ON THE SECULAR VARIATION OF RAINFALL AT ADELAIDE

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Summary

A detailed analysis of the rainfall of Adelaide has established that periodic changes occur in the incidence and duration of the winter rains. These changes have a period and amplitude of approximately 23 years and 30 days respectively, and superimposed on them is a long-term trend which is manifested by protraction of the latter half of the season, spring rains now occurring about 3 weeks later than they did just over 100 years ago. The total quantity of rain precipitated has shown no statistically significant changes.

I. Introduction

The daily rainfall observations recorded at Adelaide during the 95 years 1839–1933 inclusive have been subjected to detailed analysis, and the results reported (Cornish 1936). This investigation pointed to the occurrence of periodic changes in the incidence and duration of the winter rains, with a period of approximately 23 years and amplitude of 30 days, but no statistically significant changes were demonstrable in the amount of rain precipitated. The periodic trend was very clearly defined from 1839 to about 1912, but thereafter the observations showed some evidence of a breakdown in the law of change. With the accumulation of further data, the opportunity has been taken to re-examine this point and to revise the analysis.

II. Data

On January 1, 1839, Sir George Kingston established a daily rainfall record at Adelaide on a site approximately 500 yd from the present position of the Observatory. This record was continued until November 1879. From May 1860, readings have been taken at the Observatory, so that over 19 years the two sets of observations were concurrent. During this interval, the average annual difference between the gauges was only 0·26 in. (Kingston's being the greater), and, considering the close proximity of the sites, it may be assumed that the two series in combination give a continuous and practically uniform record of the Adelaide rainfall. No definite statement could be found regarding the diameter of the gauge employed by Kingston, nor, in fact, the size of the gauge used in the early days of the existence of the Observatory. It is fairly certain, however, that no radical departure could have been made from the standard 8-in. gauge which has been in use since 1870.

The analysis previously reported has now been extended to include the rainfall observations of 1934–1950 inclusive.

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III. Analysis

The basic data were obtained by a method devised by Fisher (1924). The rainfall of each year was divided into 61 six-day totals, and to each of these annual sequences a series of orthogonal polynomial functions of the fifth degree in time was fitted, thus furnishing six quantities with which the amount and distribution of rain in each year could be represented. These distribution constants, designated a', b', . . ., f', have been tabled (Cornish loc. cit.) for the years 1839–1933 inclusive, and supplementary data for the period 1934–1950

Table 1

RAINFALL DISTRIBUTION VALUES FOR 1934–1950

Unit: 10^{-3} in.

Year	a'	b'	c'	d'	e'	f'
1934	332	+96	—27	—47	—16	+ 3
1935	384	+23	—64	— 6	+17	— 5
1936	317	+44	— 1	+ 1	+31	—11
1937	378	+15	—30	— 7	+18	+ 1
1938	316	—46	—49	+26	+ 5	— 1
1939	381	— 6	—48	— 9	— 1	—12
1940	265	+ 9	—23	— 2	+16	—16
1941	370	—13	—26	—42	— 1	+27
1942	417	+ 6	—94	— 9	+27	— 9
1943	291	—27	—27	— 5	— 1	+13
1944	281	+20	—18	+33	+ 2	—18
1945	293	+64	— 9	—26	— 4	— 2
1946	370	—23	—11	+17	+10	+13
1947	359	+41	—47	— 3	+ 6	+16
1948	351	+33	—51	+15	—14	—23
1949	299	+43	—29	—29	—27	+ 1
1950	263	+29	—52	—10	+11	— 9
Mean 1839–1950	345·35	+14·80	—56·07	— 2·72	+15·67	— 3·28

inclusive are given in Table 1. For convenience of presentation in tabular form, the unit is 10^{-3} in. These quantities are actually proportional to the coefficients of the polynomial terms, and to obtain the latter they must be divided by factors of the form

$$\{(r!)^2 60.59 \ldots (61-r)\}/(2r+1)!,$$

where r is the degree of the term fitted.

The first constant a' represents the average rainfall in 10^{-3} in. per 6-day period, b' is proportional to the linear term of the seasonal sequence, c' is proportional to the parabolic term, and so on, taking more complex features of the seasonal distribution into account.

In Figures 1, 2, and 3, the courses of the secular changes in a', b', and c' respectively have been depicted by plotting running 10-year means of each.

The diagram of a' shows quite marked changes in the amount of rain precipitated, with two short periods of high rainfall centred at 1850 and 1920, when the mean annual rainfall was approximately 24 in., separated by a long interval in which the mean fell to about 20 in.

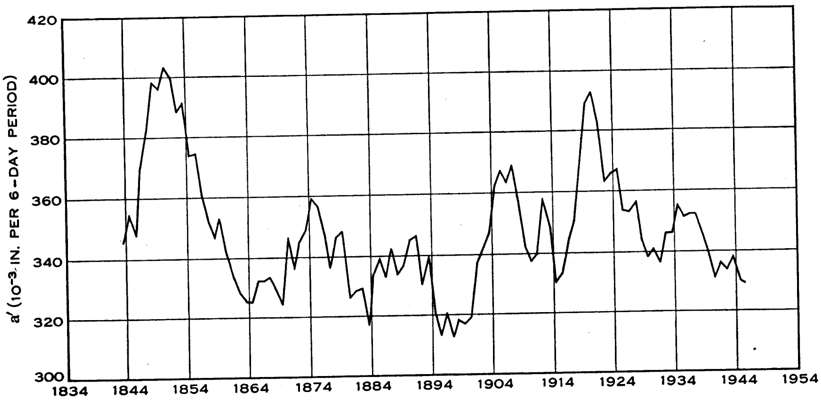

Fig. 1.—Ten-year means of the distribution value a'.

Changes in the course of b' are also prominent, and even more complex than those observed with a', the diagram showing clearly that the swing in the mean is again becoming regular, following the departure observed between 1910 and 1930. As will appear later, the effects of this disturbance are not so serious as

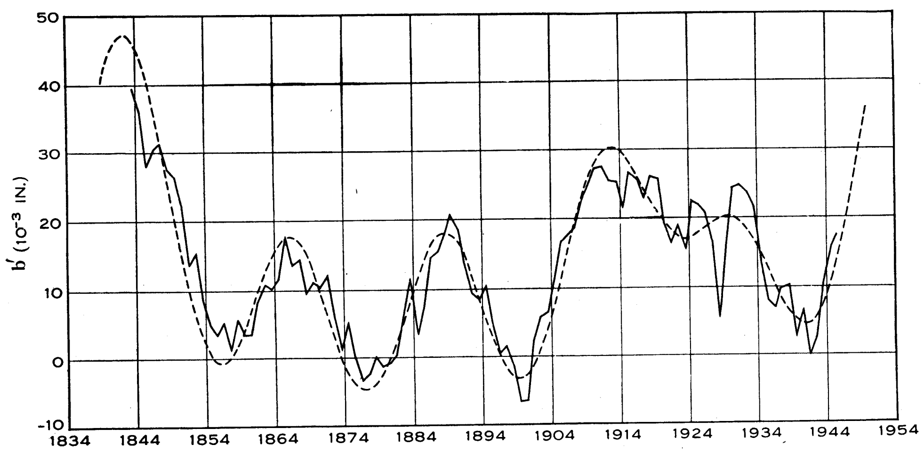

Fig. 2.—Ten-year means and harmonic curve of the distribution value b'.

the figure might indicate. By comparison, c' follows a simple course with a distinct and apparently uniform increase, the mean having increased from -67 in 1839 to -45 in 1950 (both values approximate).

The secular changes observed in the six distribution constants were then examined in more detail by fitting each series of 112 values with an orthogonal polynomial of the fifth degree in time, and the significance of the polynomials re-tested on all the available data. The procedure adopted in this step may be illustrated by the calculations relating to c'. The following values were obtained, the unit being 10^{-3} in.:

Mean		$-56\cdot07143$	x'_1	$-593\cdot40$	$(x'_1)^2$	$352{,}123\cdot56$
Coefficient of 1st degree		$3\cdot67983$	x'_2	$68\cdot06$	$(x'_2)^2$	$4{,}632\cdot16$
,, ,, 2nd ,,		$1\cdot76860$	x'_3	$42\cdot99$	$(x'_3)^2$	$1{,}848\cdot14$
,, ,, 3rd ,,		$0\cdot25608$	x'_4	$7\cdot56$	$(x'_4)^2$	$57\cdot15$
,, ,, 4th ,,		$1\cdot03557$	x'_5	$35\cdot95$	$(x'_5)^2$	$1{,}292\cdot40$
,, ,, 5th ,,		$-1\cdot07097$	x'_6	$-42\cdot98$	$(x'_6)^2$	$1{,}847\cdot28$

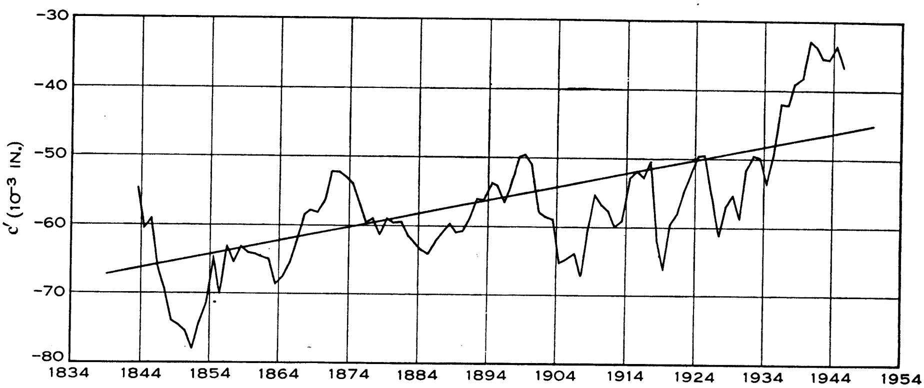

Fig. 3.—Ten-year means and linear regression of the distribution value c'.

The quantities $x'_1, \ldots, x'_6$ are orthogonal and normal linear functions of the 112 values of c', obtained from the corresponding quantities in the first column by multiplying the coefficient of the term of degree r by

$$\left\{\frac{(2r+1).112.113.\ldots(112+r)}{111.110.\ldots(112-r)}\right\}^{\frac{1}{2}},$$

and $x'_2, \ldots, x'_6$ represent the several components of secular change in the c' sequence. The last column gives the squares of these functions. The total variation of the c' values from their mean may be divided into two portions:

(1) a sum of squares associated with regression on time, and due to a comparatively simple temporal trend predominating over the random fluctuations,

(2) the remaining sum of squares which may be attributed to random annual variation,

and the first of these components may be partitioned further to give the individual contributions of the terms of the several degrees to the total for regression. The analysis of variance given in Table 2 establishes the strong significance ($P<0\cdot01$) of the linear term, thus confirming the feature noted in Figure 3.

Table 3 provides a summary analysis of all six distribution constants, the unit being 10^{-3} in.

No significant changes have occurred in a', d', e', and f', which thus merely fluctuate with large standard deviations about their respective mean values. This confirms the previous findings with respect to a', d', and e'. With f', however, the original analysis indicated an apparently significant parabolic

TABLE 2

ANALYSIS OF VARIANCE OF THE c' SERIES

Variation Due to	Degrees of Freedom	Sum of Squares	Mean Square	Variance Ratio
Linear component	1	4,632·16	4,632·16	6·94
Quadratic „	1	1,848·14	1,848·14	n.s.
Cubic „	1	57·15	57·15	n.s.
Quartic „	1	1,292·40	1,292·40	n.s.
Quintic „	1	1,847·28	1,847·28	n.s.
Deviations from regression . .	106	70,714·30	667·12	
Total	111	80,391·43		

component, but this result had been treated with caution since the f' series had shown a considerable departure from normality. It is of some interest to see that, with more extensive data, this term now degenerates to non-significance.

The significance of the linear term in the course of c' is a new feature, and the remaining significant effects, namely, the quadratic and cubic components

TABLE 3

SUMMARY POLYNOMIAL ANALYSIS OF RAINFALL DISTRIBUTION VALUES

Unit: 10^{-3} in.

	a'	b'	c'	d'	e'	f'
Mean	345·35	14·80	—56·07	— 2·72	15·67	— 3·28
x_2'	— 45·22	— 7·49	68·06	—16·98	—30·87	15·94
x_3'	17·46	62·70	42·99	—12·63	—23·20	22·72
x_4'	— 72·95	—64·85	7·56	14·91	—32·91	—27·55
x_5'	—116·24	36·96	35·95	3·02	—21·84	— 6·49
x_6'	65·14	19·22	—42·98	11·88	1·20	—16·77
Standard residual . .	68·36	29·88	25·83	18·91	17·55	14·10
t (106 deg. freedom)		5·24	22·97	1·52	9·45	2·46

of the changes in b', confirm the conclusion drawn earlier. The analysis of the b' sequence was actually carried as far as the term of the ninth degree, but since coefficients of terms beyond the fifth degree are quite insignificant they have not been quoted.

Oscillations in the course of b' not associated with the polynomial were conspicuous, and it thus appeared desirable to test whether a harmonic series would take account of them. The analysis was taken as far as the fifth harmonic, yielding the series

$$14\cdot80+6\cdot72\cos\theta-5\cdot00\sin\theta+2\cdot46\cos2\theta+4\cdot68\sin2\theta+6\cdot96\cos3\theta$$
$$+5\cdot02\sin3\theta+4\cdot89\cos4\theta-0\cdot12\sin4\theta+3\cdot96\cos5\theta+10\cdot03\sin5\theta,$$

where $\theta=0$ in 1839. The significance of this series is due to the fifth harmonic. The fitted curve is given in Figure 2, and evidently follows the oscillations not represented by the polynomial. For this reason, the harmonic series was used in place of the polynomial fitted to b' in subsequent statistical tests.

An attempt was made to provide a more precise test of the significance of the harmonic series fitted to the b' sequence and the polynomials fitted to the remaining distribution constants, by subjecting the whole of the original data, comprising 6832(112 $\times$ 61) six-day totals of rainfall to one comprehensive analysis of variance. With rainfall measured in inches, the primary subdivision of the total variation is as follows:

Variation Due to	Degrees of Freedom	Sum of Squares
Between 6-day means ..	60	159·3083
Between years	111	31·7711
Residual	6660	1412·7703
Total	6831	1603·8497

In the further subdivision of these portions of the total variation, to show the contributions of the harmonic and the polynomials, the individual sums of squares must be placed on a comparable basis. This is accomplished by normalizing the distribution constant corresponding to the original polynomial of degree r using the factor

$$\left\{\frac{(2r+1).61.62\ .\ .\ .\ (61+r)}{60.59.\ .\ .\ .\ (61-r)}\right\}^{\frac{1}{2}},$$

or, more simply, by multiplying the appropriate sum of squares in the analysis, by the square of the above quantity. This test confirmed the significances obtained above, but, in the presence of such excessive annual variation, is still not sensitive enough to detect any other secular effects.

The last line of Table 3 gives the results of tests of significance on the mean values of the constants, with the exception of a'. Apart from the mean value of d', the remainder are significant and thus may be regarded as permanent characteristics of the season. The average seasonal distribution, represented by these mean values, is given in Figure 4, where it is contrasted with the means of the 6-day totals (on a per-day basis) and the distribution obtained by fitting the average 6-day totals with a harmonic series which takes the form

$$5\cdot77-3\cdot38\cos\theta-0\cdot384\sin\theta+0\cdot290\cos2\theta-0\cdot636\sin2\theta,$$

where $\theta=0$ at the mid point of the first 6-day interval in the year. This curve is identical with what would have been obtained if harmonics of this form had been fitted to the 61 values of each year, and averaged over the 112 years. Further discussion of the polynomial and harmonic representations is given below.

IV. Representation of the Season

When the original investigation was reported (Cornish loc. cit.), Whipple (1936) criticized the representation of individual years of rainfall and the average annual seasonal distribution by means of polynomials. The polynomials were designed to represent individual seasons, and are appropriate for this purpose because they do not impose the condition of periodicity on the weather, which does not, in fact, repeat itself year by year. On the other hand, the *average* rainfall sequence, being periodic, is properly represented by a harmonic curve.

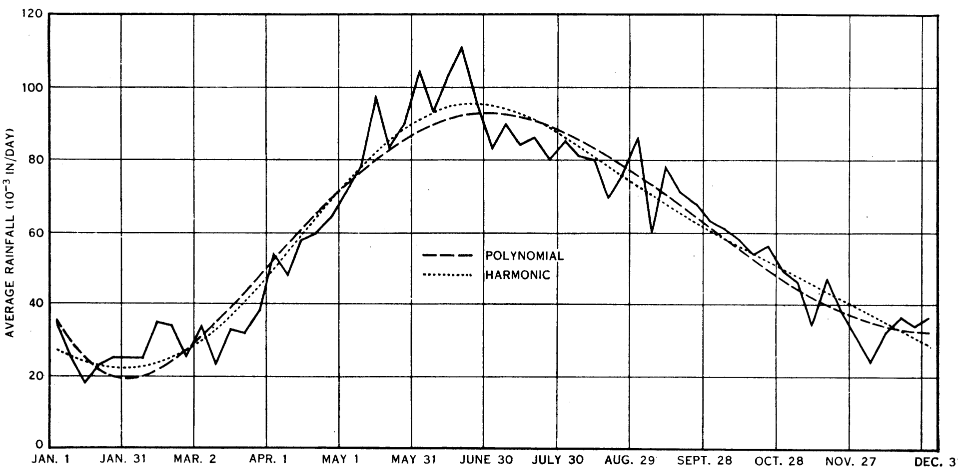

Fig. 4.—Seasonal variation of average daily rainfall.

As polynomials had been used for the annual observations, it was a natural step to show how they represented the average season, but at no stage was it claimed that they were designed for this purpose, or that their average was superior to a harmonic series. Whipple stated " the 12 monthly totals would give twice as much information about the distribution of rain through the year as six terms of the series of polynomials ", but gave no reason why they should do so when fitted with a harmonic series.

Instead, however, of using mean monthly rainfall, advantage may be taken of the finer detail supplied by the 61 six-day totals, since the superiority of either method of representation will be demonstrated more clearly with these totals than with the monthly means. The primary subdivision of the sum of squares given in Section III forms the starting point. The contribution made by the average polynomial is 145·8501, and the sum due to the harmonic is 148·2759, so that, after making allowance for the secular changes in the distribution constants, the two curves can be properly contrasted in the following analysis of variance :

Variation	Polynomial Curve			Variation	Harmonic Curve		
	Degrees of Freedom	Sum of Squares	Mean Square		Degrees of Freedom	Sum of Squares	Mean Square
Polynomial . .	5	145·8501	29·1700	Harmonic . .	4	148·2759	37·0690
Residual . .	55	13·4582	0·2447	Residual . .	56	11·0324	0·1970
Between 6-day means . .	60	159·3083			60	159·3083	
Residual . .	6630		0·2114				

Neither mean square, 0·2447 and 0·1970, differs significantly from 0·2114, and consequently both curves represent the average sequence well. The harmonic gives slightly the better representation, since its residual variance is smaller (and it absorbs one less degree of freedom), but comparison of the respective mean squares does not substantiate Whipple's estimate. The amount of information is the inverse of the variance, and the figures are 4·09 units for the polynomial and 5·08 units for the harmonic, the latter being only 25 per cent. greater. It must be emphasized again that the test has been made merely to examine the force of Whipple's statement.

TABLE 4

TEST OF NORMALITY OF RAINFALL DISTRIBUTION VALUES

Measure of Departure	a'	b'	c'	d'	e'	f'	Standard Deviation
g_1	0·2873	0·1666	0·0400	0·0781	0·2398	0·3900	0·2284
g_2	—0·3400	—0·1270	—0·2474	0·3485	0·4626	2·6497	0·4531

V. Frequency Distribution of $a', b', \ldots, f'$

All tests of significance employed are based upon the assumption of normality of the distribution constants $a', b', \ldots, f'$. Table 4 summarizes a test of normality, and shows the measures of departure with standard deviations appropriate to a normal distribution. These tests confirm the results previously given, f' again being the only quantity to show a significant departure from normality ; the positive value of g_2 indicates a symmetrical departure such that the apex and two tails of the distribution are increased at the expense of the shoulders.

VI. Correlations of the Distribution Constants

It has been observed that in the South Australian environment intra-station correlations of rainfall for subdivisions of the season are very weak, and in this connexion it is of some interest to compare these observations with correlations obtained from the extensive Adelaide record. The distribution constants have been calculated from uncorrelated functions of time, so that if rainfall at any time of the season is correlated with rainfall at any other time, the relation should appear on correlating them. After making allowance for secular changes, the correlations were determined and their inverse hyperbolic transformations $z=\tanh^{-1} r$, where r is the correlation coefficient, are presented in Table 5. The variate z is very nearly normally distributed, each value in the table having a standard deviation of 0·0958. Judging the data as a whole, the covariation is undoubtedly real ($\chi^2=81\cdot35$ with 15 degrees of freedom), but obviously from the individual values of z the correlations are all very weak, a result which confirms the observations made for stations distributed throughout the winter rainfall zone in South Australia.

TABLE 5

INTER-CORRELATIONS OF RAINFALL DISTRIBUTION VALUES

Values of $z=\tanh^{-1} r$: standard deviation 0·0958

	a'	b'	c'	d'	e'
b'	0·02536				
c'	—0·5565	0·02709			
d'	0·006209	—0·2769	0·06659		
e'	0·2420	0·1539	—0·2536	—0·03694	
f'	—0·2665	0·1121	0·2785	—0·06205	—0·2030

VII. Changes in the Rainfall Sequence

To express in simpler form the changes in the rainfall sequence represented by the secular trends of b' and c', the data were next presented from another standpoint. Examination of the monthly records disclosed that the main fluctuation seemed to concern the date of incidence of the winter rains, and it therefore appeared desirable to ascertain to what extent the oscillation in this date and the mean date of attainment of the yearly maximum of rainfall would account for the disturbances observed. In addition, there was definite evidence that spring and early summer rains had advanced toward the end of the year.

Consider the total rainfall of any two successive periods of 183 days. If the date dividing them falls in the spring, the second will generally contain the smaller quantity of rain, and vice versa if the day of division falls in the autumn. When daily differences of such totals are taken, a sequence of values should be obtained which changes sign regularly twice per rainfall year, first in the winter from negative to positive, and secondly in the summer from positive to negative.

These differences were determined for each day of the year in the 112 years of observations. With the exception of several years, the change of sign in winter and summer was defined very clearly. In exceptional years, the differences alternated in sign for short periods of varying lengths before definitely adopting the opposite sign, and in such cases the day of zero difference was obtained by smoothing the series with successive 10-day means.

The date in the winter at which the preceding 6 months had received as much rain as the 6 months following may be regarded as an empirical median of the rainfall sequence, and, in the original investigation of the changes in the mean value of b', two quartiles were located, one on either side of the median, between each of which and the median, one-quarter of the rainfall of the year surrounding the median date, had fallen. To these dates, three others corresponding to the octiles $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$ have been added. These additional figures

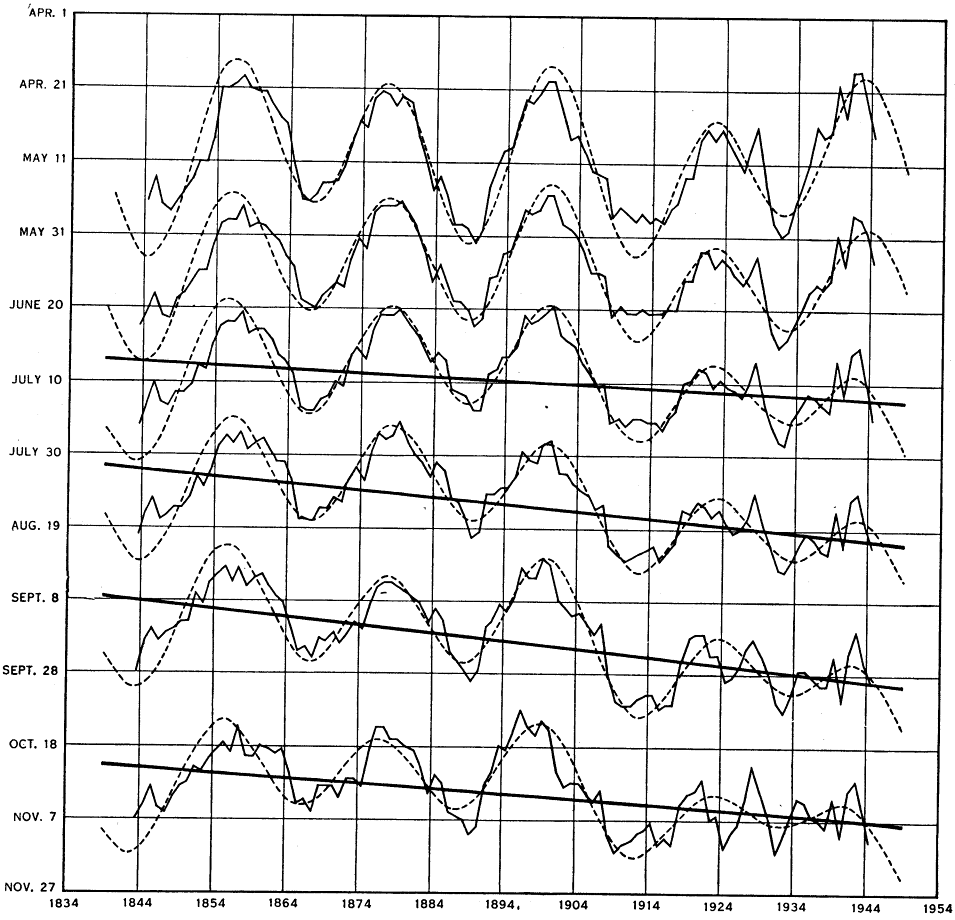

Fig. 5.—Ten-year means and harmonic curves of the octiles of the rainfall year, with linear components in the dates from the median onward.

serve to provide more detail of the changes expressed by b', but their principal use is to illustrate more clearly the linear trend represented by c'. The whole series is given in Figure 5 by plotting 10-year means of the several dates, and superimposing the harmonic curves *fitted to the original data.*

The six graphs all agree with the course of b'. The periods closely approximate each other, and the movements are in phase, but the displacements are in opposite directions, which must necessarily follow from the physical nature of the coefficient b'. The abnormal behaviour of the latter between 1910 and 1930 is reflected in all the curves of Figure 5, though not nearly to the same degree in the first quartile and third octile.* The reason for this is that the curves of Figure 5 are based on the rainfall year, which is a natural unit, whereas b' is dependent upon rain falling in the calendar year. There is definite evidence also from the curves of the first quartile and third octile that the course of b' is again becoming regular.

The changes expressed by b' are quite distinct from those of c', and represent a regular oscillation, of which the period and amplitude are approximately 23 years and 30 days respectively, in the dates of incidence of the winter rains and attainment of the yearly maximum of rainfall. This oscillation has been confined principally to a portion of the seasonal distribution, for the date of the minimum has remained practically constant throughout the 112 years, the section exhibiting greatest movement being that extending from April to November.

The amplitude of cycles occurring after 1910 is gradually reduced in the curves from the median date onward, and in the seventh octile has been almost entirely eliminated. This is undoubtedly due to the progressive change represented by c'; evidently some considerable time had to lapse before the cumulative effect became dominant, because the first three oscillations are clearly defined. The linear components in the curves from the median onward are all strongly significant, and are given in Figure 5. As the diagrams indicate, the changes expressed by c' have been confined to the latter half of the season, and amount to protraction of the later rains, the median, fifth octile, third quartile, and seventh octile having advanced by 11, 21, 24, and 17 days respectively in the period under review. The record is too short to permit any definite statement regarding this remarkable feature of the data, but it probably constitutes portion of a long-term oscillation, of which all that can be said at the moment is that the period (if it is periodic) is not less than 225 years.

The mutual agreement of the curves substantiates the view that a shift of date, regardless of quantity of seasonal precipitation, provides an adequate description of the changes in progress. In absolute terms, neither the amplitude of the cyclic movement, nor the total extension of the season, is great, but relative to a season of short duration such as occurs over a large proportion of the winter rainfall zone in South Australia, they may well assume considerable significance, particularly from the standpoint of agriculture.

VIII. Concluding Remarks

Whipple (loc. cit.) was the first to draw attention, in publication, to the extraordinary coincidence of this oscillation in the rainy season, with the alternation, from cycle to cycle, of the magnetic polarity of sunspots in both solar hemispheres, at sunspot minima.

* It will be observed that the median and quartiles of Figure 5 do not agree with the curves given previously (Cornish loc. cit.). When the original calculations were conducted, an error was made in finding the median date about 1900, which was automatically transmitted to the two quartiles and became progressively worse. This has now been rectified.

The latter phenomenon, now well known from the work of Hale and others, was first observed in 1912 (see, e.g. Chapman and Bartels 1940). Sunspots generally occur in pairs, the two spots being of opposite polarity. During any 11-year spot cycle, the polarities of the leading spots are usually the same for all pairs on the same side of the solar equator, but of opposite polarity on the other side. At the commencement of each new cycle there is a reversal of polarity, between leading and following spots in the pairs, and between northern and southern solar hemispheres. The two types of 11-year cycle, designated as *P* and *E* cycles, occurred as follows :

P Cycles	*E* Cycles
1843–1856	1856–1867
1867–1878	1878–1889
1889–1901	1901–1913
1913–1923	1923–1933
1933–1944	1944–

During a *P* cycle, the polarity of the leading spot of a pair in the northern hemisphere of the Sun is the same as that of the magnetic pole in the southern hemisphere of the Earth. Comparison of these intervals with those obtainable from Figure 5 shows how closely seasons having progressively early winter rains correspond with the *P* cycles, and seasons having progressively late rains correspond with the *E* cycles.

The oscillations in the Adelaide records must be due to secular changes in the latitudinal paths of anticyclones (with their attendant cyclones) across southern Australia, superimposed on the normal seasonal variation. Kidson (1925) demonstrated the pronounced seasonal variation in the latitude of the mean monthly tracks, and attempted to relate latitudinal departures from normal, at various longitudes, ranging from 120 °E. to 170 °E., to variations during the sunspot cycle. He made no differentiation of the cycles, as indicated above, so that the opposing changes noted herein have probably tended to annul each other, at 138 °E., the approximate longitude of Adelaide. In the absence of Kidson's original data, and later observations in convenient form, it is not possible to examine this point further.

The presence of a progressive change in the season at Adelaide agrees with a general observation made recently by Deacon (1953). Deacon's evidence was derived from mean differences in summer temperature, summer and winter precipitation, and barometric pressure between the two epochs 1880–1910 and 1910–1940, for stations in south-eastern Australia, and he concluded that a major climatic change has been operating progressively since 1880 (the earliest record he used), contemporaneously with changes occurring in the northern hemisphere.

The gradual advance in the latter half of the season at Adelaide also agrees with an increase in late spring and early summer rainfall at other stations as found by Mason (personal communication 1953).

IX. Acknowledgments

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