# THE MASSES OF THE MAGELLANIC CLOUDS FROM RADIO OBSERVATIONS

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*Summary*

The problem of deriving the masses of the Magellanic Clouds is discussed, and provisional estimates are obtained from 21 cm measurements of the distribution of radial velocity.

Various lower and upper limits to the mass of the Large Cloud are found, varying from $1\cdot4\times10^9$ to $4\cdot4\times10^9$ solar masses. The best estimate on present evidence is $3\cdot0\times10^9$. Apart from theoretical considerations, the major uncertainty arises from the sensitivity of the mass to the tilt angle, which is here taken as 65°; the actual value may be somewhat higher and the mass estimate accordingly larger. A provisional value for the mass of the Small Cloud, which can only be estimated by analogy with the Large Cloud, is $1\cdot3\times10^9$.

With these masses the observed differential velocity indicates that the Clouds probably do not describe a closed orbit around each other.

The mass-luminosity ratio appears to be somewhat lower than corresponding values for other galaxies, supporting the view that the Clouds are young systems.

## I. Introduction

This paper is the third of a series based on a survey of 21 cm line radiation from interstellar hydrogen in the Magellanic Clouds. The results of the survey were presented and the brightness observations discussed in paper I (Kerr, Hindman, and Robinson 1954); the motions of the Clouds were then derived from the observed radial velocities in paper II (Kerr and de Vaucouleurs 1955); we now consider the derivation of the masses of the Clouds from the rotational and random motions.

No reliable determinations of the masses of the Clouds are yet available, but rough estimates have been made by Oort (1940), Shapley (1950), and Holmberg (1952). Holmberg's values, which are the most recent, are derived (*a*) from the dispersion of the optical radial velocities measured by Wilson (1917) and (*b*) from an assumed mass-luminosity ratio for a type I system. After correction for the revised distance scale (double the traditional scale), they are of the order of $2\times10^9$ solar masses for the Large Cloud, and $0\cdot6\times10^9$ for the Small Cloud. These values are unreliable, however, because (*a*) the optical velocities are few and of low accuracy, and also the main source of the dispersion is not random motions in the Cloud (see paper II), and (*b*) the mass-luminosity

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ratio derived from large spirals cannot be safely applied to the Clouds which are so different in their characteristics.

A lower limit to the mass of each Cloud is provided by the mass of neutral hydrogen, which has been deduced from the 21 cm line observations (paper I). These values are $0 \cdot 6 \times 10^9$ for the Large Cloud, and $0 \cdot 4 \times 10^9$ for the Small Cloud.

In other galaxies, the most satisfactory estimates of mass have been derived from a study of rotational motions. The systems for which the most detailed information has been obtained are the large spirals M31 and M33 (Babcock 1939 ; Mayall and Aller 1942 ; Wyse and Mayall 1942 ; Mayall 1950 ; Baade and Mayall 1951 ; Mayall and Eggen 1953 ; Lohmann 1954 ; Schwarzschild 1954) and the E7 galaxy NGC 3115 (Oort 1940 ; Schwarzschild 1954). In each of these cases the rotational velocity curve has been used to derive the total mass and the mass distribution.

The radio observations of the rotational and random motions are used in the present paper to obtain estimates of the mass of each Cloud and the mass distribution in the Large Cloud. Discussion of the problems arising in the course of this study indicates that more precise values require an extension of the theory, together with the more detailed observations which can now be made.

## II. The Mass Derivation Problem

The total mass and the mass distribution can be derived for the Clouds from the rotational velocity curves presented in paper II, following the methods used by previous authors for regular spiral galaxies. The Clouds are, however, more complex dynamical systems than the regular spirals ; their spiral structure is asymmetrical and irregular and the random motions (as indicated by the widths of the line profiles) are comparable with the rotational motions. As far as we are aware, no theory is yet available for such a system.

We will first obtain estimates of the masses of the Clouds by a direct application of the theoretical methods which have been developed for the regular spirals, and then consider the factors which require an extension of the theory. Particular attention will be paid to the case of the Large Cloud, for which the most detailed information is available ; the Small Cloud will be considered only by analogy with the Large Cloud.

The discussion of the rotational motions of the Clouds in paper II was largely based on the median radial velocity of each line profile ; this is the simplest and most objective single parameter for describing a profile whose details and interpretation are not precisely known. After extensive rotation curves had been derived from the median velocities, the rotation could then be considered in more detail. By comparison with the few available optical velocities (Wilson 1917), it was possible to show that the median velocities refer to regions away from the equatorial plane of each system, whereas the peaks of many of the profiles could be associated with that plane. The peak velocities should therefore be used as far as possible in deriving the mass of each Cloud.

A mass estimate obtained from the rotation curve alone will be only a *lower* limit, for at least two different reasons. Firstly, the rotation curve is very insensitive to the mass distributed in the outlying parts of the Cloud, so that the derived value only allows satisfactorily for the mass in the inner regions. We will therefore use other means to extrapolate the mass distribution curve to infinity (Section V (*a*)) ; a reasonable estimate of the required correction can be obtained from the observed distributions of neutral hydrogen and light as a function of radial distance from the centre.

The second shortcoming of a method based on the rotation curve arises from the neglect of the random motions. A mass derivation from a rotational analysis assumes that a system is in dynamic equilibrium under the centripetal force of its own gravitation and the centrifugal force of the rotation. The close connexion between the rotational velocities and the total mass, which is postulated by the theory, probably holds in the large, highly flattened spirals, including our own Galaxy, where the rotational velocity is much larger than the random motions, except near the centre. Even in these cases however the validity of the method has been questioned (Zwicky 1937).

The assumption and theory are much less justified in the case of Magellanic-type galaxies. Apart from the possibility that the Clouds have not yet reached equilibrium, the radio observations have shown that the random and rotational motions are of the same order of magnitude. Such a system is prevented from collapsing by the kinetic energies of both the rotational and random motions, so that both must be taken into account in determining the total mass. Following the treatment based on rotation alone, we will obtain another *lower* limit to the mass from the velocity dispersion alone, neglecting the rotation (Section VI (*a*)).

Finally we will consider the case where the two types of motion are present simultaneously, in order to find closer approximations to the actual mass (Section VI (*b*)).

The main rotational treatment is based on the methods of Wyse and Mayall (1942) and Perek (1948, 1950), but two alternative interpretations are briefly discussed in Section V (*b*).

Theoretical considerations aside, precise values for the masses of the Clouds cannot be expected at this stage, owing to uncertainties in the data, especially in the tilt angle of the Large Cloud ; this angle is taken, in the main discussion, as $i=65°$, but final results will also be given for $i=70$ and $75°$. Shapley and Nail (1955) have recently questioned the description of the Large Cloud as a flattened, tilted system (de Vaucouleurs 1955*a*), but the demonstration of rotational motion given in paper II appears to us to confirm that the Cloud is, in fact, a flattened, rotating system, tilted to the line of sight.

For the Small Cloud, uncertainty in the tilt angle is unimportant, but allowance for the asymmetrical "tidal" prominence (de Vaucouleurs 1955*b*) raises a serious difficulty. However, the less extensive velocity curve for the Small Cloud does not permit a full independent treatment, and its mass can only be derived by simple comparison with the case of the Large Cloud.

## III. Adopted Rotation Curves

We must first derive the rotation curves which best fit all the available evidence. As shown in paper II, the most appropriate rotation curves are those obtained from the peak radio velocities, which are believed to refer to the equatorial planes of the two systems. On present information, however, such a curve can only be drawn over a limited range ($1° < r < 5°$), but in the case of the Large Cloud we can deduce its probable course in the outer parts with the help of the more extensive median velocity data.

The observed rotation curves are somewhat smoothed, owing to the finite extent of the aerial beam, and we must first attempt to correct for this effect. Bracewell (1955) has given an approximate method for correcting a brightness distribution which has been blurred by a Gaussian aerial beam. In this method

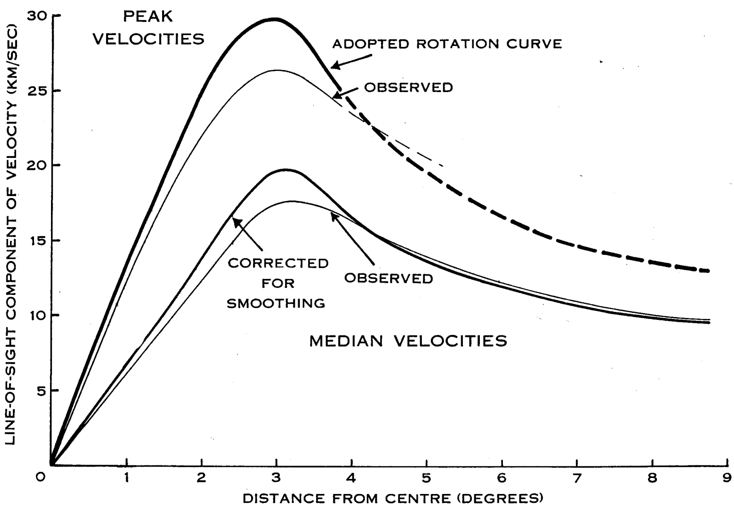


Fig. 1.—Velocity curves for the Large Cloud.

a two-dimensional array of values of brightness-temperature is tabulated at intervals of $\sqrt{2}$ times the standard deviation of the Gaussian curve. Then the correction to be applied to the observed value at any point is the difference between that value and the mean of the four surrounding values.

In the present case, the aerial beam does not depart far from a Gaussian shape, with a standard deviation of $0 \cdot 65°$. The method, as described for a brightness distribution, can be applied directly to a velocity distribution if the brightness is uniform over the region considered, but must be modified when, as in the Clouds, the brightness decreases from the centre outwards. This has the effect of giving greater weight to the velocities in the portions of the aerial beam which are nearest the centre of the Cloud. In this case the correction to be applied to an observed velocity can be approximated by weighting the four neighbouring values according to their associated brightness ; this procedure is not formally precise, but gives a sufficiently good approximation under the present circumstances.

This modification of Bracewell's method has been used to correct the rotation curves for the effects of aerial smoothing. The weights were obtained from the mean radial distribution of 21 cm brightness, and idealized velocity patterns over the Clouds were derived from the observed mean rotation curves, assuming tilt angles of 65 and 30° for the Large and Small Cloud respectively. The corrected curves are shown in Figures 1 and 2, together with the observed curves taken (except for the peak velocity curve for the Small Cloud) from Figures 4 and 5 of paper II.

In the case of the Large Cloud, the corrected curve for the peak radio velocities was then extrapolated in the region $4° < r < 8°$, by using the shape of the median velocity curve and assuming that the ratio of peak-to-median velocity tends to unity as $r \to \infty$.*

The finally adopted curves for the two Clouds are shown by the heavy lines in Figures 1 and 2.

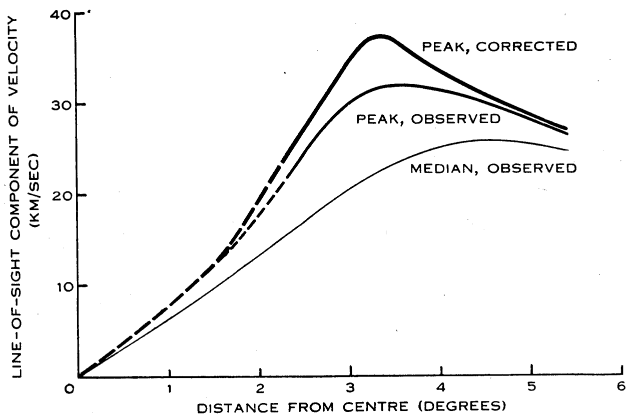


Fig. 2.—Velocity curves for the Small Cloud.

## IV. Mass Estimates from Rotation Curve

We now fix attention on the Large Cloud, applying to the rotation curve methods which have already been used for other galaxies.

### (a) *Point Mass: Keplerian Branch*

A first useful indication of the mass can be obtained from consideration of the Keplerian branch alone. The outer parts of the velocity curve for the Large Cloud agree closely with the curve which would be produced by a central point mass of $1 \cdot 5 \times 10^9$ solar masses.

### (b) *Wyse and Mayall's Method: Thin Disk Approximation*

In the method developed by Wyse and Mayall (1942) it is assumed that a galaxy can be approximated by an infinitely thin disk having a suitable radially symmetric density distribution. The radial distribution of density in this disk

* The trend of the ratio towards unity was found to be approximately linear with $1/\sqrt{r}$ over the region where both velocities were available; this trend was continued in the extrapolation.

is expressed in the form of a power series extending to the fifth power of the radius and thus involving six parameters. The gravitational force curve corresponding to any density distribution is then derived by a lengthy mathematical process. The force curves produced by a number of special density models are computed and their coefficients tabulated, so that for these particular cases the mass distribution can be readily derived from an observational force curve of the same type. More generally, when none of the special models can be used, five points of a force curve can be used to set up five simultaneous equations, which, together with the condition that the density must be zero outside the disk, yield the density distribution.

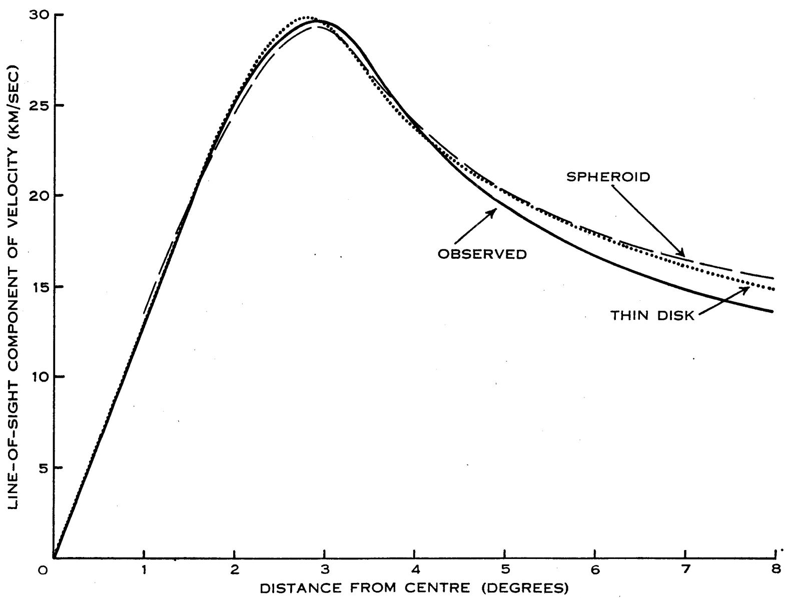


Fig. 3.—Theoretical velocity curves for the Large Cloud for the best fitting thin disk and spheroid models, compared with the adopted observed curve.

An application of the full treatment to the observed velocity curve (Fig. 3) has led to unsatisfactory results for the inferred density distribution, mostly on account of the alternating character of the series involved ; the desired quantities are obtained as the differences between much larger coefficients of opposite signs. This defect had already been noted by Wyse and Mayall.

In view of this a simpler treatment was preferred, based on the theoretical distributions for some of Wyse and Mayall's thin disk models. It was found by trial and error that two of their models could be combined to give a force curve in fair agreement with the observed one. The models used were those with an " oblate spheroid " density distribution, $\sigma=\sigma_c[1-(r/R)^2]^{\frac{1}{2}}$ and a " Gaussian " distribution $\sigma=\sigma_c \exp\{-4(r/R)^2\}$. The best fit was obtained with a composite model, consisting of three spheroids with $R=2\cdot2$, $2\cdot9$, $3\cdot5°$, and $\sigma_c=33$, 67, 33 solar units per square parsec, together with a " Gaussian " distribution with $R=5{\cdot}37°$ and $\sigma_c=22$. The theoretical velocity curve for this model is shown in Figure 3, in comparison with the observed curve, and the distribution of projected density in Figure 4. The model leads to a mass of $1{\cdot}73\times10^9$ solar masses for the Large Cloud and a projected central density of 155 suns/pc².

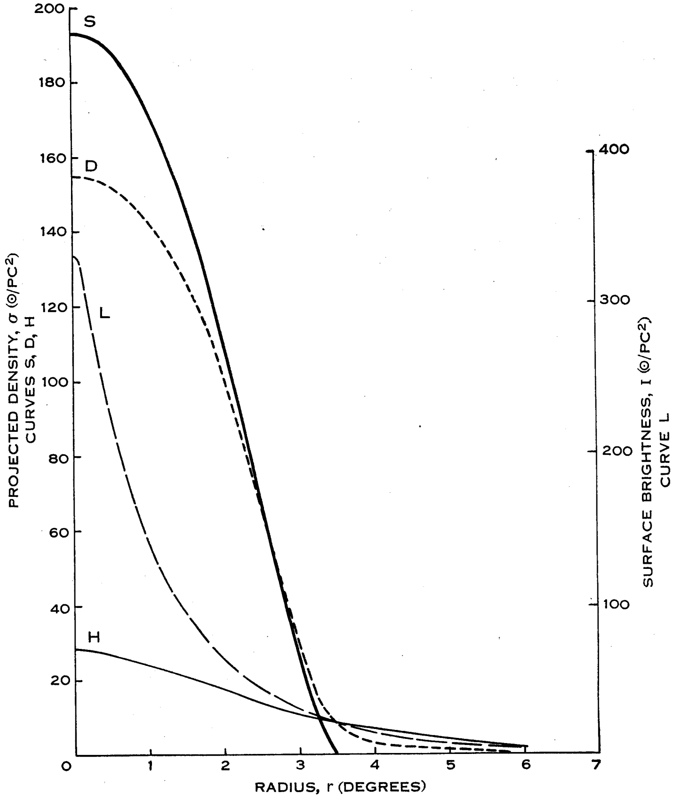


Fig. 4.—Projected density distribution in the Large Cloud, for H I, red light, and rotational mass according to the two models :

——— Hydrogen, corrected for aerial smoothing (⊙/pc²) ;
— — Red light (⊙/pc², right-hand scale) ;
- - - - Thin disk (⊙/pc²) ;
——— Spheroid (⊙/pc²).

### (c) *Perek's Method: Oblate Spheroid Approximation*

The thin disk treatment has given a good fit with the observational results, but the method has the disadvantage that it neglects the thickness of the system and thus prevents the derivation of space densities. In the case of NGC 55 (Plate 3, paper II), an edgewise system believed to be of Magellanic type, it can be seen that the thickness at right angles to the equatorial plane is by no means negligible ; a microphotometric investigation (de Vaucouleurs 1956) indicates that the ratio $c/a$ of the axes of the outer isophotes of NGC 55 is close to 1/5. We will adopt this value for the Large Cloud.

A better approximation may then be obtained by following Perek (1948, 1950) who has discussed theoretically the velocity curves of non-homogeneous spheroids for various density laws and produced a convenient nomogram and table for the rapid derivation of the force curves as a function of $c/a$, when the density law is of the form

$$\rho=\rho_c(1-m^2)^n,$$

where

$$m^2=(x^2+y^2)/a^2+z^2/c^2.$$

In the present case a simple model with $n=1$ was found to give a good fit with the observed velocity curve (see Fig. 3) for

$$a=3\cdot5^\circ=2\cdot8 \text{ kpc},$$
$$\rho_c=1\cdot77\times10^{-23} \text{ g/cm}^3=0\cdot26 \text{ sun/pc}^3.$$

The corresponding values for the projected central density $\sigma_c$ and total mass $\mathbf{M}$ are

$$\sigma_c=0\cdot266a\rho_c=195 \text{ suns/pc}^2,$$
$$\mathbf{M}=0\cdot335a^3\rho_c=1\cdot93\times10^9 \text{ solar masses}.$$

The total mass is in good agreement with the value derived from the thin disk model. The projected central density $\sigma_c$ is slightly higher, as could be expected from the change to a model with matter outside the equatorial plane, but we obtain in addition the central space density $\rho_c$, which is equivalent to 11 hydrogen atoms per cubic centimetre (either condensed into stars or in interstellar space).

A defect of the model is that it leaves no room for matter beyond $r=3\cdot5^\circ$ where both radio and optical observations detect considerable radiation. Experiments with other values of $c/a$ indicate that non-zero densities in the outer parts can be obtained if $c/a$ is less than 1/5 and the thin disk approximation gives then a fair idea of the sort of distributions which can be obtained. For $c/a=1/5$, matter can be spread beyond $r=3\cdot5^\circ$ by using more elaborate density laws, but the accuracy of the present data was not considered sufficient to justify this refined treatment. In any case the total mass and central projected density cannot be changed much and the values above should be sufficient for the present, since more serious sources of uncertainty exist, as described earlier.

For the succeeding discussion, we will adopt $1\cdot8_5\times10^9$ solar masses for the mass of the Large Cloud indicated by the rotation curve, for a tilt angle $i=65^\circ$.

## V. Discussion of Mass Estimates Derived from Rotation

### (a) *Extrapolation to Infinity*

The mass derived from the radio rotation curve is probably only a lower limit, since the velocity curve is insensitive to the density in the outlying regions. This shortcoming has already been experienced by Wyse and Mayall (1942) in their discussion of M33. The density in these outer regions is low, but the total volume of space concerned is large, so that the contribution to the mass might be quite substantial.

Evidence that the thin disk and spheroid models provide too little mass in the outer parts of the Large Cloud, in spite of the good fit which they give to the observed velocity curve, is shown in Figure 4. The mean radial distributions of projected mass corresponding to these models are compared with those for neutral hydrogen, and for the bulk of the stellar population as indicated by observations of the surface brightness in red light (unpublished data). Both the latter distributions indicate that there is more mass in the outlying parts than has been accounted for by the models ; the divergence becomes serious at about 3°.

Our problem then is to form some estimate of the factor by which the mass of the models must be increased to allow for the mass distributed in the outer parts. A lower limit to this additional mass can be obtained directly by integrating the hydrogen density in excess of the density given by the models. Another estimate may be obtained by assuming that the distribution of the combined mass (stellar and interstellar) in the outer parts is intermediate between the hydrogen and light distributions. This assumption is plausible, since the hydrogen is known to comprise by far the largest fraction of the interstellar mass, and the faint stars which contribute most of the red light carry most of the stellar mass.

TABLE 1

OVERALL MASSES OF NEUTRAL HYDROGEN

| | LMC | SMC |
|---|---|---|
| Measured .. .. .. | $0 \cdot 57 \times 10^9$* | $0 \cdot 43 \times 10^9$† |
| Extrapolated .. .. | $0 \cdot 7 : \times 10^9$ | $0 \cdot 5 : \times 10^9$ |

* $A = 165$ sq. deg. ($\bar{r} = 7 \cdot 3°$).
† $A = 104$ sq. deg. ($\bar{r} = 5 \cdot 7°$).

Before these estimates can be formed, however, we note that the hydrogen density, as indicated by the trend of the curve in Figure 4, does not fall abruptly to zero beyond the last measurable contour. An attempt has been made to allow for the mass of hydrogen beyond the observed limits by an extrapolation method similar to that used in the determination of the total (integrated) luminosities in galaxies (Redman 1936 ; de Vaucouleurs 1948).

Graphical integration of the H I brightness contours (paper I) gives the total mass of hydrogen within successive contours ; if the logarithm of this mass is plotted as a function of the area $A$ limited by the contour (e.g. against $1/\sqrt{A}$), it is possible to obtain by a short extrapolation the limit towards which the mass of hydrogen tends as $A$ is indefinitely increased. In this way allowance may be made for the contribution of the very faint outer regions where the radiation falls below the threshold of detectability.

Extrapolation of the measured masses of neutral hydrogen leads to the rather conservative estimates of the overall masses which are shown in Table 1.

Once in possession of the overall mass the fraction of the total within any distance from the centre can be readily computed, leading to the relative integrated mass curves shown in Figure 5.

The same procedure has been applied to an unpublished set of isophotes in red light extending to an average radius $\bar{r}=7\cdot7°$ ($A=185$ sq. deg.) and including about 85 per cent. of the total (integrated) luminosity (see Fig. 5). Note, however, that the centre of these isophotes is displaced from the centre of rotation or the hydrogen centre.

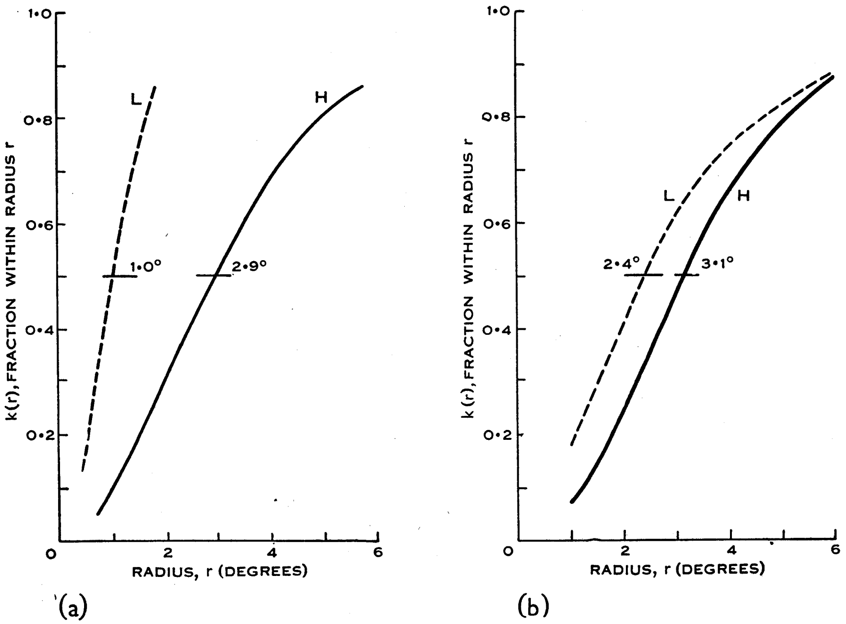


Fig. 5.—Relative integrated intensity distributions in the Clouds for neutral hydrogen and red light. The curves give the fraction $k(r)$ projected within a circle of radius $r$ centred on $C_H$ or $C'$. The effective radii $r_e$ are shown on the curves at $k(r)=0\cdot5$. (*a*) Small Cloud, (*b*) Large Cloud.

We can now estimate the mass of hydrogen which has not been accounted for by the models. In this way, we obtain $0\cdot3\times10^9$ solar masses as a lower limit to the excess mass. The contribution from the stars cannot be directly estimated in a similar manner, because the optical surface brightness is probably not proportional to the projected surface density of stellar mass (cf. Section V (*b*) below).

As a first approximation, however, we can assume that the combined mass (gas + stars) is distributed in the same way as the mean of the hydrogen and light, and that the value of the mass derived from the rotational analysis is unaffected by the mass of the outlying regions. The curves of Figure 4 indicate that the total mass of hydrogen is $2\cdot1$ times that within $r=3°$ from the centre; the corresponding factor for the luminosity is $1\cdot7$. For an estimate of the mass we will take a factor of $1\cdot9$.

The mass within $r=3°$ indicated by the rotational analysis is $1\cdot6\times10^9$ for the thin disk model and $1\cdot8\times10^9$ for the spheroid model. The corrected mass, extrapolated to infinity according to the above assumption, is therefore $1\cdot9\times1\cdot7\times10^9=3\cdot2\times10^9$ solar masses.

This estimate has been tested by comparing a theoretical velocity curve for the extrapolated mass distribution with the observed curve. The theoretical curve was derived by approximating the mass distribution by a series of straight-line segments, and then applying Wyse and Mayall's solution for a linear distribution of projected density. The velocities obtained in this way were considerably higher than the observed velocities in the outer parts of the Cloud, e.g. 27 and 26 km/sec at $r=5°$ and 7°, where the observed values were $19\cdot7$ and $13\cdot7$ km/sec. The observed velocities are of low accuracy in the outer regions, but the above differences are substantially greater than the estimated probable error.

The above estimate of the extrapolated mass must therefore be too high. There are two possible reasons for this, one or both of which may be operating: either the assumption of a nearly constant mass-luminosity ratio is at fault (see also the discussion of Schwarzschild's method in Section V (*b*)), or else the shortcomings of the simple rotational analysis may be connected with the neglect of the random motions (see Section VI).

For the present, we can only conclude that the extrapolated mass is $>2\cdot1$ and $<3\cdot2\times10^9$, say $2\cdot5\times10^9$ solar masses.

### (*b*) *Other Interpretations of the Velocity Curve*

Our mass estimates have been obtained through two related interpretations of the velocity curve (thin disk and spheroid models), based on some *assumed* density distribution laws selected for simplicity. The rather good fit they give to the observed velocity curve does not necessarily imply that the actual density distribution closely follows the assumed laws everywhere. In fact Figure 4 shows conclusively that it does not, at least in the outer parts.

It is therefore desirable to consider two alternative interpretations of the velocity curve which have been proposed for other galaxies:

(1) that the force curve can be accurately represented by the simple interpolation formula due to Bottlinger (Lohmann 1954);

(2) that the projected surface brightness is proportional to the projected mass density, i.e. that the mass-luminosity ratio is constant throughout the system (Schwarzschild 1954).

According to Lohmann, following Bottlinger, the gravitational force in the equatorial plane of a galaxy can be represented by

$$F=ar/(1+br^3),$$

and

$$v^2=ar^2/(1+br^3),$$

whence

$$\mathbf{M}=a/bG \quad (G=\text{gravitational constant}).$$

A model for which $v_{\text{max.}}=30\times\sec 65°=71$ km/sec at $r=3°=2\cdot4$ kpc ($a=2\cdot67\times10^3$ (km/sec)$^2$(kpc)$^{-2}$, $b=0\cdot147$ (kpc)$^{-3}$) gives a total mass

$\mathbf{M} = 4 \cdot 2 \times 10^9$ solar masses, but as shown in Figure 6 it does not give a good fit with the observed velocity curve. If the theoretical curve is adjusted to fit at 2 or 4°, the corresponding masses are $3 \cdot 5$ and $2 \cdot 9 \times 10^9$.

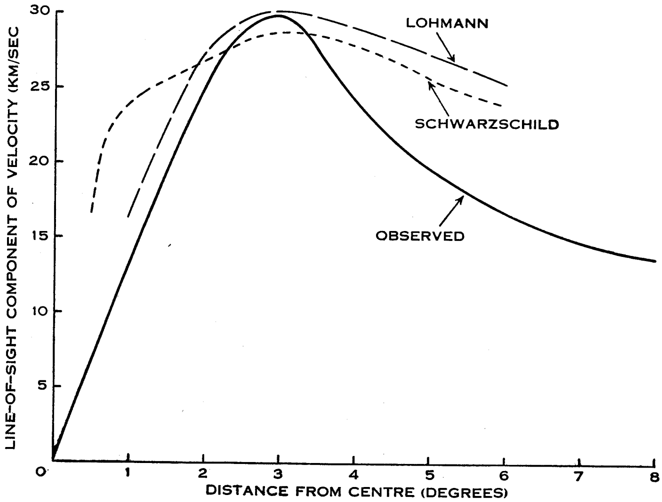


Fig. 6.—Comparison between theoretical velocity curves derived according to Lohmann's and Schwarzschild's assumptions and the observed velocity curve.

Schwarzschild assumes that the projected mass distribution $\sigma(r)$ is proportional to the luminosity distribution $I(r)$; the latter is approximated by a series of straight segments, so that

$$I(r) = \sum_n a_n (1 - r/R_n) \quad \text{and} \quad \sigma(r) = \sum_n A_n (1 - r/R_n),$$

TABLE 2

STRAIGHT SEGMENT APPROXIMATION TO LMC LUMINOSITY CURVE

| $n$ | $R_n$ (pc) | $a_n$ ($\odot$/pc$^2$) | $R_n^2 a_n$ |
|---|---|---|---|
| 1 | 400 | 40 | $6 \times 10^6$ |
| 2 | 800 | 80 | $51 \times 10^6$ |
| 3 | 1800 | 140 | $455 \times 10^6$ |
| 4 | 3200 | 53 | $510 \times 10^6$ |
| 5 | 4800 | 18 | $440 \times 10^6$ |
| 6 | 7200 | 9 | $465 \times 10^6$ |

where $A_n = f \times a_n$, if $f$ is the supposedly constant mass-luminosity ratio. Wyse and Mayall's results for a linear density distribution then lead to a simple solution of the velocity curve for the thin disk model, and to a total mass

$$\mathbf{M} = \tfrac{1}{3}\pi \sum_n A_n R_n^2 \quad \text{(in solar units).}$$

This treatment was applied to an unpublished luminosity curve in red light for which the adopted straight segment approximation is given in Table 2.

Allowing for various numerical factors this leads to a total mass $\mathbf{M}=2\cdot 0\times 10^9\times f$ (in solar units) and a computed velocity curve

$$v(r)=0\cdot 131\sqrt{f}[r\sum_n a_n g(r/R_n)]^{\frac{1}{2}},$$

where $g(r/R_n)$ is the function $M(\alpha)$ and $m(\beta)$ tabulated by Wyse and Mayall. Approximate agreement with the observed maximum of the velocity curve is obtained for $f=1\cdot 5$ (if $i=65°$), so that $\mathbf{M}=3\cdot 0\times 10^9$ solar masses, but elsewhere the fit is very poor (Fig. 6).* The best average fit over a larger portion of the velocity curve would give $\mathbf{M}=2\times 10^9$.

These two alternative interpretations which take some account of the mass in the outlying parts are useful in confirming the order of magnitude of the derived mass ; we see from the velocity curves, however, that their assumptions do not fit the observations as closely as do those of the earlier models, and we will base our conclusions on the results obtained in Sections IV and V (*a*).

## VI. Mass Estimates including Random Motions

### (*a*) *Spherical Approximation*

As mentioned in Section II above, the random, or peculiar, motions in the Large Cloud appear to be comparable with the rotational motions. This suggests that another lower limit to the mass of the Cloud may be obtained from the velocity dispersion through the virial theorem, as applied to spherically symmetrical systems in which rotation is negligible (spheroidal galaxies, globular clusters). In a sense this represents the limiting case at the opposite extreme from the thin disk approximation in the rotational analysis ; the actual system is in an intermediate situation.

The mean value of the observed velocity dispersion (the half half-width of the line) in the Large Cloud is about 25 km/sec, with slightly higher values in the central regions and slightly lower values in the outer parts. This velocity spread must be due in part to systematic mass motions in the equatorial plane, as shown by the irregularity of many of the profiles, and also to the variation of rotational speed with depth in the Cloud. There is also a small broadening due to the finite receiver bandwidth. Hence the true " random " velocities must be somewhat smaller than 25 km/sec. The various effects cannot be separated on present evidence, but for definiteness we will take a value of 20 km/sec for the r.m.s. random velocity.

For a numerical application of the virial theorem, we use the simple equation (Chandrasekhar 1942, p. 200)

$$\mathbf{M}=\frac{2\overline{V^2}R}{G},$$

where $\overline{V^2}=\overline{u^2}+\overline{v^2}+\overline{w^2}=3\overline{w^2}$, $R$ is a suitably defined effective radius, and $G$ is the gravitational constant. Here we may take $(\overline{w^2})^{\frac{1}{2}}=20$ km/sec and $R=3°=2\cdot 4$ kpc, which gives $\mathbf{M}=1\cdot 4\times 10^9$ solar masses.

* Reference to Figure 3 in Schwarzschild's paper indicates that his attempt to fit the M33 data led to the same sort of discrepancy. It seems clear that the assumption $f=$const. is not justified for late-type systems.

This value must be greater than the mass which can be related to the random motions, since the Cloud is far from spherical, but it must be taken as a lower limit, as the rotational motions have been neglected.

### (b) *Combined Effect of Rotational and Random Motions*

Direct addition of the masses obtained by considering rotational and random motions separately leads to a value of

$$2 \cdot 5 + 1 \cdot 4 = 3 \cdot 9 \times 10^9 \text{ solar masses.}$$

This can be regarded as an extreme upper limit, as the effects are obviously not additive and their interaction must be taken into account. We will estimate the combined centrifugal effect of the two types of motion from an approximate physical picture.

When random motions are superposed on the observed rotation, each particle will execute an orbit which is in general elliptical and inclined to the equatorial plane. The components of the random velocities which are directed towards the centre (the $x$-direction) will average to zero in their centrifugal effect ; they can therefore be neglected in this discussion, and the orbits regarded as circular. The velocity of a particle "rotating" in such an inclined orbit will in general be greater than the equatorial plane component of rotational velocity which was derived in paper II. The higher rotational velocities can support, by their centrifugal action, a greater mass.

To a first order, the representative velocity, which is to be associated with the mass of the system, can be taken as that derived from the equatorial plane rotational velocity and the r.m.s. value of the component of random velocity directed away from the plane, the $z$-component. The $x$-component has no net centrifugal effect, as explained above : the $y$-(tangential) component will not to the first order affect the result, since the mean rotational velocity as measured is a fair approximation to the vector sum of the pure rotational velocity and the random $y$-component.

We again take 20 km/sec as the r.m.s. random velocity in the line of sight for the Large Cloud. In a spherical system, the three components of the random motions would be equal, and equal to the line-of-sight component. Since the Cloud appears to be flattened, the $z$-component of the random velocities is presumably somewhat smaller than the $x$- and $y$-components, but for the present approximate treatment we will regard the three components as equal to 20 km/sec. Further, the observations suggest that the random velocities decrease on going outwards from the centre of the Cloud ; they also presumably decrease on going away from the equatorial plane, but for the moment we will neglect both these variations.

The maximum rotational velocity in the equatorial plane curve is 71 km/sec (for a tilt angle of 65°), but we should take here a mass-weighted mean value for the rotational velocity, say 45 km/sec. Compounding this with the random $z$-component of 20 km/sec gives a correcting factor of about $1 \cdot 2$. The previous value of $1 \cdot 8 \times 10^9$ solar masses, without the incompleteness correction, is thereby increased to $2 \cdot 2 \times 10^9$. If, finally, we apply the incompleteness correction, we get a value for the total mass of $3 \cdot 0 \times 10^9$ solar masses, with limits of $2 \cdot 2 + 0 \cdot 3 = 2 \cdot 5 \times 10^9$ and $2 \cdot 2 \times 1 \cdot 9 = 4 \cdot 2 \times 10^9$.

Oort (1940), following Jeans (1922), has given a treatment for a system in a dynamically steady state. He obtained a relation between the "circular" velocity in the absence of random motions, $V_c$, and the observed mean rotational velocity when random motions are present, $V_0$, which may be written, in our notation,

$$V_c^2 - V_0^2 = \overline{u^2}\left[-\frac{\partial \log \rho}{\partial \log r} - \frac{\partial \log \overline{u^2}}{\partial \log r} - \left\{1 - \frac{\overline{v^2}}{\overline{u^2}}\right\}\right],$$

where $u,v,w$ are the $x,y,z$ components of the random velocity, and $\rho$ the space density.

The assumption of strict equilibrium is not applicable to the Large Cloud, and also the expression requires a greater knowledge of the random velocities through the system than is at present available, but an attempt to estimate the factors involved leads to a mean value of $V_c^2/V_0^2$ in rough agreement with the correction factor derived above.

## VII. Best Estimate of the Masses

### (*a*) *Large Cloud*

In Table 3 are collected the various upper and lower limits for the mass of the Large Cloud which have been derived in the preceding sections. The rotational treatment leads in the first instance to a mass of $1 \cdot 8 \times 10^9$ solar masses, and $2 \cdot 2 \times 10^9$ when allowance is made for the random motions. If the estimated extrapolation correction is applied to the latter value, a mass of

Table 3

ESTIMATES OF THE MASS OF THE LARGE CLOUD

(in $10^9$ $\odot$, for $i = 65°$)

| Method | Inner Parts ($r < 3°$) | Main Body (models) | Whole Cloud (extrapolated) |
|---|---|---|---|
| Rotation only . . . . | $\geqslant 1 \cdot 7$ | $\geqslant 1 \cdot 8$ | $> 2 \cdot 5$ |
| Random motions only | $\geqslant 1 \cdot 4$ | | |
| Rotation + random . . | | $> 2 \cdot 2$ | $3 \cdot 0$ ($\leqslant 4 \cdot 2$) |
| Lohmann . . . . | | | (3–4) |
| Schwarzschild . . . . | | | (2–3) |
| Best estimate . . . . | | | $3 \cdot 0$ |

$3 \cdot 0 \times 10^9$ is obtained. This is the best estimate of the mass of the Large Cloud which can be made on the present evidence, and subject to the various assumptions which have been introduced. As pointed out in Section V (*a*), however, the extrapolation correction is very uncertain, although it is clear that such a correction must be applied. Other treatments of the data which make some allowance for the outlying mass (Section V (*b*)) give values of the same order, $3 \times 10^9$ solar masses.

The greatest uncertainty in the data lies in the tilt angle of the Large Cloud ; the whole analysis has been carried out with a tilt angle of 65°, the provisional value derived from the ellipticity of the outer optical isophotes (de Vaucouleurs 1955*a*), but the radio data suggest that the tilt may well be greater than this. Since the square of the secant varies rapidly in this range of angle, the consequent uncertainty in the mass is very large. For tilt angles of 70 and 75°, the corresponding values for the mass would be $4 \cdot 3 \times 10^9$ and $7 \cdot 1 \times 10^9$ solar masses respectively.

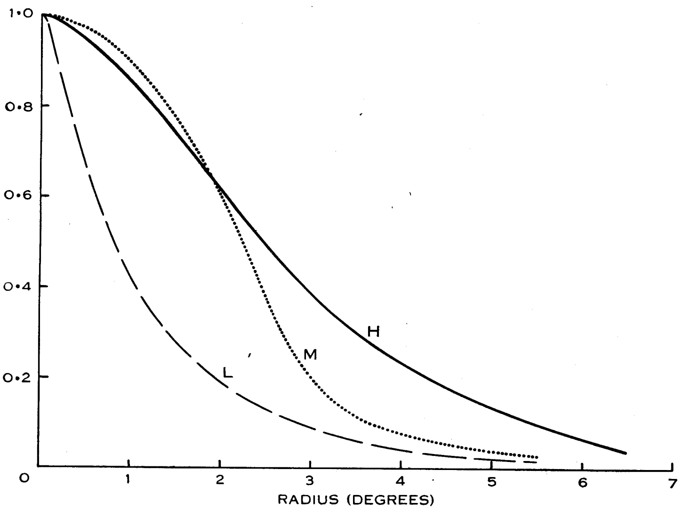


Fig. 7.—Comparison between the approximate mass distribution in the Large Cloud, derived from the present treatment, and the observed distributions for hydrogen and red light.

The mass distribution cannot be determined in the outer parts, but a distribution curve which is consistent with the above estimate for the extrapolated mass is shown in normalized form in Figure 7, together with the hydrogen and luminosity distributions.

### (*b*) *Small Cloud*

We now discuss briefly the Small Cloud. The rotation curve in this case does not extend sufficiently far from the centre for an independent treatment to be possible, but an approximate mass can be obtained from the Large Cloud discussion if we assume that both Clouds are basically built on the same model. The optical observations of spiral structure give some support to this assumption but the presence of the asymmetrical prominence and the displacement of the centre of rotation from the optical centre detract from its validity.

For a given model, $\mathbf{M} \propto (v_m)^2 \times r_m$, where $v_m$ is the maximum velocity, reached at a distance $r_m$ from the centre. From the adopted rotation curve for the Small Cloud (Fig. 2), $v_m = 37$ km/sec at $r_m = 3 \cdot 3°$, allowing for a tilt angle of 30°. If similar models are taken, the mass is therefore 45 per cent. of that for the Large Cloud, when the Large Cloud tilt is taken as 65°. This leads to a value for the mass of the Small Cloud of $1\cdot3\times10^9$ solar masses. Although the value was arrived at through a comparison with the Large Cloud, it is not itself affected by the uncertainty in the tilt angle for the Large Cloud, and the effect of the uncertainty in the tilt of the Small Cloud amounts to only $\pm10$ per cent.

A ratio of 45 per cent. between the masses of the two Clouds appears to be somewhat high, since the total luminosity of the Small Cloud is only a fifth of that of the Large Cloud, according to the most recent determinations ; also, if the Small Cloud prominence is due to tidal perturbation by the Large Cloud, the mass of the Small Cloud must be considerably lower than that of the Large Cloud. The structures of the two Clouds may well be sufficiently different for the mass of the Small Cloud to have been overestimated by the above procedure. Also, the discrepancy would be reduced by using a larger tilt angle for the Large Cloud, as shown in Table 4, which also gives the results of comparisons between the two Clouds from other methods of observation.

Table 4

COMPARISONS BETWEEN THE SMALL AND LARGE CLOUDS

| Quantity | SMC/LMC |
|---|---|
| Mass (assumed $i$ for LMC$=65°$) .. .. | 0·45 |
| ( ,, ,, ,, $=70°$) .. .. | 0·31 |
| ( ,, ,, ,, $=75°$) .. .. | 0·19 |
| Neutral hydrogen (including SMC prominence) | 0·71 |
| ,, ,, (excluding SMC prominence) | 0·33 |
| Luminosity .. .. .. .. .. .. | 0·19 |

The Small Cloud presents a special problem through its large prominence, which is of much greater relative importance in the radio than in the optical view. In fact, the greater part of the gas appears to be outside the main body of the Cloud as defined by the distribution of stars.

An attempt has been made to obtain separate values for the mass of hydrogen in the prominence and in the main stellar system. The results are necessarily rough since the two parts are not clearly distinguishable. In particular, optical studies indicate that the prominence is not in the equatorial plane of the flattened stellar system (de Vaucouleurs 1955*b*).

For this purpose, the main stellar system was taken to be limited by an ellipse (dashed contour on Plate 2 of paper II) centred on the optical centre, $C$, with major axis 9° and minor axis 4°, the approximate " overall " dimensions indicated by the star counts at $m=16$ (de Vaucouleurs 1955*b*). Integration over the hydrogen contours gave $0\cdot2\times10^9$ solar masses for the mass of hydrogen in the main stellar system, and $0\cdot22\times10^9$ in the prominence. Star counts indicate that 70 per cent. of the stars brighter than $m=14\cdot3$ are in the main system, and only 30 per cent. in the prominence. Counts to $m=16$ do not yet allow a similar comparison, but it is clear that the percentage of the total population located within the main stellar system is even higher. Thus the ratio of gas to stars is very different in the two parts of the Small Cloud. The great difference in the distributions of gas and stars in the Small Cloud is evident from Figure 5 ; in the Large Cloud, the distributions are much more similar.

## VIII. Orbital Motion of the Clouds

It was shown in paper II that the differential radial velocity of the Clouds is about 50–60 km/sec, the exact value depending on the galactic rotation velocity at the Sun. If the two Clouds were moving as an isolated system, their combined mass $\mathbf{M}$ would be related to the relative orbital velocity $v$ by the expression

$$\mathbf{M}=\frac{v^2}{G(2/r-1/a)},$$

where $G$ is the gravitational constant, $r$ the distance between the two Clouds ($\geqslant 16$ kpc, the projected distance), and $a$ the semi-major axis of the relative orbit. The minimum value of $\mathbf{M}$ for a closed orbit would correspond to a parabolic orbit ($a=\infty$), seen edgewise, and with the system at periastron. The observed value of differential radial velocity could represent motion in a closed orbit only if the sum of the masses of the Clouds exceeded $5\times10^9$ solar masses. For a circular orbit, the combined mass would be greater than $10\times10^9$ solar masses. The sum of the masses derived above is less than $5\times10^9$ solar masses, but would exceed this value if the tilt angle of the Large Cloud is increased from 65° to 70° or greater. The total mass is, however, hardly likely to approach $10\times10^9$. We must therefore conclude that the Clouds are moving in a hyperbolic or nearly parabolic orbit relative to one another.

The Clouds cannot in fact be considered independently of the Galaxy, since the galactic field at the Clouds is of the same order of magnitude as that of either Cloud at the other. The group can only be treated as a three-body system. The evidence that the Clouds are probably moving in an open orbit carries a broader inference in suggesting that caution must be adopted in any study in which the combined masses of double galaxies are derived from their relative motions, particularly when the galaxies are located in a cluster.

## IX. Comparisons with Other Galaxies

As noted in Section I, only two other external galaxies, M31 and M33, both regular spirals, have been previously investigated for rotation and mass distribution in anything like the detail given by the radio observations in the Magellanic Clouds. To these may be added the still fragmentary information available on our own Galaxy. It is therefore important to compare the new results with these earlier data to see whether they throw additional light on the dynamics and evolution of stellar systems.

The theoretical difficulties discussed in earlier sections all apply, to a greater or less extent, to the regular spirals considered by other authors. Values derived on a similar basis must therefore be selected in making any comparison between the mass of either Cloud and the mass of some other galaxy. No correction for " incompleteness " or for random motions has been made in the treatments used for other galaxies ; the latter correction will, however, be quite small for the regular spirals, so need not be considered here.

An illustration of the effect of the various treatments on mass estimates may be obtained from a comparison of some values computed for M33 from the same set of optical data (Mayall and Aller 1942). The data are collected in Table 5 ; Schwarzschild's and Lohmann's values are as published ; Wyse and Mayall's values from thin disk models are corrected for the doubling of the distance scale. The value corresponding to the present treatment of the Large Cloud was obtained as follows. Since spheroid models give values 10–20 per cent. higher than thin disk models (cf. Section IV), we may take $4 \cdot 0 \times 10^9$ as our corresponding estimate of the mass within $r = 18'$. From the observed luminosity distribution in M33 (Patterson 1940), we compute that about 75 per cent. of its total luminosity is comprised within this same radius ; hence the total mass extrapolated as explained in Section V (*a*) is about $5 \cdot 5 \times 10^9$.

TABLE 5

MASS ESTIMATES FOR M33

| | |
|---|---|
| Wyse and Mayall (1942) (corrected) $r < 18'$ .. | $3 \cdot 5 \times 10^9$ |
| $r < 30'$ .. | $4 \cdot 0 \times 10^9$ |
| Present treatment .. .. .. .. | $5 \cdot 5 \times 10^9$ |
| Schwarzschild (1954) .. .. .. .. | $5 \times 10^9$ |
| Lohmann (1954) .. .. .. .. .. | $10 \times 10^9$ |

Various comparisons between the Clouds and other galaxies are summarized in Table 6.

Consider first the periods of rotation. Assuming circular motions the period of rotation at any point is

$$P = 2\pi r / v_r = 6 \cdot 16 \times 10^9 / \omega \quad \text{(years)}$$

if the angular velocity $\omega$ is expressed in km sec$^{-1}$ kpc$^{-1}$. The values of $P$ listed in Table 6 correspond to the maximum of the velocity curve, except for the last two cases. The periods found for the Clouds, 350 and ~150 million years, are as long or longer than those in the corresponding parts of regular spirals and elliptical galaxies. According to some provisional data of Mayall (1948) the periods of rotation in the nuclei and " main bodies " of regular galaxies (corrected for the revision of the distance scale) are as follows :

| | |
|---|---|
| E7, S0 .. .. | 5–10 million years |
| Sa .. .. | 10–20 million years |
| Sb, SBb .. | 10–40 million years |
| Sc .. .. | 20–80 million years. |

There is, however, considerable scatter and some late Sc and SBc galaxies indicate periods well in excess of 100 million years.

With the small sample of galaxies for which reliable mass estimates are available at present, the probable correlations between period of rotation, size,

TABLE 6

ELEMENTS OF SOME GALAXIES

| Object | Type | Diameter ("main body") (kpc) | Absolute Magnitude | Luminosity **L** ($10^9$ ⊙) | Period of Rotation[1] $P$ ($10^6$ years) | Mass **M** ("main body") ($10^9$ ⊙) | **M**/**L** | Projected Central Density (⊙/pc$^2$) |
|---|---|---|---|---|---|---|---|---|
| SMC[2] . . . . | Mpec | 4 | $-16\frac{1}{2}$ | 0·5 | 350 | ~1 | ~2 | ? |
| LMC[2] . . . . | M | 8 | $-18\frac{1}{2}$ | 2·5 | 200 to 100[2] | 2·5 to 5[2] | 1 to 2[2] | 200 to 400[2] |
| M33 . . . . | Sc | 8 | −18 | 2·0 | 100 | 5 | 2 to 3 | 500 ? |
| Galaxy . . . . | Sbc ? | 20 | ? | ? | 185[3] | 100 | (4)[4] | ? |
| M31 . . . . | Sb | 20 | −20 | 12 | 200 | 150 | 12 | 2000 ? |
| NGC 4594[5] . . | Sa | 5: | −19: | 5: | 40:[6] | > 60: | > 12: | ? |
| NGC 3115[5] . . | E7 | 5: | −18: | 2: | 10:[6] | > 40: | > 20[5] | > 4000 ? |

References: Oort (1940, Table 4); Mayall and Aller (1942, pp. 20–21); de Vaucouleurs (1953, Table IV); Lohmann (1954, Table 3); Schwarzschild (1954, Table VII).

[1] At $r = r_m$.

[2] For range of $i$ from 65 to 75°.

[3] From 21 cm rotation curve (Kwee, Muller, and Westerhout 1954).

[4] Near the Sun.

[5] The masses of NGC 4594 and 3115, being "rotational" masses, are lower limits only. Their distances, which are highly uncertain, have been taken here as 4 Mpc in the revised scale. The value **M**/**L**=100, obtained by Schwarzschild (1954) for NGC 3115, is for 2 Mpc, corresponding to 50 at 4 Mpc.

[6] In these cases the periods of rotation refer to the inner regions showing solid body rotation. (Detailed velocity curves have not been published.)

and mass cannot be eliminated, but existing information is consistent with the assumption that spin increases along the sequence (M)→(S)→(E); this may therefore represent an evolutionary sequence as suggested by Shapley (1950). Further, the situation in Magellanic-type systems is compatible with some views put forward by Hoyle (1951) and von Weizsäcker (1951) on the origin of rotation in galaxies: this is indicated by the slow rotation and large random motions which result in irregularity of the spiral pattern, together with the frequent occurrence of such systems in close pairs showing clear signs of strong interaction (de Vaucouleurs 1954). If this is substantially correct, then the Magellanic Clouds appear to be galaxies of recent formation and still in the process of condensation and organization; thus the dynamical evidence supports the interpretation suggested by the detailed study of the physical content of the Clouds, especially of the Large Cloud, which shows large numbers of super-luminous and presumably "young" stars of short lifetime.

Additional evidence in support of the interpretation that the Clouds are young systems is found in the low values obtained for the mass-luminosity ratio and central densities as compared with the regular spirals or the solar neighbourhood (Table 6). In combination with other optical data this suggests that stars fainter than the Sun, for which the mass-luminosity ratio exceeds unity, are relatively rare in the Clouds. According to current ideas (Strömgren 1952), such low luminosity stars have very long lifetimes and are not likely to condense out of interstellar matter under present conditions; the near absence of such "old" stars in the Clouds adds colour to the hypothesis that such systems are of comparatively recent formation.

The 21 cm results have provided estimates of both the mass of neutral hydrogen and the total mass. For the Large Cloud, assuming a tilt angle $i=65°$, the ratio of these quantities is 23 per cent. for the extrapolated values, or 19 per cent. when the more reliable results for $r<3°$ are considered. (The ratio will be smaller for $i>65°$.) These values may be compared with the corresponding figures for the relative gas density in the solar neighbourhood, for which two recent estimates are 15 per cent. (Oort 1952) and 20 per cent. (Bok 1954).

## X. Conclusion

The survey on which this study was based was of a preliminary nature, but, even at the present stage, the velocity curve for the Large Cloud is more extensive than, and probably as accurate as, that available for any other external galaxy. A reasonable estimate of the total mass of the Large Cloud has been obtained, but the evidence on mass distribution is weak, as is shown by the discussion on extrapolation to infinity of the results for the inner regions. Also, little is yet known about the detailed internal motions of the Clouds.

The shortcomings in each case are partly observational and partly theoretical. The observational requirement is for greater resolution, both in angle and velocity, in the main body of each Cloud, and for higher sensitivity in the outer regions; on the theoretical side, a better treatment is needed for a thick system containing both rotational and random motions.

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