Is there a Metallic State in Two Dimensions?*

M. Y. Simmons,A,B  A. R. Hamilton,A,B, M. Pepper,B, E. H. Linfield,B, P. D. Rose,B and D. A. Ritchie,B

A School of Physics, University of New South Wales, Sydney, NSW 2052, Australia.
email: Michelle.Simmons@unsw.edu.au
B Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 OHE, UK.

Abstract
This paper reviews a series of experimental results on the metallic behaviour recently discovered in high quality, two-dimensional (2D) GaAs hole transistors. In particular, we address the question of what has happened to the two quantum corrections to the resistivity due to weak localisation and weak electron–electron interactions in the so-called metallic state. Detailed magnetoresistance data are presented just on the metallic side of the apparent metal–insulator transition, which show that both weak localisation (observed via negative magnetoresistance) and weak hole–hole interactions (giving a correction to the Hall constant) are present in the ‘metallic’ phase. The results suggest that as $T \to 0$ the resistivity will stop decreasing but turn up and tend towards infinity, in agreement with the early predictions of the one parameter scaling theory of localisation. The implication is that, even at high $r_s$, there is no metallic phase at $T=0$ in two dimensions. Other unexplained features of the anomalous ‘metallic’ state are also discussed, such as the destruction of metallic behaviour by a parallel magnetic field.

1. Introduction
The discovery of a metallic state of matter in exceptionally high quality two-dimensional (2D) silicon transistors (Kravchenko et al. 1994), and more recently in high quality GaAs hole transistors (Simmons et al. 1997, 1998; Hanein et al. 1998), has sparked intense international interest into the method of electrical conduction in semiconductor systems. Early, well-established theoretical works, known as the one parameter scaling theory of localisation (Abrahams et al. 1979; Gor’kov et al. 1979) predicted that there could be no metallic state in 2D systems. In these theories they proposed that if an electron in a 2D plane travelling in the y-direction hit an impurity it had a high probability of back-scattering with the same phase, i.e. returning in the direction it came, thereby causing an increase in the resistivity. Even if two-dimensional electron systems were conducting at high temperatures, as the temperature was decreased this phase coherent scattering would take over giving a logarithmically diverging increase in the resistance. It was also shown that weak electron–electron interactions between the conducting particles would give rise to a similar logarithmic correction to the resistivity (Altshuler et al. 1980). This theory was extensively supported by experimental evidence (Uren et al. 1980; Bishop et al. 1980) and it was generally believed that the presence of any amount of disorder within a 2D system would give rise to these logarithmic corrections. The unexpected discovery of metallic

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behaviour in high quality two-dimensional systems has therefore challenged our basic understanding of conduction in highly pure quantum transistors. Whilst many recent theories have been proposed to explain this effect, at present there remains no consensus as to the nature of the newly discovered metallic state.

It is important to be clear about the difference between a metal and an insulator. The distinction is only properly made at the absolute zero of temperature, since thermal excitations can permit an insulator to carry a current. If the resistance is finite at $T = 0$ then the system is metallic, otherwise it is insulating. Obviously it is impossible, experimentally, to reach the absolute zero of temperature and therefore very difficult to prove that a sample has a finite resistance at $T = 0$. How then is it possible that experimentalists claim that they observe a metallic state in 2D systems?

In 1994 Kravchenko et al. presented evidence that a metal–insulator transition existed at very low electron densities in Si MOSFET samples that had a much higher mobility ($35,000 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$) than previous experimenters had used. In this work they measured the resistivity of the electron gas as a function of temperature for several different electron densities. The results reflected two regimes—a regime of low electron density where the resistivity tends to infinity as the temperature is reduced (characteristic of insulating behaviour), and a regime of higher electron density where the resistivity tends to a finite value as $T \to 0$ (characteristic of metallic behaviour). Using a single scaling parameter it was possible to collapse all the resistivity versus temperature data for different carrier densities onto two separate curves, one in the insulating regime and the other in the metallic regime. The collapse of all the data onto two separate branches has been taken as evidence that this is a true quantum phase transition between 2D insulating and metallic states. Furthermore, the ability to scale the data over several orders of magnitude of temperature in the metallic branch showed that as $T \to 0$ the resistivity remains finite, consistent with the definition of a metallic state, but in dramatic contrast to the early theoretical predictions (Abrahams et al. 1979; Gor’kov et al. 1979; Altshuler et al. 1980).

Until recently the metal–insulator transition was only observed in silicon based transistors (Kravchenko et al. 1994; Simonian et al. 1997a; Popovic et al. 1997; Coleridge et al. 1997) which raised concern that somehow the effect was an artefact of the silicon material system. The first observation of a metal–insulator transition in high mobility 2D hole gases in GaAs/AlGaAs heterostructures (Simmons et al. 1997) confirmed that the metallic state is a universal property of high mobility 2D systems.

In all reports of the forbidden metal–insulator transition there are two common factors: firstly, the mobility is high and, secondly, the transition occurs at low carrier densities. This suggests that the novel metallic phase is driven by strong interactions between the electrons, the strength of which is characterised by the parameter $r_s$, defined as the interparticle spacing in units of the Bohr radius. As the carrier density is reduced, $r_s$ increases, until at $r_s > 5$ the interactions between electrons becomes sufficiently strong that the metallic phase forms. It is the advances in semiconductor crystal growth quality in the last few years that have made it possible to achieve transistors with such high values of $r_s$, where the levels of disorder are low enough that electron–electron interactions (not included in original scaling arguments) become important. Further evidence that this is no ordinary metal comes from the observation that it is destroyed by the application of a magnetic field parallel to the 2D plane.

This paper will review recent experimental results on the metallic behaviour in high quality GaAs hole systems. Firstly, we look at what has happened to phase coherent back scattering (weak localisation) and electron–electron interactions in the so-called metallic
state. We then look at the unexplained effect that a parallel magnetic field has on the metallic behaviour. Our results show that despite strong electron–electron interactions, quantum corrections to the conductivity are still present in the metallic phase. The consequences of this observation to the existence of a metallic state and hence a true quantum phase transition will be discussed.

2. Experimental Details

The sample used here was grown by MBE on a (311)A GaAs substrate and consisted of a 200 Å GaAs quantum well, modulation doped on one side with Si as the acceptor. The carrier density \( p_s \) was varied with a p\(^+\) back-gate, formed using a combination of in-situ ion-implantation and MBE regrowth, 360 nm below the quantum well. Four terminal magnetoresistance measurements were performed at temperatures down to 100 mK using low frequency (4 Hz) ac lock-in techniques and currents of 0.1–5 nA to avoid electron heating. The hole density could be varied in the range \( 0 – 3.5 \times 10^{11} \text{ cm}^{-2} \), with a peak mobility of \( 2.5 \times 10^5 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1} \). Only the heavy hole (\( M_J = \frac{1}{2} \)) subband is occupied, although there is some mixing between light and heavy hole bands for \( |k| > 0 \). The large effective mass (\( m^* \approx 0.3 m_e \)) quenches the kinetic energy thereby enhancing the effects of Coulomb interactions. It should also be noted that the asymmetric confining potential in our samples leads to a partial lifting of the twofold Kramer (spin) degeneracy away from \( k || = 0 \).

Fig. 1. (a) Temperature dependence of the resistivity at densities from \( p_s = 3.2–5.6 \times 10^{10} \text{ cm}^{-2} \). (b) The same data with each trace scaled along the x-axis to collapse onto metallic and insulating branches.

3. Evidence for Metallic-like Behaviour in Two-dimensional Hole Systems

Initially we demonstrate that the high quality hole sample that we are studying exhibits all the characteristics that were originally taken as evidence for the 2D metal–insulator quantum phase transition. Fig. 1a shows the temperature dependence of the resistivity, in zero magnetic field, plotted for carrier densities close to the transition, from \( p_s = 3.2 – 5.6 \times 10^{10} \text{ cm}^{-2} \). At the lowest densities, strongly localised behaviour is observed with resistivity taking the familiar form for variable range hopping: \( \rho(T) = \rho_{\text{VRH}} \exp[(T/T_{\text{VRH}})^{-m}] \). Analysis of the data shows that \( m = \frac{1}{3} \) far from the transition and \( m = \frac{1}{2} \) close to the transition. At a critical density \( p_c = 4.6 \times 10^{10} \text{ cm}^{-2} (r_s = 12) \) the resistivity is almost temperature independent marking the transition region from insulating to metallic behaviour. Above this critical density the resistivity drops markedly as the temperature is reduced, although this drop is somewhat masked by the logarithmic axis of Fig. 1a.
The metallic behaviour can be seen more clearly in the ‘scaled’ data shown in Fig. 1b. Here each \( \rho(T) \) trace was individually scaled along the \( T \)-axis in order to collapse all the data onto one of two separate branches. These data clearly separate into two regimes. In the insulating branch the resistivity tends towards infinity as the temperature is reduced, whereas in the metallic branch the resistivity tends towards a finite value as \( T \to 0 \). It is this ability to scale the data both in the strongly localised and metallic branches that has been taken as evidence for a quantum phase transition between insulating and metallic states in a 2D system (Kravchenko et al. 1994). However, this ‘scaling’ procedure does not explain what has happened to the quantum corrections to the resistivity which should take over as \( T \to 0 \).

4. Observation of Weak Localisation Corrections

Early studies of weakly interacting, disordered 2D systems (\( r_s \sim 4 \)) (Uren et al. 1980) demonstrated that both weak localisation and weak electron–electron interactions caused a logarithmic reduction of the conductivity as \( T \to 0 \). More recently it has been shown that the same interaction effects occur in slightly less disordered samples (\( r_s \sim 6 \)) that exhibit

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Fig. 2. The left-hand panels show resistivity at \( B = 0 \) versus temperature data, illustrating the transition from insulating to metallic behaviour as the density increases. The right-hand panels show the corresponding magnetoresistance traces for temperatures of 147, 200, 303, 510, 705 and 910 mK.
‘metallic behaviour’, at high carrier densities, far from the metal–insulator transition (Hamilton et al. 1999). We now turn to look at what has happened to these logarithmic corrections at lower densities near the metal–insulator transition, and whether they still exist in the metallic regime.

Fig. 2 shows a detailed study of the temperature dependence of the $\mathcal{E} = 0$ resistivity (left-hand panel) and magnetoresistance (right-hand panel) at different densities on both sides of the ‘metal’–insulator transition. In Fig. 2a we are just on the insulating side of the transition. The left-hand panel shows that $\rho(T)$ is essentially $T$-independent down to 300 mK and then increases by 2.5% as the temperature is further reduced. This weak increase in the resistivity has been previously taken as evidence for weak localisation and weak electron–electron interaction effects (Hamilton et al. 1999; Pudalov et al. 1998). It is, however, not possible to determine the precise origins of this weak increase in resistivity solely from the $\mathcal{E} = 0$ data, and we therefore look at the magnetoresistance shown in the right-hand panel of Fig. 2a.

A characteristic signature of weak localisation is a strong temperature dependent negative magnetoresistance. At high temperatures, where the phase coherence length is short the probability of phase coherent backscattering is small and a very weak increase in the resistivity is observed around $\mathcal{E} = 0$. As we reduce the temperature the phase coherence length increases and the probability of phase coherent backscattering increases. This is reflected by the increase in the resistivity around $\mathcal{E} = 0$. The application of a perpendicular magnetic field breaks time reversal symmetry, removing the phase coherent backscattering and causing the resistance to decrease as we move away from $\mathcal{E} = 0$. As observed previously there is no evidence of weak localisation for temperatures down to 300 mK in these high quality samples (Simmons et al. 1998). However, as $T$ is lowered below 300 mK a strong negative magnetoresistance peak develops as phase coherent effects become important, mirroring the small increase in the resistivity at $\mathcal{E} = 0$.

Increasing the carrier density brings us into the metallic regime (Fig. 2b) where the exponential drop in the resistivity with decreasing temperature starts to become visible. The upturn in $\rho(T)$ marked by the arrow has moved to lower temperatures and the negative

![Figure 3](image-url)

**Fig. 3.** Fitting of the magnetoconductivity $\sigma_{xx}$ data using equation (1) at a density of $4.7 \times 10^{10}$ cm$^{-2}$ (just on the metallic side of the transition) for temperatures of 147, 200, 303, 510 and 705 mK.
magnetoresistance in the right-hand panel has become less pronounced. Further increasing the density (Fig. 2c) causes the metallic behaviour to become stronger, with the upturn in \(\rho(T)\) moving to even lower temperatures, until at \(p_x = 5 \times 10^{10} \text{ cm}^{-2}\) the upturn is no longer visible within the accessible temperature measurement range. However, the magnetoresistance still exhibits remnants of the weak localisation temperature dependent peak at \(B = 0\). The weak localisation is therefore always present and is neither destroyed in the metallic regime, nor is it ‘swamped’ by the exponential decrease in resistivity with decreasing temperature. Instead, what can clearly be seen in the left-hand panel of Fig. 3 is that the upturn in \(\rho(T)\) due to weak localisation (marked by the arrows) moves to lower \(T\) as the carrier density is increased. This is not surprising since, as we move further into the metallic regime, both the conductivity and therefore the mean free path increase (\(l \propto \sigma/\sqrt{\rho_x}\)), such that the weak localisation corrections are only visible at lower temperatures (larger \(l_0\)).

In order to observe what happens to the phase relaxation time as we cross from insulating to metallic behaviour, we transpose the data to obtain \(\sigma_{xx}\) and \(\rho_{xy}\). We can then fit the temperature dependent magnetoconductance data to the following formula to extract the phase relaxation time (Hikami et al. 1980):

\[
\Delta \sigma(B) = -\frac{e^2}{\pi \hbar} \left[ \Psi \left( \frac{1}{2} + \frac{B_0}{B} \right) - \Psi \left( \frac{1}{2} + \frac{B_1}{B} \right) \right],
\]

where \(\Psi(x)\) is the digamma function, and \(B_0\) and \(B_1\) are characteristic magnetic fields related to the transport scattering rate and the phase relaxation rate. Using equation (1) we fit the experimental data just on the metallic side of the transition (\(\rho_x = 4.7 \times 10^{10} \text{ cm}^{-2}\)) for different temperatures as shown in Fig. 3. These fits are in excellent agreement with the experimental data, and from this it is possible to extract the fitting parameter \(B_0\) and thus the phase relaxation time \(\tau_0\). Repeating this fitting procedure for all different carrier densities and temperatures allows us to observe the change in the phase breaking rate around the transition.

Fig. 4 shows the temperature dependence of the phase breaking rate \(1/\tau_0\) for three different densities on both sides of the ‘metal’–insulator transition. The phase breaking rate falls approximately linearly with decreasing temperature in each case. The measured phase breaking rate is nearly five times larger than that expected theoretically from the Nyquist noise (Eiler 1984). However, we note that a similar, unexplained enhancement of the phase breaking rate has been observed in other studies of both 2D electron (Taboryski et al. 1990) and hole (Pedersen et al. 1999) systems. The linear dependence agrees well with that predicted by Altshuler et al. (1980), where \(1/\tau_0 \sim 2k_BT/hk_fl\). The phase breaking mechanism therefore only depends on \(k_fl\) and not on the carrier density, mobility or interaction strength. It is therefore instructive to compare the results obtained in these low disorder, low density GaAs hole systems (r_s \(\sim 10\)) with earlier data from disordered high density electrons in silicon MOSFETs (Davies et al. 1983). The phase breaking rates obtained in these low density p-GaAs samples, with 2.5 < \(k_fl\) < 5, are almost identical to those found in n-type silicon MOSFETs with \(k_fl\) \(\sim 1\), despite a factor of 20 difference in the carrier densities (see the data in Fig. 4). This agreement with scattering limited electron lifetime suggests that the hole states are only mildly perturbed by the strong interactions, and the 2D hole system essentially remains Fermi liquid-like.

Another important feature of these results is that there is little variation in \(\tau_0\) with density and in particular there is no dramatic change in \(\tau_0\) as we cross from insulating
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$\rho(\tau) = 4.5 \times 10^{10} \text{ cm}^{-2}$ to metallic behaviour ($\rho_T = 5.2 \times 10^{10} \text{ cm}^{-2}$). There is therefore no reflection of the exponential decrease of $\rho(T)$ with decreasing temperature in the phase breaking rate. This implies that whatever mechanism is causing the metallic behaviour does not suppress weak localisation as originally believed, and is further evidence that the system is behaving as a Fermi liquid. Since all models of the resistivity in the metallic phase (Pudalov et al. 1997; Mills et al. 1999) predict that the exponential drop saturates at low temperatures, our data predicts that localisation effects will again take over as $T \to 0$.

5. Role of Electron–Electron (Hole–Hole) Interactions in the Metallic Phase

We now address the role of electron–electron (hole–hole) interactions in the 2D metallic phase. Unlike weak localisation, interactions not only affect the $B = 0$ resistivity, but also cause a correction to the Hall resistance $R_H$ given by

$$\frac{\Delta R_H}{R_H} = -2\frac{\Delta \sigma_T}{\sigma}. \tag{2}$$

By measuring the low field Hall effect it is thus possible to distinguish between weak localisation and interaction effects (Uren et al. 1980). Fig. 5a shows the Hall resistivity $\rho_{xy}$ measured on the metallic side of the transition (i.e. where the zero field resistivity shows an exponential drop with decreasing temperature as shown in Fig. 2c).

The data reveal a small, but significant decrease of the Hall slope with increasing temperature. Whilst a series of different temperatures traces from 100–700 mK were taken only three of these traces are presented for clarity. From these data we can plot the change in the Hall slope with magnetic field, shown in Fig. 5b. This clearly shows that the Hall slope varies as $\log T$. We can then extract the interaction correction to the zero field conductivity $\Delta \sigma_T$ from the temperature dependent Hall data using equation (2). Fig. 6 shows a plot of the correction to the conductivity due to interactions as a function of
temperature for several different carrier densities on both sides of the transition. Remarkably, the data for the different densities collapse onto a single line, clearly demonstrating a log\(T\) dependence of \(\Delta\sigma_I\), which will act to reduce the conductivity to zero as \(T \to 0\).

Logarithmic corrections to the Hall resistivity have previously been observed in studies of interaction effects in high density electron systems (Uren et al. 1980), where interaction effects are weak. It is perhaps therefore surprising that results observed in, and derived from, weakly interacting systems apply to our system where interactions are known to be strong \((r_s > 10)\). Nevertheless, we find reasonable agreement (within a factor of 2) between the magnitude of the logarithmic corrections due to interactions in our system and those predicted by Altshuler et al. (1980). As with the phase coherent effects this logarithmic correction due to hole–hole interactions is independent of whether we are in the

![Fig. 5.](image1)

(a) Hall resistivity for different temperatures at a density of \(4.7 \times 10^{10}\) cm\(^{-2}\). (b) Logarithmic correction to the Hall resistivity as a function of temperature at the same density.

![Fig. 6.](image2)

Fig. 6. Logarithmic correction to the conductivity as a function of temperature for densities close to the transition.
insulating or metallic phase and is present despite the exponential drop in resistivity. This strongly suggests that electron–electron interactions are not responsible for the 2D ‘metal’–insulator transition observed in high mobility (low $E_F$) systems (Simmons et al. 2000).

6. Destruction of Metallic Behaviour in a Parallel Magnetic Field

If the metallic behaviour is not due to phase coherence effects or interaction effects, what then is the origin of the widely observed metallic-like drop in resistance in 2D systems? In order to gain further insight we examine the evolution of the resistivity in a parallel magnetic field. This reveals another unexplained feature of the so-called metallic state. The application of a parallel magnetic field $B_\parallel$ couples directly to the spin, altering many-body interactions by introducing a spin-splitting of the ‘spin-up’ and ‘spin-down’ particles. Although the in-plane factor $g_{\|}$ is zero for purely heavy hole states, mixing between the light and heavy hole bands at non-zero $k_{\|=}$ leads to a finite $g_{\|}$. We have therefore measured the four terminal resistivity $\rho = 1/\sigma$ as a function of $B_\parallel$, and observe a negative magnetoconductance for all carrier densities on both sides of the transition (Fig. 7). Whilst the decrease may appear small, the conductivity is on a log axis, such that for a carrier density of $p_s = 6.4 \times 10^{10}$ cm$^{-2}$ the conductivity changes from $0.9 e^2/h$ at zero magnetic field to $0.23 e^2/h$ at 10 T.

In a system with weak electron–electron interactions and strong spin–orbit scattering the destruction of metallic behaviour by the application of a parallel magnetic field can occur as the spin degeneracy is lifted and a transition from weak antilocalisation to weak localisation occurs. However, in Section 4 we have shown that there is no evidence for weak antilocalisation. The destruction of metallic behaviour in a parallel field both in Si MOSFETs (Simonian et al. 1997b; Pudalov et al. 1997) and in our samples does however point to a spin related origin of the metallic behaviour. More work is needed to ascertain the true nature of the metallic behaviour and why it should be destroyed by a parallel magnetic field.

7. Summary

In summary we have presented a comprehensive study of localisation, interaction effects and parallel field studies of a high mobility two-dimensional hole gas sample that shows all the signatures of a $B = 0$ ‘metal’–insulator transition. The results reveal that neither phase coherent effects nor electron–electron interactions are responsible for the apparent
2D metal–insulator transition. Both of these effects are present in the metallic regime and both give rise to localising corrections to the conductivity at low temperatures. Instead, these results strongly suggest that the metallic behaviour is a finite temperature effect, and that as $T \to 0$ the old results of the one parameter scaling theory of localisation and weak electron–electron interactions remain valid—there is no genuine 2D metallic phase. However, the physical reasons behind the anomalous metallic behaviour remain uncertain. There is still a long way to go before we can answer why it appears stronger in some material systems than others and why it is destroyed by the application of a parallel magnetic field.

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