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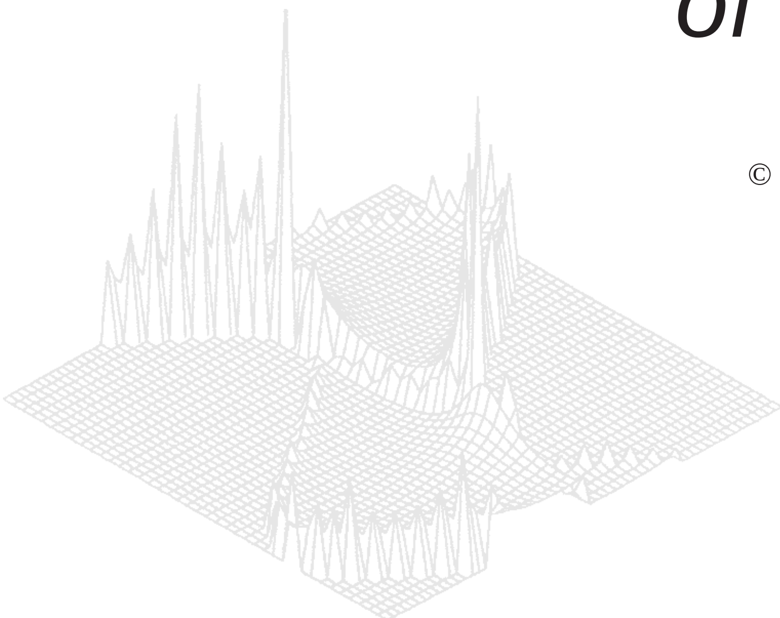

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Quark Triangle Diagrams and Radiative Meson Decays*

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Abstract

The radiative meson decays $V \to P\gamma$ and $P \to \gamma\gamma$ are analysed using the quark triangle diagram. Experimental data yield well determined estimates of the universal quark–antiquark–meson couplings $g_{Vq\bar{q}'}$ and $g_{Pq\bar{q}'}$ for the light meson sector. Also predictions for the ratios of neutral to charged heavy meson decay coupling constants are given and await experimental confirmation.

1. Introduction

Over the past few years the group at Hobart have examined the strong and electromagnetic interactions of the hadrons in the context of a supermultiplet theory which unifies the mesons and baryons of different spins and flavours. This unification is achieved in the meson sector by use of a relativistic spinor field $\Phi^{b\beta}_{a\alpha}$:

$$\Phi^{b\beta}_{a\alpha}(p) = (\not p + m)[\gamma^\mu \phi^b_{\mu a} - \gamma_5 \phi_{5a}{}^b]^\beta_\alpha/2m \qquad (\phi_\mu p^\mu = 0),$$

where a, b are flavour indices and α, β are Dirac spin indices. Such a field describes the pseudoscalar and vector meson nonets, and each member has the correct spin, parity and flavour degrees of freedom, hence the name supermultiplet. This formalism was first developed around 1965 [1], but it turns out to be a sensible approximation for the heavy flavoured hadrons, as demonstrated by the success of heavy quark symmetry over the past five years or so. To account for two–body meson decays the effective interaction Lagrangian

$$\mathcal{L}_{int} = G\Phi^B_A(p_1)[\Phi^C_B(p_2)\Phi^A_C(p_3) + (p_2 \leftrightarrow p_3)]$$

with $A = (a, \alpha), B = (b, \beta)$ and $C = (c, \gamma)$ was also proposed. Note that the supermultiplet theory need only assume a single coupling constant and therefore

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implies relations between the standard covariant couplings of mesons with different spins and flavours. For example, it predicts $2g_{VPP} = m_1 g_{VVP}$ where g_{VPP} is the covariant coupling constant corresponding to the decay of a vector meson with a mass m_1 into two pseudoscalar mesons, while g_{VVP} is the covariant coupling constant corresponding to the decay of a vector meson into a vector and pseudoscalar meson.

Table 1. Supermultiplet predictions for g_{VVP}

Decay mode	g_{VVP} $\times 10^{-2}$ MeV^{-1}
$\rho^0 \to \pi^0\gamma$	$1\cdot 216 \pm 0\cdot 156$
$\rho^0 \to \eta\gamma$	$0\cdot 984 \pm 0\cdot 094$
$\omega \to \pi^0\gamma$	$0\cdot 878 \pm 0\cdot 033$
$\omega \to \eta\gamma$	$0\cdot 919 \pm 0\cdot 165$
$\phi \to \pi^0\gamma$	$0\cdot 731 \pm 0\cdot 040$
$\phi \to \eta\gamma$	$0\cdot 603 \pm 0\cdot 020$
$\phi \to \eta'\gamma$	$< 1\cdot 69$
$K^{*\pm} \to K^{\pm}\gamma$	$0\cdot 905 \pm 0\cdot 045$
$K^{*0} \to K^0\gamma$	$0\cdot 733 \pm 0\cdot 036$
$J/\psi \to \eta_c\gamma$	$0\cdot 308 \pm 0\cdot 073$

Recent work [2] compares the supermultiplet theory with the most recent measurements using channels $\rho \to \pi\pi, K^* \to K\pi$ and $\phi \to KK$. They determined $g_{VPP} \approx 4\cdot 58$ to within 10%. If we then take an average vector meson mass of 866 MeV (obtained from ρ, ω, K^* and ϕ with equal weights) we find $2\langle g_{VPP}\rangle/\langle m_1\rangle \approx 1\cdot 057 \times 10^{-2}$ MeV^{-1}. The only direct measure of g_{VVP} comes from the channel $\phi \to \rho\pi$ which yields $g_{VVP} = 1\cdot 062 \times 10^{-2}$ MeV^{-1}. This compares very favourably with the prediction, but unfortunately there are no more channels to obtain other measurements of g_{VVP}. Instead, by incorporating vector meson dominance (VMD) in the supermultiplet scheme, $V \to P\gamma$ decays gave an estimate of g_{VVP}. The extra determinations of g_{VVP} offered from the $VP\gamma$ decays are given in Table 1 and lie in a relatively wide range with some of them significantly different from the supermultiplet prediction. One notable observation is that from a given parent vector meson, the g_{VVP} determined from channels involving that parent meson are all relatively similar *except* for $K^* \to K\gamma$ (the neutral to charged modes differ by at least 4 standard deviations for their values of g_{VVP}). We point out that there is some uncertainty in using VMD to account for these radiative decays which take place at $q^2 = 0$, the mass–shell of the photon. As we are only able to calculate VMD from $V \to \ell^+\ell^-$ which occurs at $q^2 = m_V^2$, the mass–shell of the vector meson, there is some ambiguity about the correct extrapolation from one mass–shell case to the other.

To understand the radiative processes further we sought an alternative method and found a quark triangle scheme [3] which accurately predicted the ratio $\Gamma_{K^{*0}\to K^0\gamma}/\Gamma_{K^{*\pm}\to K^{\pm}\gamma}$. We decided to apply it to all the radiative processes considered previously. We shall begin with the quark triangle method and formalism we have developed and then apply it to known radiative decays. Finally we provide some predictions in the heavy meson sector.

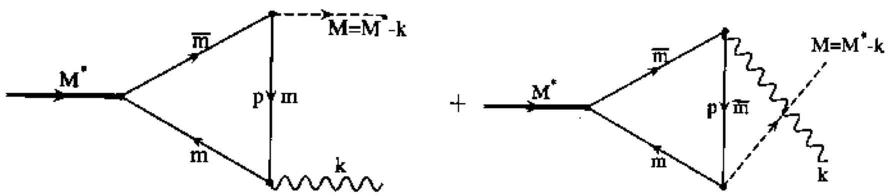

Fig. 1. Quark triangle diagrams contributing to $V \to P\gamma$.

2. Quark Triangle Diagrams

We shall examine the triangle diagram for the radiative decay of a vector meson into a pseudoscalar meson as given in Fig. 1. According to these loop diagrams, we write the decay amplitude as

$$A(V \to P\gamma) = -N_C g_{Vq\bar{q}'} g_{Pq\bar{q}'} eQ\kappa^\mu \epsilon^\nu$$

$$\times \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\gamma_\nu \frac{1}{\not{p} - m}\gamma^5 \frac{1}{\not{p} + \not{M}^* - \not{k} - \overline{m}}\gamma_\mu \frac{1}{\not{p} - \not{k} - m}\right] + (m \leftrightarrow \bar{m}, Q \leftrightarrow \bar{Q}), \tag{1}$$

where $\kappa^\mu(\epsilon^\nu)$ is the vector meson (photon) polarization vector, $M^*(M)$ is the vector (pseudoscalar) mass and $g_{Vq\bar{q}'}(g_{Pq\bar{q}'})$ is the vector (pseudoscalar) meson–quark–antiquark coupling. Here Q is the charge of the quark of mass m. We used dressed quark propagators and as a result, m and $\bar{m}$ are the constituent masses. The loop integral over the momentum p may be carried out with standard techniques. In the end we obtain

$$A(V \to P\gamma) = iN_C e g_{Vq\bar{q}'} g_{Pq\bar{q}'} \epsilon_{\mu\nu\rho\sigma} \kappa^\mu \epsilon^\nu P^\rho k^\sigma [QJ/m + \bar{Q}\bar{J}/\overline{m}]/4\pi^2\,, \tag{2}$$

with $J = m^2(J_1^* - J_1 + J_2^* - J_2)/(M^{*2} - M^2)$, where

$$J_1 = \begin{cases} \mathrm{Li}_2(v_+, 0) + \mathrm{Li}_2(v_-, 0) \text{ if } \lambda \geq 0 \\ 2\mathrm{Li}_2(R, \phi) \text{ if } \lambda < 0, \end{cases}$$

$$J_2 = \frac{m - \overline{m}}{mM^2}\left[(\overline{m}^2 - m^2 + M^2)\ln\frac{\overline{m}}{m} - \sqrt{\lambda}\mathrm{arctanh}\left(\frac{\sqrt{\lambda}}{\overline{m}^2 + m^2 - M^2}\right)\right],$$

and $v_\pm = [m^2 - \overline{m}^2 + M^2 \pm \sqrt{\lambda}]/2m^2$, $R = M/m$, $\cos\phi = (m^2 - \overline{m}^2 + M^2)/2\,Mm$.

The dilogarithm function is defined by $\mathrm{Li}_2(R, \phi) = -\int_0^R \ln(1 - 2\cos\phi\, u + u^2)/2u\, du$, and the triangle function for the meson–quark–antiquark vertex by $\lambda = [M^2 - (m + \bar{m})^2][M^2 - (m - \bar{m})^2]$. Here J_1^* and J_2^* are obtained from J_1 and J_2 by substitution of M with M^*. Also $\bar{J}$, associated with the crossed diagram, is derived from J by the interchange $m \leftrightarrow \bar{m}$. We also note that we have removed the imaginary part of the amplitude as it is irrelevant to the decay process.

If we compare the quark triangle amplitude to the general covariant form for such a decay we find $g_{VP\gamma} = N_C e g_{Vq\bar{q}'} g_{Pq\bar{q}'}(QJ/m + \bar{Q}\bar{J}/\bar{m})/4\pi^2$, and as such we have now related the couplings at the meson level, $g_{VP\gamma}$, to couplings at the quark level, $g_{Vq\bar{q}'}, g_{Pq\bar{q}'}$ and eQ.

3. Analysis of $K^* \to K\gamma$

The experimentally observed widths [4] of radiative K^* decays are $\Gamma_{K^{*0}\to K^0\gamma} = 116 \cdot 2$ keV and $\Gamma_{K^{*\pm}\to K^\pm\gamma} = 50 \cdot 3$ keV. Using $m_d = 340$ MeV and $m_s = 510$ MeV ($\approx M_\phi/2$) in the J and $\bar{J}$ above, we obtain the product of coupling constants $g_{Vd\bar{s}} g_{Pd\bar{s}} = 8 \cdot 43 \pm 0 \cdot 37$ from the neutral channel data, and with $m_u = 340$ MeV we extract $g_{Vu\bar{s}} g_{Pu\bar{s}} = 8 \cdot 21 \pm 0 \cdot 37$ from the charged channel and observe that isospin symmetry is obeyed in the meson–quark–antiquark coupling. Therefore, when taking the ratio of neutral to charged decay widths we may safely cancel $g_{Vq\bar{q}'} g_{Pq\bar{q}'}$ terms and end up with a result solely dependent on the triangle diagram. Our numerical result using the above quark masses is $\Gamma_{K^{*0}\to K^0\gamma}/\Gamma_{K^{*\pm}\to K^\pm\gamma} = 2 \cdot 29$ which is in good agreement with the data $2 \cdot 31 \pm 0 \cdot 29$ [4]. The experimental uncertainty in the ratio permits the s quark mass to range between 475 and 545 MeV for $m_{u,d} = 340$ MeV and a range of 225 to 385 MeV for $m_{u,d}$ when m_s is fixed at 510 MeV.

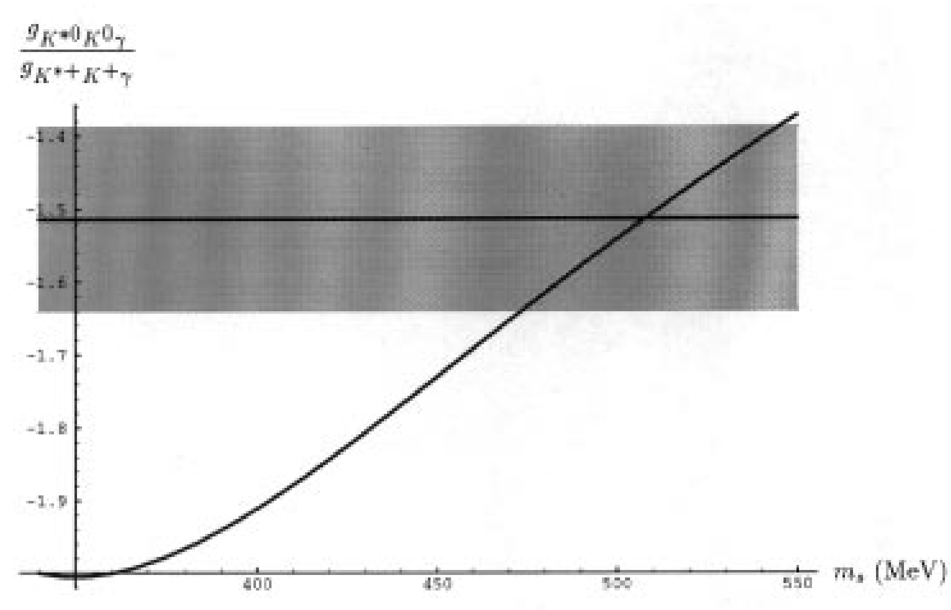

Fig. 2. Breaking of SU(3) V by s quark mass. The experimental measurement is shown as the shadow band.

Interestingly, $\mathrm{SU}(3)_V$ predicts $\Gamma_{K^{*0}\to K^0\gamma}/\Gamma_{K^{*\pm}\to K^\pm\gamma} = 4$, a result in stark contradiction to experiment. Nonetheless, the quark triangle method matches the $\mathrm{SU}(3)_V$ prediction if the s quark mass is set equal to the u and d quark masses as shown in Fig. 2. Hence the violation of constituent quark masses from

$\mathrm{SU}(3)_V$ symmetry is responsible for the large deviation in K^* radiative decays from the expected symmetry. As $m_s \approx M_\phi/2$ has produced an adequate match between theory and experiment we will continue to use this value. The quark triangle scheme is also insensitive to the K meson mass and thus J may be simplified under a chiral symmetry limit [3].

4. Unflavoured Processes

There are numerous decay channels $V \to P\gamma$ and corresponding data from which we can determine the product $g_{Vq\bar{q}'}g_{Pq\bar{q}'}$. To do this we firstly interpret individual meson–meson–photon couplings in terms of meson–quark–antiquark couplings, deriving relations between them as shown in Table 2. Following this we extract individual meson–meson–photon couplings from the most recently measured decay widths $\Gamma_{V\to P\gamma} = (m_V^2 - m_P^2)^3 g^2_{VP\gamma}/12\pi m_V^3$. The results are listed in the first column of Table 3. As shown, they scatter over a relatively wide range.

If we assume isospin symmetry there are only three unknown products of couplings involved in the light meson sector; $g_{Vu\bar{u}}g_{Pu\bar{u}}$, $g_{Vs\bar{s}}g_{Ps\bar{s}}$ and $g_{Vu\bar{s}}g_{Pu\bar{s}}$. We are able to determine $g_{Vu\bar{u}}g_{Pu\bar{u}}$ solely from any one of the processes $\rho \to \pi(\eta)\gamma$, $\omega(\phi) \to \pi^0\gamma$. In addition, the decays $\omega \to \eta\gamma$ and $\phi \to \eta\gamma$ simultaneously obtain $g_{Vu\bar{u}}g_{Pu\bar{u}}$ and $g_{Vs\bar{s}}g_{Ps\bar{s}}$. Our numerical results are shown in the second column of Table 3 where we use the same quark masses as previously along with standard mixing angles. The values of $g_{Vu\bar{u}}g_{Pu\bar{u}}$ turn out to lie in a quite small range, except for that from the $\phi \to \pi^0\gamma$ process. Nonetheless, it would fall into this range had we changed θ_V by $4\cdot6°$. Such a high sensitivity to mixing angle suggests it is reasonable to exclude $\phi \to \pi^0\gamma$ from our analysis.

Given the product $g_{Vq\bar{q}'}g_{Pq\bar{q}'}$, knowledge of either $g_{Pq\bar{q}'}$ or $g_{Vq\bar{q}'}$ would completely determine the other. We are able to calculate values of $g_{Pq\bar{q}'}$ by applying the triangle diagram to $P \to \gamma\gamma$ and hence ultimately we can also determine $g_{Vq\bar{q}'}$. To obtain the amplitude for the decay of a pseudoscalar meson into two photons we make the following substitutions in the $V \to P\gamma$ amplitude (2): $M^* \to M$, $M \to 0$, $g_{Vq\bar{q}'} \to g_{Pq\bar{q}'}$ and $g_{Pq\bar{q}'} \to eQ$ and since all pseudoscalar mesons involved in $P \to \gamma\gamma$ decays must be quark flavour singlets, $m = \overline{m}$, $Q = \bar{Q}$, $J = \bar{J}$. Thus from the decay amplitude, $\Gamma_{P\to\gamma\gamma} = M^3 g^2_{P\gamma\gamma}/64\pi$ we find $g_{P\gamma\gamma} = 2N_C g_{Pq\bar{q}'} e^2[Q^2 J/m]/4\pi^2$, where

$$J = \begin{cases} \dfrac{m^2}{M^2}[\mathrm{Li}_2(v_+, 0) + \mathrm{Li}_2(v_-, 0)] \text{ if } M \geq 2m \\[2ex] 2\dfrac{m^2}{M^2}\mathrm{Li}_2(R, \phi) \text{ if } 0 \leq M \leq 2m \end{cases}$$

and $v_\pm = M[M/m \pm (M^2/m^2 - 4)^{\frac{1}{2}}]/2m$, $R = M/m$, $\cos\phi = R/2$.

Starting with pion decay, we use the experimental value $\Gamma_{\pi^0\to\gamma\gamma} = 7\cdot95 \pm 0\cdot55$ eV [4] to calculate $g_{Pu\bar{u}} = 5\cdot14 \pm 0\cdot19$. The Goldberger–Treiman (GT) relation at the quark–level, gives us a good check of our result. For the pion, the relation reads $f_\pi g_{Pu\bar{u}}/\sqrt{2} = m_u$, and using our coupling value along with $m_u = 340$ MeV

Table 2. Radiative decays of ground state mesons and relations between covariant couplings $g_{P\gamma\gamma}$, $g_{VP\gamma}$ or $g_{PV\gamma}$ and meson–quark–antiquark couplings $g_{V_{q\bar{q}'}}$, $g_{P_{q\bar{q}'}}$

Process	Relation between covariant couplings and meson–quark couplings
$\pi^0 \to \gamma\gamma$	$g_{\pi^0\gamma\gamma} = \frac{e^2}{2\sqrt{2}\pi^2} g_{Pu\bar{u}} J_u[\pi^0]/m_u$
$\eta \to \gamma\gamma$	$g_{\eta\gamma\gamma} = \frac{e^2}{6\sqrt{6}\pi^2}\{5(\cos\theta_P - \sqrt{2}\sin\theta_P) g_{Pu\bar{u}} J_u[\eta]/m_u - \sqrt{2}(\sin\theta_P + \sqrt{2}\cos\theta_P) g_{Ps\bar{s}} J_s[\eta]/m_s\}$
$\eta\prime \to \gamma\gamma$	$g_{\eta\prime\gamma\gamma} = \frac{e^2}{6\sqrt{6}\pi^2}\{5(\sin\theta_P + \sqrt{2}\cos\theta_P) g_{Pu\bar{u}} J_u[\eta\prime]/m_u + \sqrt{2}(\cos\theta_P - \sqrt{2}\sin\theta_P) g_{Ps\bar{s}} J_s[\eta\prime]/m_s\}$
$\eta_c \to \gamma\gamma$	$g_{\eta_c\to\gamma\gamma} = \frac{2e^2}{3\pi^2} g_{Pc\bar{c}} J_c[\eta_c]/m_c$
$\rho^0 \to \pi^0\gamma$	$g_{\rho^0\to\pi^0\gamma} = \frac{e}{4\pi^2} g_{Vu\bar{u}} g_{Pu\bar{u}} J_{u,u}[\rho^0, \pi^0]/m_u$
$\rho^+ \to \pi^+\gamma$	$g_{\rho^+\to\pi^+\gamma} = \frac{e}{4\pi^2} g_{Vu\bar{d}} g_{Pu\bar{d}} J_{u,d}[\rho^+, \pi^+]/m_u$
$\rho^0 \to \eta\gamma$	$g_{\rho^0\to\eta\gamma} = \frac{\sqrt{3}e}{4\pi^2}(\cos\theta_P - \sqrt{2}\sin\theta_P) g_{Vu\bar{u}} g_{Pu\bar{u}} J_{u,u}[\rho^0, \eta]/m_u$
$\omega \to \pi^0\gamma$	$g_{\omega\to\pi^0\gamma} = \frac{\sqrt{3}e}{4\pi^2}(\sin\theta_V + \sqrt{2}\cos\theta_V) g_{Vu\bar{u}} g_{Pu\bar{u}} J_{u,u}[\omega, \pi^0]/m_u$
$\omega \to \eta\gamma$	$g_{\omega\to\eta\gamma} = \frac{e}{12\pi^2}\{(\sin\theta_V + \sqrt{2}\cos\theta_V)(\cos\theta_P - \sqrt{2}\sin\theta_P) g_{Vu\bar{u}} g_{Pu\bar{u}} J_{u,u}[\omega, \eta]/m_u + 2(\cos\theta_V - \sqrt{2}\sin\theta_V)(\sin\theta_P + \sqrt{2}\cos\theta_P) g_{Vs\bar{s}} g_{Ps\bar{s}} J_{s,s}[\omega, \eta]/m_s\}$
$\phi \to \pi^0\gamma$	$g_{\phi\to\pi^0\gamma} = \frac{\sqrt{3}e}{4\pi^2}(\cos\theta_V - \sqrt{2}\sin\theta_V) g_{Vu\bar{u}} g_{Pu\bar{u}} J_{u,u}[\phi, \pi^0]/m_u$
$\phi \to \eta\gamma$	$g_{\phi\to\eta\gamma} = \frac{e}{12\pi^2}\{(\cos\theta_V - \sqrt{2}\sin\theta_V)(\cos\theta_P - \sqrt{2}\sin\theta_P) g_{Vu\bar{u}} g_{Pu\bar{u}} J_{u,u}[\phi, \eta]/m_u - 2(\sin\theta_V + \sqrt{2}\cos\theta_V)(\sin\theta_P + \sqrt{2}\cos\theta_P) g_{Vs\bar{s}} g_{Ps\bar{s}} J_{s,s}[\phi, \eta]/m_s\}$
$K^{*0} \to K^0\gamma$	$g_{K^{*0}\to K^0\gamma} = \frac{e}{4\pi^2} g_{Vd\bar{s}} g_{Pd\bar{s}} (J_{d,s}[K^{*0}, K^0]/m_d + J_{s,d}[K^{*0}, K^0]/m_s)$
$K^{*+} \to K^+\gamma$	$g_{K^{*+}\to K^+\gamma} = \frac{e}{4\pi^2} g_{Vu\bar{s}} g_{Pu\bar{s}} (2J_{u,s}[K^{*+}, K^+]/m_u - J_{s,u}[K^{*+}, K^+]/m_s)$
$J/\psi \to \eta_c\gamma$	$g_{J/\psi\to\eta_c\gamma} = \frac{e}{\pi^2} g_{Vc\bar{c}} g_{Pc\bar{c}} J_{c,c}[J/\psi, \eta_c]/m_c$

we predict $f_\pi = 93{\cdot}5 \pm 3{\cdot}46$ MeV which compares well with the experimental result $f_\pi = 92{\cdot}4 \pm 0{\cdot}26$ MeV [4] determined from charged pion decays.

We also use $\eta \to \gamma\gamma$ and $\eta\prime \to \gamma\gamma$ channels to simultaneously determine $g_{Pu\bar{u}}$ and $g_{Ps\bar{s}}$. The results are given in Table 3. From these it appears $g_{Pu\bar{u}}$ differs as determined from π^0, η and $\eta\prime$ processes. Since the η meson is about four times as massive as the pion, it may be appropriate to allow for some mass dependency in the coupling constant. Also included in Table 3 is the estimate of $g_{Pc\bar{c}}$ using a charm quark mass of $m_c = 1550$ MeV, along with the experimentally determined width [4] of $\Gamma_{\eta_c\to\gamma\gamma} = 7{\cdot}0 \pm 2{\cdot}6$ keV.

Now that we have found values of $g_{Pu\bar{u}}$, we may use these to determine $g_{Vu\bar{u}}$ from the products $g_{Vu\bar{u}} g_{Pu\bar{u}}$ as listed in Table 3. On average we find

Table 3. Determination of meson–quark–antiquark couplings

Experimental result ($\times 10^{-4}$ MeV^{-1})[4]	Meson–quark–antiquark coupling[A] $\theta_P = -10\cdot 5°$	$\theta_P = -20°$
$\lvert g_{\pi^0\gamma\gamma}\rvert = 0\cdot 2516 \pm 0\cdot 0091$	$g_{Pu\bar{u}} = 5\cdot 14 \pm 0\cdot 19$	
$\lvert g_{\eta\gamma\gamma}\rvert = 0\cdot 239 \pm 0\cdot 011$, $\lvert g_{\eta'\gamma\gamma}\rvert = 0\cdot 312 \pm 0\cdot 016$	$g_{Pu\bar{u}} = 4\cdot 03 \pm 0\cdot 14$, $g_{Ps\bar{s}} = 6\cdot 42 \pm 0\cdot 67$	$g_{Pu\bar{u}} = 3\cdot 56 \pm 0\cdot 12$, $g_{Ps\bar{s}} = 8\cdot 19 \pm 0\cdot 65$
$\lvert g_{\eta_c\gamma\gamma}\rvert = 0\cdot 07297 \pm 0\cdot 01366$	$g_{Pc\bar{c}} = 2\cdot 03 \pm 0\cdot 38$	
$\lvert g_{\rho^0\to\pi^0\gamma}\rvert = 2\cdot 96 \pm 0\cdot 38$	$g_{Vu\bar{u}}g_{Pu\bar{u}} = 14\cdot 82 \pm 1\cdot 90$	
$\lvert g_{\rho^+\to\pi^+\gamma}\rvert = 2\cdot 24 \pm 0\cdot 13$	$g_{Vu\bar{u}}g_{Pu\bar{u}} = 11\cdot 32 \pm 0\cdot 64$	
$\lvert g_{\rho^0\to\eta\gamma}\rvert = 5\cdot 67 \pm 0\cdot 53$	$g_{Vu\bar{u}}g_{Pu\bar{u}} = 10\cdot 96 \pm 1\cdot 02$	$g_{Vu\bar{u}}g_{Pu\bar{u}} = 9\cdot 56 \pm 0\cdot 89$
$\lvert g_{\omega\to\pi^0\gamma}\rvert = 7\cdot 04 \pm 0\cdot 21$	$g_{Vu\bar{u}}g_{Pu\bar{u}} = 12\cdot 53 \pm 0\cdot 38$	
$\lvert g_{\omega\to\eta\gamma}\rvert = 1\cdot 83 \pm 0\cdot 23$, $\lvert g_{\phi\to\eta\gamma}\rvert = 2\cdot 117 \pm 0\cdot 052$	$g_{Vu\bar{u}}g_{Pu\bar{u}} = 12\cdot 3 \pm 1\cdot 5$, $g_{Vs\bar{s}}g_{Ps\bar{s}} = 7\cdot 08 \pm 0\cdot 17$	$g_{Vu\bar{u}}g_{Pu\bar{u}} = 10\cdot 7 \pm 1\cdot 3$, $g_{Vs\bar{s}}g_{Ps\bar{s}} = 8\cdot 66 \pm 0\cdot 21$
$\lvert g_{\phi\to\pi^0\gamma}\rvert = 0\cdot 417 \pm 0\cdot 021$	$g_{Vu\bar{u}}g_{Pu\bar{u}} = 25\cdot 1 \pm 1\cdot 3$[B]	
$\lvert g_{K^{*0}\to K^0\gamma}\rvert = 3\cdot 84 \pm 0\cdot 17$	$g_{Vd\bar{s}}g_{Pd\bar{s}} = 8\cdot 43 \pm 0\cdot 37$	
$\lvert g_{K^{*+}\to K^+\gamma}\rvert = 2\cdot 534 \pm 0\cdot 115$	$g_{Vu\bar{s}}g_{Pu\bar{s}} = 8\cdot 21 \pm 0\cdot 37$	
$\lvert g_{J/\psi\to\eta_c\gamma}\rvert = 1\cdot 67 \pm 0\cdot 26$	$g_{Vc\bar{c}}g_{Pc\bar{c}} = 1\cdot 87 \pm 0\cdot 30$	

[A] $m_u = m_d = 340$ MeV, $m_s = 510$ MeV, $m_c = 1550$ MeV, $\theta_V = 219\cdot 4°$.
[B] $g_{Vu\bar{u}}g_{Pu\bar{u}} = 11\cdot 9 \pm 0\cdot 6$ for $\theta_V = 224°$.

$g_{Vu\bar{u}} = 2\cdot 40 \pm 0\cdot 08$ (weighted average) for $\theta_P = -10\cdot 5°$ and $g_{Vu\bar{u}} = 2\cdot 35 \pm 0\cdot 08$ (weighted average) for $\theta_P = -20°$. It differs from $g_{Pu\bar{u}}$, revealing a substantial violation of the spin symmetry in the triangle scheme. We repeat this procedure in the analysis of $g_{Vs\bar{s}}$, but with fewer channels to determine a result. Consequently we have $g_{Vs\bar{s}} = 1\cdot 10$ for $\theta_P = -10\cdot 5°$ and $g_{Vs\bar{s}} = 1\cdot 06$ for $\theta_P = -20°$, indicating a large SU(3)$_V$ symmetry breaking once again. Estimates for $g_{Vc\bar{c}}$ using the $J/\psi \to \eta_c\gamma$ channel yield $g_{Vc\bar{c}} = 0\cdot 92 \pm 0\cdot 23$.

Since $K^0 \to \gamma\gamma$ decay is not mediated by pure electromagnetic interactions we have no means of getting $g_{Pd\bar{s}}$ for the kaon in the triangle scheme. But if we assume the GT relation at the quark level, $f_K g_{Pd\bar{s}}(m_K^2) = (m_u + m_s)/2$, and we find $g_{Pd\bar{s}} = 3\cdot 77 \pm 0\cdot 03$ using $f_K = 113\cdot 0 \pm 1\cdot 0$ MeV [4]. Subsequently, we have $g_{Vd\bar{s}} = 2\cdot 21 \pm 0\cdot 10$ (averaged over the charged and neutral processes).

5. $D^* \to D\gamma$ and $B^* \to B\gamma$ Coupling Ratios

We now make predictions for the radiative decays of the heavy flavoured mesons. We assume $g_{V(P)u\bar{Q}} = g_{V(P)d\bar{Q}}$ where Q is either the c or b quark to obtain the ratios

$$\frac{g_{D^{*0}D^0\gamma}}{g_{D^{*+}D^+\gamma}} = \frac{2(J_{u,c}[D^{*0}, D^0]/m_u + J_{c,u}[D^{*0}, D^0]/m_c)}{-J_{d,c}[D^{*+}, D^+]/m_d + 2J_{c,d}[D^{*+}, D^+]/m_c}, \tag{3}$$

$$\frac{g_{B^{*0}B^0\gamma}}{g_{B^{*+}B^+\gamma}} = \frac{J_{d,b}[B^{*0}, B^0]/m_d + J_{b,d}[B^{*0}, B^0]/m_b}{J_{u,b}[B^{*+}, B^+]/m_u - 2J_{b,u}[B^{*+}, B^+]/m_b}. \tag{4}$$

Relations (3) and (4) allow us to examine the coupling constant ratios as a function of the c and b quark mass, respectively. In order to give actual values we use a c quark mass of 1550 MeV ($\approx M_{J/\psi}/2$) yielding $g_{D^{*0}D^0\gamma}/g_{D^{*+}D^+\gamma} = 6\cdot 47$.

For the B^* decays we use a b quark mass of 4730 MeV ($\approx M_\Upsilon/2$) which gives $g_{B^{*0}B^0\gamma}/g_{B^{*+}B^+\gamma} = 0{\cdot}018$.

6. Conclusions

We have formulated $V \to P\gamma$ and $P \to \gamma\gamma$ processes in a quark triangle diagram scheme. By comparison with available experimental data, we found that this scheme works well for all radiative processes involving the light mesons, except for $\phi \to \pi^0\gamma$ decay.

The scheme produces well determined estimates of the meson–quark–antiquark couplings for the light mesons. The large difference between $g_{Vq\bar{q}'}$ and $g_{Pq\bar{q}'}$ indicates a substantial violation of spin symmetry in the quark triangle formalism. We also observed a relatively weak $\mathrm{SU}(3)_A$ chiral symmetry breaking due to the finite masses of the Goldstone–type pseudoscalar mesons, along with a more apparent $\mathrm{SU}(3)_V$ symmetry breakdown arising from the difference in light constituent quark masses.

Our prediction for $g_{D^{*0}D^0\gamma}/g_{D^{*\pm}D^\pm\gamma}$ is within range of other theoretical estimates ($1{\cdot}66 \sim 12{\cdot}9$) [5, 6] while $g_{B^{*0}B^0\gamma}/g_{B^{*\pm}B^\pm\gamma}$ is small compared with the few results in the literature (near $0{\cdot}6$) [6, 7]. However, a b quark mass of around 5000 MeV places our result near these predictions. We expect future measurements of these radiative decays will distinguish between these predictions.

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