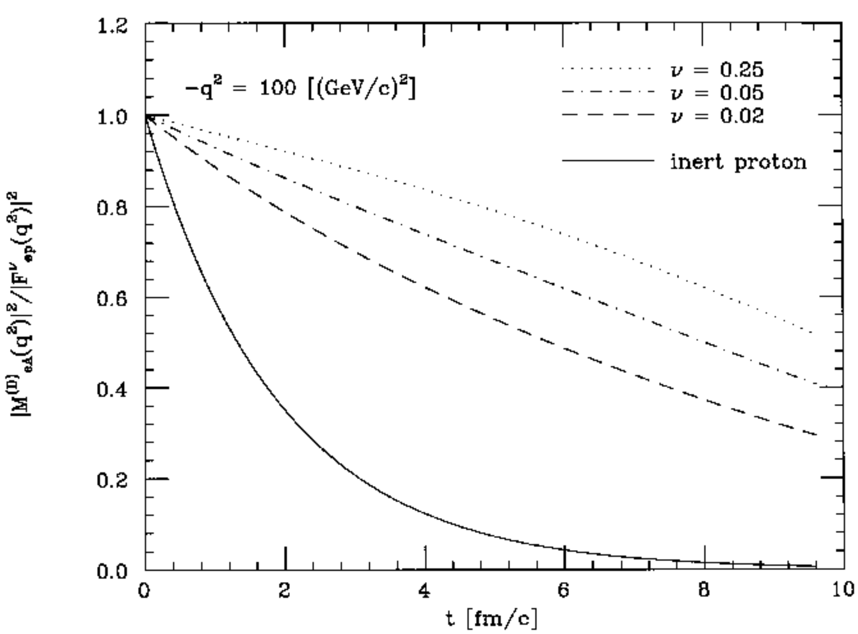

Fig. 3. Time development of the survival amplitude squared divided by the form factor squared for $-q^2 = 100$ (GeV/c)^2. The curves are the same as in Fig. 2.

strength c_0 is determined by $2c_0\langle\phi_0;\mathbf{P}_f|(\mathbf{r}_\perp^2+\mathbf{s}_\perp^2)|\phi_0;\mathbf{P}_f\rangle = v_q/\lambda$, where v_q is the velocity of the struck proton. Here $\lambda = 1/\sigma^r_{\rm NN}\,\rho_{NM}$ is the mean-free path of the proton in nuclear matter, and we get $\lambda = 2\cdot 1$ fm when we use $\sigma^r_{\rm NN} = 28$ mb, and $\rho_{NM} = 0\cdot 17\ {\rm fm}^{-3}$. The use of σ^r_{NN} is a result from GIA.

The numerical results are shown in the Figs 1–3. Fig. 1 shows a comparison of the electromagnetic form factor of the original version of the harmonic oscillator model and that of the modified version. We could determine α to reproduce the RMS radius of the proton, but here we fix α to reproduce the behaviour of the observed form factor for each fixed ν. We choose α by requiring that the form factors pass through the same selected data point. We should require that ν is small enough to maintain the power-law behaviour as much as possible.

Figs 2 and 3 show the time development of the survival amplitude squared divided by the form factor squared. By comparing the two figures, we can conclude that the colour transparency occurs for large $-q^2$ in the modified harmonic oscillator model. This is a reflection of the longitudinal–transverse correlation, and of the Lorentz covariance. Once we take a finite ν, we obtain the colour transparency.

5. Summary and Conclusion

We have applied a relativistic quark model to calculate the nuclear transparency in order to preserve the causality. Unfortunately the original version of the harmonic oscillator model does not cause the colour transparency, because the model has no longitudinal–transverse correlation, and the correlation plays a crucial role in exciting the transverse modes.

We have put the correlation into the model by hand. The modified model causes a non-exponential t-dependence of the survival amplitude, which is a signature of the colour transparency. The modification violates the counting rule for the form factor, but does not affect the form factor in the relevant region.

We will further study the nuclear transparency for finite nuclei and the target mass number dependence (A-dependence). The numerical calculations are in progress.

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