LUNAR VARIATIONS IN THE IONOSPHERE

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Summary

The global pattern of the observed lunar variations of the height and electron density of the F_2 region is briefly summarized, new analyses being presented for Canberra (f_0F_2), Brisbane ($h'F_2$), and Washington (h' and f_0F_2). It is concluded that the height variation has an amplitude of from 1 to 3 km and a phase giving maximum height at 06 lunar hours at moderate latitudes and at 09 lunar hours at the geomagnetic equator. The critical frequency variation has an amplitude of from 2 to 4 per cent., maximum critical frequency occurring at about 09 lunar hours at moderate geomagnetic latitudes and 04 lunar hours at equatorial geomagnetic latitudes.

A theory of lunar ionospheric variations is then presented. The current system which Chapman has shown could be responsible for the observed lunar geomagnetic field variations is taken as a starting point: it is considered that this current must flow at a height of about 100 km. The tidal winds needed to drive the current, the potential distribution which will be set up in the dynamo layer, and the resulting periodic vertical drifts of ionization in the higher layers are calculated. It is shown that the divergence of drift velocity is too small to account for the lunar variations in f_0F_2. These are calculated taking into account the probable height variations of recombination coefficient and ionization production rate.

The conclusions thus reached are in good agreement with the observed variations in the F_2. It is concluded that the amplitude of the lunar tidal wind near the E layer is about 45 times greater than that observed on the ground.

I. Introduction

In recent years periodic atmospheric phenomena have been intensively studied. These phenomena include variations, in both solar and lunar time, of barometric pressure, of geomagnetic field, and of the maximum electron density in the ionosphere; and the horizontal and vertical movements of electron density peaks. The lunar variations can be safely ascribed to a single cause, the Moon's gravitational field, and in this paper an attempt will be made to relate ionospheric phenomena of lunar periodicity to this field.

In 1882 Balfour Stewart suggested that the daily magnetic variations were due to electric currents generated in the upper atmosphere by the daily convective movement of ionized air across the Earth's magnetic field. Subsequently Schuster (1908) and Chapman (1919) developed this "dynamo" theory quantitatively. Two unknowns were in the theory: the tidal velocity and the conductivity in the upper atmosphere.

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From the study of barometric pressure oscillations, the lunar atmospheric tidal movements on the ground are fairly well understood, but there is no direct information about the corresponding movements in the upper atmosphere. The early workers, Laplace and Lamb, considering perfect isothermal and adiabatic atmospheres respectively, concluded that the tidal velocity at higher levels would be the same as at the ground, which is much too small for the purposes of the dynamo theory ; however, Pekeris (1937) showed that for other cases it was usual for the amplitude to increase with height. From a study of the vertical oscillation of the E layer, Martyn (1947) concluded that the amplification was probably about 200. A new estimate of the tidal velocity in the dynamo layer will be made in this paper.

Pedersen (1927) pointed out that the Earth's magnetic field impedes the movement of ions across it, and for this reason it has been difficult to find enough conductivity to account for the observed magnetic variations. However, Martyn (1948*b*) suggested that because of Hall current and consequent polarization the conductivity would not be as low as Pedersen had suggested, and later Baker and Martyn (1952, 1953) and, independently, Hirono (1950) and Fejer (1953) studied the problem of the conductivity of a thin layer of ionization in a magnetic field in detail and showed that, because of the form of the global wind pattern and the consequent inhibition of Hall current, vertical and horizontal polarization fields are set up, which increase the effective conductivity to about six times the Pedersen value over most of the Earth and to even higher values in a narrow strip over the geomagnetic equator. This removed the last objection to the dynamo theory.

The oscillations of height and ion density of the ionospheric layers remained to be explained. Martyn (1947) pointed out that these could hardly be attributed to the simple rising and falling of isobaric surfaces, and he suggested that they were due to electrodynamical interaction of the dynamo current with the geomagnetic field.

The revised dynamo theory of Baker and Martyn enables us to say with some certainty that the day-time dynamo layer is near the E region and to calculate the global distribution of electrical potential, which differs radically from that given by the Schuster theory. We shall here bring forward experimental evidence that notable lunar variations in ionospheric layer heights and electron densities occur chiefly in the day-time, when the E layer is present. These facts permit the presentation of a consistent explanation of lunar variations.

We first describe the observed ionospheric lunar variations. Then, taking the current system which Chapman has shown to be consistent with the observed lunar geomagnetic field variations, we calculate the necessary tidal winds, the potential field set up in the dynamo layer, and the periodic movements of ionization and changes in ion density which this field will produce in higher layers. The conclusions thus reached are reconciled with all available measurements of movements and electron density changes in the F_2 region of the ionosphere.

II. The Observed Lunar Variations of Electron Density and Height of the F_2 Region

Martyn (1947, 1948*a*), Appleton and Beynon (1948), Burkard (1948), and Matsushita (1949) have shown that the main lunar periodicity of F_2 electron density and height is semi-diurnal. Maximum heights generally occur at 06 lunar hours at moderate latitudes and at 08 lunar hours above the geomagnetic equator (Table 1), the only serious exceptions to this rule being Ottawa and Watheroo, where the tidal amplitudes are small and the phase determinations therefore not so reliable. Maximum electron densities occur 3–4 hr after maximum height at moderate latitudes, and about 4 hr before maximum height at Huancayo.

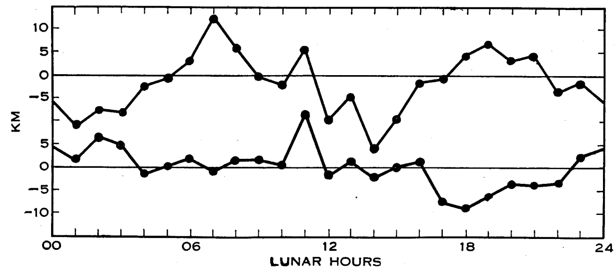

Fig. 1.—The lunar variation of $h^{\max.}F_2$ at 13 (top) and 01 (bottom) solar hours. Huancayo 1942–43–44.

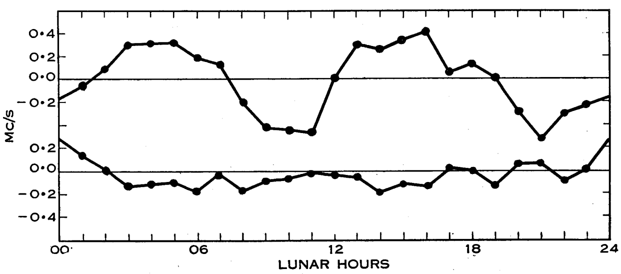

Fig. 2.—The lunar variation of f_0F_2 at 11 (top) and 23 (bottom) solar hours. Huancayo 1942–43–44.

Analyses made by the Radio Research Board, which will be published in detail later, show that large lunar semi-diurnal variations of $h^{\max.}$ and f_0F_2 occur above Huancayo only during the day (Figs. 1 and 2), and above Canberra only during the late afternoon and evening and for a very brief period at dawn.

III. The Theory of Lunar Ionospheric Variations

(*a*) *The Lunar Dynamo Current*

Chapman and Bartels (1940) have calculated the form and magnitude of the currents which must flow in the ionosphere if the observed lunar geomagnetic variations are to be explained in this manner. Figure 3 shows the current at new moon during the equinox : a similar current system centred near solar noon

TABLE 1

HARMONIC COEFFICIENTS, P_2 AND t_2, OF THE SEMI-DIURNAL LUNAR VARIATIONS OF F_2 REGION DATA

n is the number of months' data examined. Martyn's computations have been re-checked using slightly more accurate procedures, and differences, mostly trivial, will be found between the old values and those given here. Martyn's analyses for f_0F_2 Canberra, $h'F_2$ Brisbane, and f_0 and $h'F_2$ Washington have been extended to cover twice the previous number of months' data

Station	Geomagnetic Latitude	Investigator	$h'F_2$			$h^{max.}F_2$			f_0F_2		
			P_2 (km)	t_2	n	P_2 (km)	t_2	n	P_2 (Mc/s)	t_2	n
Huancayo	− 0·6	Martyn	2·3	9·2	36	5·2	8·4	30	0·14	4·3	48
Cape York	−20·7	Martyn	2·0	6·7	11	2·2	6·3	6	0·12	9·7	10
Kihei	+20·9	Martyn	2·6	7·9	15				0·17	11·4	14
Tokyo	+25·5	Matsushita	3·0	6·6	13						
Durbanville	−32·6	Martyn							0·05	11·6	8
Brisbane	−35·7	Martyn	1·3	6·4	82	2·0	6·0	33	0·08	9·3	31
Watheroo	−41·7	Martyn	0·3	5·9	85	0·6	8·0	73	0·03	9·9	112
Canberra	−44·0	Martyn	1·6	5·6	48	1·7	5·6	36	0·06	9·0	72
Christchurch	−48·0	Martyn	0·8	5·2	28				0·05	8·5	28
Washington	+50·3	Martyn	1·4	5·8	77				0·04	10·2	77
Washington	+50·3	Burkard							0·09	11·0	12
Hobart	−51·6	Martyn	2·0	4·6	10	1·1	5·6	9	0·04	9·3	10
Slough	+54·3	Appleton & Beynon				2·0	6·0		0·05	11·0	12
Ottawa	+56·9	Martyn	0·6	3·0	24				0·01	11·2	30
Burghead	+59·4	Martyn							0·02	8·3	48

in each case flows at first quarter, full moon, and last quarter. At intermediate lunar ages, there is slight distortion of this picture, the day-night boundary cutting across the current loops. As a result of more recent geomagnetic analyses, it is known that an anomalously large current of about twice the intensity shown in Figure 3 flows above the geomagnetic equator.

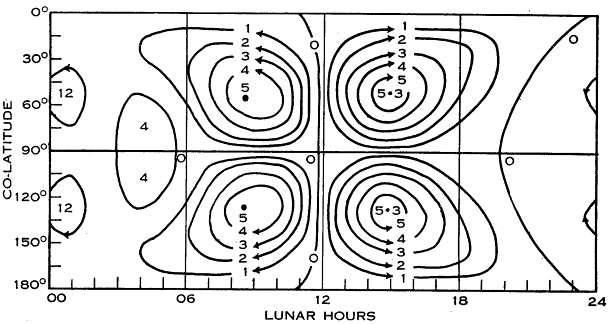

Fig. 3.—The system of ionospheric currents which could cause the lunar geomagnetic variations observed at new moon during the equinoxes. Contours 1000 A apart (after Bartels).

If we ignore the difference between the day and night intensities and the equatorial anomaly, the current system of Figure 3 is well described by the function :

$$R = 2\cdot 60 J \sin^2\theta \cos\theta \sin 2\varphi \text{ e.m.u.}, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{1})$$

$$j_\theta = \frac{\partial R}{r\sin\theta\partial\varphi}, \quad j_\varphi = -\frac{\partial R}{r\partial\theta},$$

where θ is the co-latitude.

φ is the longitude measured from the Moon,
r is the radius of the Earth,
j_θ is the southward current density,
j_φ is the eastward current density.

This function represents a current system with vortex centres at $\varphi = 45° \pm m90°$ and $\theta = 54\cdot 7°$ and $125\cdot 3°$, and with a circulation per vortex of J. A clockwise vortex with a day-time circulation of 500 e.m.u. flows in the first octant (Fig. 3) so that J has a day-time value of -500. We shall take the equatorial eastward current j_e to be double that given by (**1**), so that

$$j_e = (5\cdot 20/r) J \sin 2\varphi. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{2})$$

The E region must carry the substantial part of this current during the day and early evening, and, as detectable lunar variations in the F_2 are almost entirely confined to these hours, we shall not concern ourselves with the problem of the height distribution of the relatively small night-time dynamo currents.

(b) Lunar Tidal Winds in the Dynamo Layer

Barometric observations show a lunar pressure variation (p) on the ground of the form

$$p=\sin^3\theta\cos 2\varphi. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Fejer (1953) was the first to point out that the Earth's rotation enhances the electromotive effect of the tidal winds, as Coriolis deflection induces east-west air motion, i.e. air motion normal to the geomagnetic field, at higher latitudes. Taking account of the Earth's rotation, then, southward and eastward wind components

$$\left.\begin{aligned} v_\theta &= -\cos\theta\sin 2\varphi, \\ v_\varphi &= -(1-0\cdot59\sin^2\theta)\cos 2\varphi \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (4)$$

may be expected (Gold 1910). We are free to choose a suitable phase and amplitude for these winds in the dynamo layer.

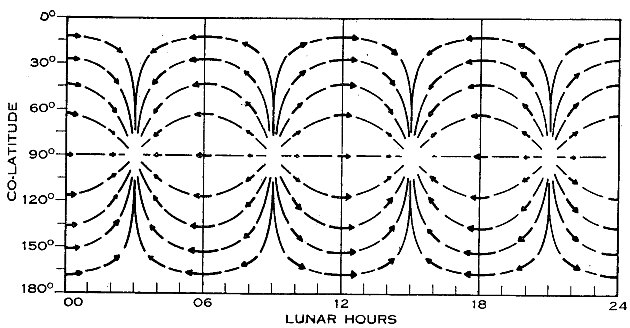

Fig. 4.—The global distribution of the lunar gravitational tidal winds in the dynamo region. An arrow one division long represents a velocity of 100 cm/sec.

Although the conduction of the ionosphere depends on the formation of Hall polarization, Baker and Martyn have shown that the dynamo current at high latitudes is parallel to the dynamo electromotive force, and maximum eastward wind must therefore occur at the time of maximum poleward current, that is at 00 and 12 lunar hours. The phase of the winds in the E layer must therefore be opposite to that on the ground with components of the form

$$\left.\begin{aligned} v_\theta &= v_{\max.}\cos\theta\sin 2\varphi, \\ v_\varphi &= v_{\max.}(1-0\cdot59\sin^2\theta)\cos 2\varphi. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (5)$$

The wind velocities may be estimated by equating the dynamo electromotive force to the ohmic potential drop around a circuit which runs along the meridian $\varphi=0$ from the equator to the pole, back to the equator along the meridian $\varphi=\frac{1}{2}\pi$, and then along the equator to the starting point: this corresponds very closely to the circuit of maximum dynamo voltages and currents. The dynamo field E_θ along the meridians is

$$E_\theta=H_r v_\varphi, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where H_r, the vertical component of the geomagnetic field, equals $-0\cdot622 \cos\theta$. No dynamo field is generated along the equator and the total e.m.f. around the circuit is therefore

$$\begin{aligned} \text{e.m.f.} &= 2\int_0^{\frac{1}{2}\pi} -0\cdot622 \cos\theta\, v_{\max.} (1-0\cdot59 \sin^2\theta) r d\theta \\ &= 1\cdot00 r v_{\max.}. \qquad (7) \end{aligned}$$

We shall accept Martyn and Baker's value of $3\cdot5\times10^{-8}$ e.m.u. for the conductivity in the non-equatorial region so that the ohmic drop along the meridians is

$$\frac{2}{3\cdot5\times10^{-8}}\int_0^{\frac{1}{2}\pi} j_\theta r d\theta,$$

i.e. from (**1**), equal to $(1\cdot49\times10^8)J$. Along the equator the conductivity is $1\cdot64\times10^{-7}$, so that from (**2**) the potential drop is

$$\frac{5\cdot20J}{r(1\cdot64\times10^{-7})}\int_{\frac{1}{2}\pi}^{0} r \sin 2\varphi d\varphi = (1\cdot58\times10^7)J.$$

Hence the total potential drop around the circuit is

$$1\cdot65\times10^8 J. \qquad (8)$$

Equating (**7**) and (**8**), we find

$$v_{\max.} = 130 \text{ cm/sec.} \qquad (9)$$

This is about 45 times as great as the lunar tidal wind observed on the ground. The E region winds are shown in Figure 4.

(c) *Polarization and Electric Fields in the Ionosphere*

The E-layer dynamo current will induce polarization, the horizontal distribution of which can be most easily described by dividing it into two parts, an equatorial Schuster polarization and a Hall polarization at the centre of each current loop.

The intensity of this polarization is independent of the diurnal variations of conductivity, as the dynamo current is proportional to the ion density and the induced polarization is proportional to this current divided by the ion density. We may therefore use noon values of current and conductivity in our computations without loss of generality.

The latitude variation of conductivity arising from the variation of solar zenith angle and magnetic dip is important. Although a more precise investigation is by no means intractable, we shall adopt the simplified picture of Baker and Martyn (1953): a region lying within 7° of the geomagnetic equator is taken to have a height integrated direct east-west noon conductivity $\Sigma_y = 1\cdot64\times10^{-7}$ e.m.u. Above the rest of the Earth, a direct conductivity $\Sigma_1 = 6\cdot4\times10^{-9}$ e.m.u., a Hall conductivity $\Sigma_2 = +1\cdot36\times10^{-8}$ e.m.u. in the northern and $-1\cdot36\times10^{-8}$ e.m.u. in the southern hemisphere, and an effective conductivity $\Sigma_3=\Sigma_1+\Sigma_2^2/\Sigma_1=3\cdot5\times10^{-8}$ e.m.u. will be assumed. The meaning of these four conductivities has already been described by Baker and Martyn and will be clear from the context.

The dynamo electromotive forces (Fig. 5) do not form closed circuits but converge on, or diverge from, four points spaced around the equator so causing " Schuster " polarization, the geographical distribution of which may be described approximately by the expression

$$V_s=V_s^{\text{max.}}\sin^2\theta\cos2\varphi. \qquad (10)$$

It is this polarization which closes the current loops by producing flow along the equatorial parallels of latitude so that

$$j_e=E\varphi\Sigma_y, \qquad (11)$$

where $E\varphi$, the eastward field, equals

$$-\frac{\partial V_s}{r\sin\theta\partial\varphi}=2V_s^{\text{max.}}\frac{\sin\theta}{r}\sin2\varphi.$$

Therefore, from (2)

$$\frac{5\cdot20J}{r}=\frac{2V_s^{\text{max.}}\Sigma_y}{r}, \qquad (12)$$

i.e. $V_s^{\text{max.}}=-79\cdot3\times10^8$ e.m.u. and the extreme potentials due to horizontal polarization above the geomagnetic equator are ±80 V.

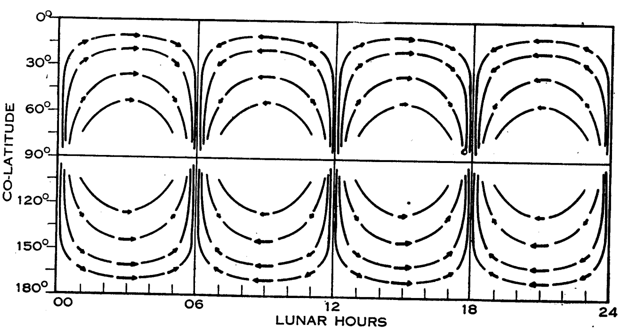

Fig. 5.—The global distribution of the electromotive forces in the dynamo region. An arrow one division long represents an electromotive field of 100 mV/km.

Because the ionosphere has a Hall conductivity, initially a component of current will flow at right angles to the dynamo electromotive forces, but as such currents all converge (or diverge) from the vortex centres, they will be rapidly countered by the accumulation of Hall polarization. As is explained in Baker and Martyn's papers, this Hall polarization then causes its own Hall current to flow around the current loop, augmenting the dynamo current and raising the effective conductivity from Σ_1 to $\Sigma_1+\Sigma_2^2/\Sigma_1$. Of the observed non-equatorial current, therefore, a part proportional to Σ_1 is due directly to the dynamo voltages, but the major part, proportional to Σ_2^2/Σ_1 (and thus a fraction $\Sigma_2^2/(\Sigma_1^2+\Sigma_2^2)=0\cdot 83$ of the whole) is caused indirectly by the Hall polarization. The Hall current function R_H is therefore from (**1**)

$$\left.\begin{aligned} R_H&=0\cdot 83R,\\ j_{H,\theta}&=0\cdot 83\frac{\partial R}{r\sin\theta\partial\varphi},\\ j_{H,\varphi}&=-0\cdot 83\frac{\partial R}{r\partial\theta}.\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (\mathbf{13})$$

This current is related to the Hall field E_H and polarization potential V_H by the relations

$$\left.\begin{aligned} j_{H,\theta}&=\Sigma_2 E_{H,\varphi} &&=-\Sigma_2\frac{\partial V_H}{r\sin\theta\partial\varphi},\\ j_{H,\varphi}&=-\Sigma_2 E_{H,\theta}&&=+\Sigma_2\frac{\partial V_H}{r\partial\theta}.\end{aligned}\right\} \quad \ldots\ldots (\mathbf{14})$$

Hence V_H varies in the same manner as R, that is,

$$V_H=2\cdot 60 V_H^{\text{max.}}\sin^2\theta\cos\theta\sin 2\varphi, \quad \ldots\ldots (\mathbf{15})$$

$$V_H^{\text{max.}}=\frac{-0\cdot 83J}{\Sigma_2}=\pm 3\cdot 05\times 10^{10} \quad \ldots\ldots\ldots\ldots (\mathbf{16})$$

in the northern and southern hemispheres respectively.

The complete expression for the global distribution of polarization potential V in the dynamo layer is therefore

$$V=[\pm 300\times 2\cdot 60\sin^2\theta\cos\theta\sin 2\varphi-80\sin^2\theta\cos 2\varphi]\times 10^8 \text{ e.m.u.} \quad \ldots\ldots\ldots\ldots\ldots\ldots (\mathbf{17})$$

This is shown in Figure 6.

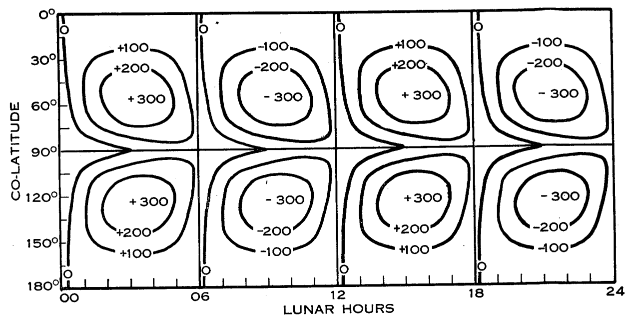

Fig. 6.—The global distribution of horizontal polarization in the dynamo region. Contours 100 V apart.

The polarization of the E region will be communicated to the higher levels along the highly conducting and thus equipotential geomagnetic field lines. Hence, generally,

$$V=[\pm 780\sin^2\theta_E\cos\theta_E\sin 2\varphi-80\sin^2\theta_E\cos 2\varphi]\times 10^8, \quad \ldots\ldots (\mathbf{18})$$

where θ_E is the co-latitude at which the region connects via the geomagnetic field lines with the dynamo layer. From the properties of the geomagnetic field

$$\sin^2 \theta_E=(r_E/r) \sin^2 \theta, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

so that the potential function for the complete ionosphere is

$$V=[\pm 780(r_e/r) \sin^2 \theta\sqrt{\{1-(r_E/r) \sin^2 \theta\}}\sin 2\varphi-80(r_E/r) \sin^2 \theta \cos 2\varphi] \times 10^8,$$
$$V\simeq[\pm 780 \sin^2 \theta\sqrt{\{1-(r_E/r) \sin^2 \theta\}}\sin 2\varphi-80 \sin^2 \theta \cos 2\varphi] \times 10^8. \quad \ldots \quad (20)$$

In the E region the total electric field will be the sum of the electromotive and electrostatic fields, but due to the greatly decreased pressure at higher levels, which will entail a much greater reduction of gravitational tidal power per unit volume than of viscous and electrodynamic damping, it is likely that in the F region gravitational tidal winds are negligible and that the electrostatic field is the only one which need be considered. There will be horizontal air movements in the F_2 due to the interaction of the electric currents produced by the electrostatic field and the geomagnetic field, but, as Baker and Martyn (1953) have shown, the e.m.f.'s developed by these will be back e.m.f.'s and will merely reduce the effective electric field. This effect will be discussed later.

(*d*) *Vertical Drift of Ionization in the* F$_2$ *Region*

Martyn has shown that, because of the geomagnetic field, an electric field in the ionosphere produces a drift of neutral ionization. This drift is the same as that given by Ampere's law; an eastward current being associated with an upward drift of ionization.

A discussion of ion drift in the E region is difficult because of the very rapid change with height, in this region, of the conductivity per ion pair; we shall confine ourselves to the very much simpler problem of ion drift in the F_2 region. Here there is no Hall current so that an eastward field will cause an eastward current. Martyn (1953) has derived the relation

$$v_r=\frac{E\varphi}{H} \cos \psi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

for the upward drift velocity (v_r), where $E\varphi$ is the eastward electric field, H the geomagnetic field intensity, and ψ the geomagnetic dip angle.

As has been pointed out (Baker and Martyn 1953), this relation will need modification if the air is set in motion by the drifting ionization. A rough measure of the time which air will take to acquire the velocity of ionization drifting through it is given by $t=\rho/(\sigma H^2)$, where ρ is the air density and σ the conductivity. If we evaluate this for the F_2 region we find that t is of the order of an hour; independent of the air pressure. Hence some air motion probably occurs. However, vertical air drift will be prevented by the Earth's gravitational field and even horizontal motion will be affected by a redistribution of air pressure because the horizontal component of the ion drift has appreciable divergence. Air motion is therefore probably a second order effect, but it should be remembered that it will reduce the vertical ion drift to some extent, particularly at high latitudes.

Going back to equation (**21**), we have from the properties of the geomagnetic field

$$\frac{\cos\psi}{H}=\frac{\sin\theta}{0\cdot 311(1+3\cos^2\theta)},$$

also,

$$E_\varphi=-\frac{\partial V}{r\sin\theta\partial\varphi},$$

so that

$$v_r=-\frac{\partial V}{0\cdot 311r(1+3\cos^2\theta)\partial\varphi}. \quad \text{..........} \quad (\mathbf{22})$$

From (**20**), then

$$v_r=80\cdot 3\frac{\sin^2\theta\sqrt{(94\cdot 5\cos^2\theta_E+1)}}{1+3\cos^2\theta}\left[\cos\left(2\varphi-\arctan\frac{-1}{-9\cdot 75\cos\theta_E}\right)\right]. \quad \text{....................} \quad (\mathbf{23})$$

Hence maximum upward drift velocity should occur at 06 lunar hours over most of the Earth. The theoretical variation of the amplitude of the drift

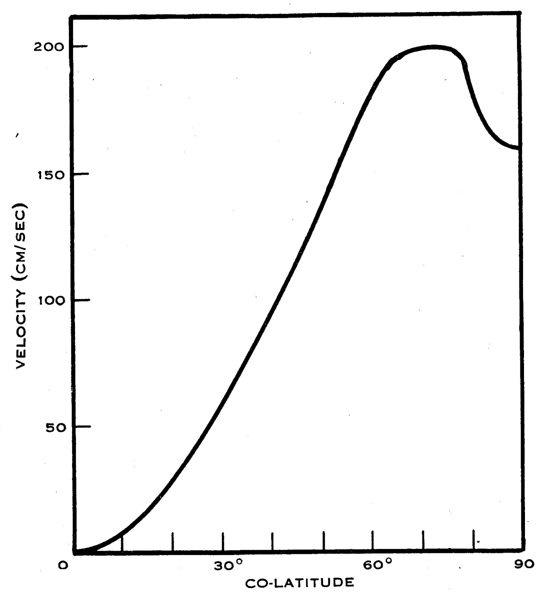

Fig. 7.—The variation with co-latitude of the vertical lunar drift velocity at 300 km.

with latitude is shown in Figure 7. Above the geomagnetic equator the time of greatest upward drift depends on the height, being 09 hours in the dynamo region (100 km), 07 hours at 300 km, and approaching 06 lunar hours at very great heights.

(*e*) *Lunar Periodicity in the Height of the* F *Region*

Integration of equation (**23**) shows that if the *F* region were translated up and down by the drift it would oscillate through about 12 km, maximum height being reached 3 hr after the time of greatest upward velocity. However, this large movement does not occur because the limited life of electrons (about an hour in the day-time F_2 region) prevents them from being shifted very far from the height at which they are produced. Even at night diffusion and the height gradient of the recombination coefficient tend to stabilize the height of the F_2 layer: if the layer is shifted upwards it tends to diffuse down again: if the layer is shifted downwards the accelerated decay of the lower edge will tend to restore it to its original height. Vertical drift, therefore, will perturb the height of an ionospheric layer rather than shift it bodily.

Suppose that we have in an ionospheric region, ionization, recombination, and a redistribution of electrons by a solar tide. The equation of continuity of the electron density is

$$\frac{\partial N}{\partial t}=I-\alpha N^2-v_r\frac{\partial N}{\partial h}-N\frac{\partial v_r}{\partial h}, \quad \ldots\ldots\ldots\ldots\ldots \quad (24)$$

where N is the electron density, t is time, I is the rate of ionization, α is the recombination coefficient, v_r is the velocity of vertical drift, and h is height. In time the electron density will approach a value N_e such that $\partial N_e/\partial t=0$. If the electron density is perturbed from this value by an amount n so that $N=N_e+n$, and the perturbing cause then removed, the equation of continuity becomes

$$\frac{\partial n}{\partial t}=-2\alpha N_e n-n\frac{\partial v_r}{\partial h}-v_r\frac{\partial n}{\partial h}.$$

The only term on the right of this equation which represents a tendency for the perturbation to decay is the first, $-2\alpha N_e n$, the other two terms merely show that the electron density perturbation is redistributed by the solar tide. The electron density perturbation n decays, then, according to the law

$$\partial n/\partial t=-2\alpha N_e n,$$

or

$$n=\exp\left(-2\alpha N_e t\right), \quad \ldots\ldots\ldots\ldots\ldots \quad (25)$$

so that $\tau=1/2\alpha N$ may be taken as the " relaxation time " of the ionosphere.

If this relaxation time is much shorter than 3 hr, we should expect an ionospheric layer to be displaced a distance $\Delta h=v_r\tau$ by a vertical drift v_r. Of all time intervals τ, that beginning $\frac{1}{2}\tau$ seconds before and ending $\frac{1}{2}\tau$ seconds after the time of maximum upward drift, covers the period of greatest upward drift, and the electrons should be at their greatest height $\frac{1}{2}\tau$ seconds after the time of maximum upward velocity. The following more rigorous treatment shows this picture to be correct.

If $\mathrm{d}N/\mathrm{d}h$ is the height gradient of the electron density, then in the presence of a uniform vertical drift v_r the equation of continuity becomes

$$\frac{\partial N}{\partial t}=I-\alpha N^2-v_r\frac{\partial N}{\partial h},$$

that is,

$$\frac{\partial N}{\partial t}=-2\alpha Nn-v_r\frac{\partial N}{\partial h}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (26)$$

When the relaxation time is short, the layer will reach a new equilibrium state so that $\partial N/\partial t=0$,
and hence

$$n=v_r\tau\frac{\partial N}{\partial h}. \qquad (27)$$

This relation implies that to the first order we can consider the ion density perturbation at a given height to be due to the layer being raised undistorted a distance Δh where

$$\Delta h=v_r\tau. \qquad (28)$$

(It should perhaps be pointed out that the electron density perturbation at a given height is deduced in (**27**) merely as a means of obtaining the height perturbation. It is not the observed quantity; the perturbation of the value of the maximum electron density. The latter will be calculated in the next section.)

It is difficult to obtain reliable estimates of τ in the F_2 but if we accept S. K. Mitra's (1952) values of recombination coefficient, we obtain a midday figure of 4000 sec. From this we should expect a day-time lunar height variation of about 6 km, maximum height occurring at 06 lunar hours at moderate latitudes and 07 lunar hours at the geomagnetic equator. Even larger amplitudes and later phases would be expected at night. However, we shall show that at F_2 heights, and particularly at the height of the night F_2 (300 km) diffusion becomes a process important enough to keep the effective relaxation time of the ionosphere short.

The greatest height at which reasonably reliable rocket measurements of temperature and mean free paths have been made is 200 km (Rocket Panel 1952), and on this evidence the coefficient of diffusion of air (K) at this level is roughly $7\cdot5\times10^8$ cm^2/sec. The diffusion coefficient will increase with height due to the increasing temperature and decreasing pressure. Pressure decreases exponentially with height and will have much the greater influence so we shall write

$$K=(7\cdot5\times10^8)e^{h/H}, \qquad (29)$$

where h/H is the reduced height above our 200 km datum level.

Huxley (1952) and others before him have shown that the electrons and positive ions in the ionosphere are held together by electrostatic attraction, the electron-positive ion gas diffusing at twice the rate appropriate to the positive ions alone. For our purposes, it will be sufficiently accurate to assume the coefficient of diffusion of the latter equal to that of neutral molecules, so that the coefficient of diffusion of the electron-positive ion gas (K_i) is

$$K_i=(1\cdot5\times10^9)e^{h/H}. \qquad (30)$$

The equation of continuity for the electron-positive ion gas is

$$\frac{\partial N}{\partial t}=\text{div}\left[K_i\frac{\partial N}{\partial h}-WN\right], \qquad (31)$$

where W is the drift velocity of the ions under gravity. It may be shown from kinetic theory that

$$W = -\frac{K_i mg}{kT} = -\frac{K_i}{H_i}, \quad \dots\dots\dots\dots\dots\dots \quad (32)$$

so that, as is well known, diffusion and gravitational drift are in equilibrium when

$$N = N_0 \exp(-h/H_i), \quad \dots\dots\dots\dots\dots\dots \quad (33)$$

where H_i, the scale height for the electron-positive ion gas, will be twice that for the neutral molecules (H).

It is clear, however, that in an ionospheric layer gravitational drift and diffusion will not generally balance one another; indeed, on the under side of the layer, the side observed, they both cause a downward flux of ionization. The total effective velocity of this flux (U) is

$$U = -K_i \frac{\partial N}{N \partial h} + W,$$

or

$$U = -K_i\left(\frac{\partial N}{N \partial h} + \frac{1}{H_i}\right). \quad \dots\dots\dots\dots\dots \quad (34)$$

We have not yet considered the effect of the geomagnetic field. Ions can gravitate and diffuse only in the direction of this field. If the dip is ψ the components of ion density gradient and gravitational field in this direction are the corresponding vertical components multiplied by sin ψ. To find the vertical component of velocity we must multiply by sin ψ once again, so that

$$\left.\begin{aligned} U &= -K_i\left(\frac{\partial N}{N \partial H} + \frac{1}{H_i}\right) \sin^2 \psi, \\ \text{that is,} \qquad U &= -2K\left(\frac{\partial N}{N \partial h} + \frac{1}{2H}\right) \sin^2 \psi. \end{aligned}\right\} \quad \dots\dots\dots \quad (35)$$

At night we may consider the F_2 layer to consist of a relatively thick bank of ionization in the upper region of rapid diffusion and low decay, with a lower boundary whose position is determined by the opposing processes of downward diffusion and gravitational drift on one hand, and the relatively rapid decay of the lower edge on the other. A reasonable value for the scale height of the F_2 region is 30 km. We shall take the semi-thickness of the F_2 to be 60 km so that the mean value of $\partial N/\partial h$ is $N_{\text{max.}}/(6 \times 10^6)$. The mean value of N may be taken as $\frac{1}{2}N_{\text{max.}}$, so that

$$U = -(1 \cdot 5 \times 10^9)\left[\frac{1}{3 \times 10^6} + \frac{1}{6 \times 10^6}\right] \sin^2 \psi e^{h/H},$$

that is,

$$U = -750 \sin^2 \psi e^{h/H}, \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (36)$$

and hence

$$\frac{dU}{dh} = -25 \times 10^{-5} \sin^2 \psi e^{h/H}. \quad \dots\dots\dots\dots\dots\dots\dots \quad (37)$$

An upward electrodynamic drift (v_r) will push the F_2 layer up until the downward diffusion velocity ($-U$) is increased by an amount equal to v_r. Hence, if in the relation

$$\mathrm{d}h = -\frac{10^5}{25} \operatorname{cosec}^2 \psi \mathrm{e}^{-h/H} \mathrm{d}U \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (38)$$

we put $-\mathrm{d}U$ equal to the upward drift v_r, $\mathrm{d}h$ is the displacement which this drift will cause, that is,

$$\Delta h = \frac{10^5}{25} \operatorname{cosec}^2 \psi \mathrm{e}^{-h/H} v_r, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (39)$$

or

$$\Delta h = \frac{10^5}{25} \frac{1+3\cos^2\theta}{4\cos^2\theta} \mathrm{e}^{-h/H} v_r. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (40)$$

The F_2 layer is about two scale heights above the 200 km level. We shall put therefore,

$$h/H = 2,$$

and

$$v_r = 155 \text{ cm/sec},$$

corresponding to the situation at co-latitude 55°.

This gives $\Delta h = 1 \cdot 26$ km.

This crude argument does not take account of the height gradient of recombination, but it indicates approximately how much diffusion will limit the amplitude of the lunar height variations.

It seems therefore that, at moderate latitudes, the amplitude of both the day and night lunar variation of $h'F_2$ should be about a kilometre, and that maximum h' and $h^{\max.}F_2$ should occur close to the time of maximum upward drift velocity ; 06 lunar hours. Table 1 shows that, at most moderate latitude stations, maximum F_2 heights occur close to 06 lunar hours, and that the amplitude of the variation is from 1 to 3 km, in fair agreement with these arguments.

Above the geomagnetic equator vertical diffusion is inhibited and this will lead to large F_2 height variations. The maximum drift velocity here is 160 cm/sec, so that at midday when the relaxation time is about 4000 sec, a maximum positive height variation of about 6 km should occur close to the time of maximum upward drift ; 07 lunar hours. Towards sunset, when the relaxation time is of the order of hours, even larger amplitudes and a phase of 10 lunar hours can be expected.

Martyn (1947) has shown that the observed variation at Huancayo does in fact exhibit these features.

(*f*) *Lunar Periodicity in the Electron Density of the* F_2 *Region*

Periodic drift may result in periodic variations in electron density in two ways :

(i) The drift velocity may have a gradient, resulting in the periodic concentration and rarefaction of the ionosphere.

(ii) The drift may shift the layer to a height where a different ionization and recombination rate prevails.

Let us first consider the effect of drift velocity gradients. The drift velocity $\mathbf{v}$ is given in magnitude and direction by the relation:

$$\mathbf{v}=\frac{1}{H^2}(\mathbf{E}\times\mathbf{H}). \qquad (41)$$

The divergence of the drift is therefore,

$$\begin{aligned}\text{div }\mathbf{v}&=\nabla\cdot\left[\frac{1}{H^2}(\mathbf{E}\times\mathbf{H})\right]\\ &=\frac{1}{H^2}\nabla\cdot(\mathbf{E}\times\mathbf{H})+\nabla\frac{1}{H^2}\cdot(\mathbf{E}\times\mathbf{H})\\ &=\frac{1}{H^2}\,(\text{curl }\mathbf{E}.\mathbf{H}-\mathbf{E}.\text{curl }\mathbf{H})+\left(\text{grad }\frac{1}{H^2}\right).(\mathbf{E}\times\mathbf{H}).\end{aligned}$$

Now the first term is zero as both $\mathbf{E}$ and $\mathbf{H}$ can be derived from scalar potentials so that

$$\text{div }\mathbf{v}=\left(\text{grad }\frac{1}{H^2}\right).(\mathbf{E}\times\mathbf{H}). \qquad (42)$$

As both H_φ and $\partial(1/H^2)/\partial\varphi$ are zero, this reduces to

$$\text{div }\mathbf{v}=E_\varphi\left[-\frac{\partial(1/H^2)}{\partial r}H_\theta+\frac{\partial(1/H^2)}{r\partial\theta}H_r\right]. \qquad (43)$$

The geomagnetic field is such that

$$H_r=\frac{2M\cos\theta}{r^3},$$

$$H_\theta=\frac{M\sin\theta}{r^3},$$

$$H^2=\frac{M^2}{r^6}\,(3\cos^2\theta+1).$$

where M is the Earth's magnetic moment. Hence,

$$\frac{\partial(1/H^2)}{\partial r}=\frac{6r^5}{M^2\,(3\cos^2\theta+1)},$$

and

$$\frac{\partial(1/H^2)}{r\partial\theta}=\frac{6r^5\cos\theta\sin\theta}{M^2\,(3\cos^2\theta+1)^2}.$$

Substituting these values in (**43**) we find that

$$\text{div }\mathbf{v}=-\frac{6r^2\sin\theta\,(\cos^2\theta+1)}{M\,(3\cos^2\theta+1)^2}E_\varphi. \qquad (44)$$

It is useful to obtain this expression in terms of the vertical component of drift (v_r). From (**21**)

$$v_r=-\frac{r^3\sin\theta}{M\,(3\cos^2\theta+1)}E_\varphi,$$

therefore

$$\operatorname{div} \mathbf{v} = +\frac{6\,(\cos^2\theta+1)}{r\,(3\cos^2\theta+1)}v_r.$$

$(1/N)(\partial N/\partial t) = -\operatorname{div} \mathbf{v}$ and, if the F_2 relaxation time is τ, the critical frequency variation will be

$$\frac{\Delta f_0}{f_0} = -\frac{3(1+\cos^2\theta)}{r(1+3\cos^2\theta)}v_r\tau = -\frac{3(1+\cos^2\theta)}{r(1+3\cos^2\theta)}\Delta h. \quad \ldots\ldots (\mathbf{45})$$

Thus the critical frequency variation due to drift divergence will be in phase opposition to the height variation.

Martyn (1955) has calculated the vertical gradient of the vertical component of the drift velocity ($\partial v_r/\partial r$) on the assumption that the geomagnetic field intensity is uniform, and he has equated this to $-\partial N/N\partial t$. Clearly, we should

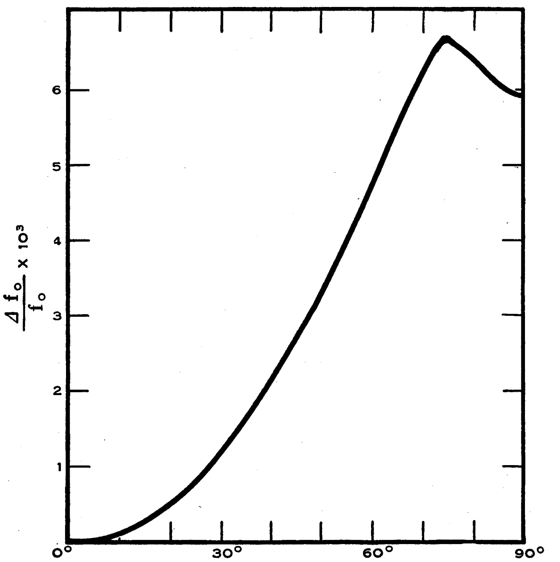

Fig. 8.—The variation with co-latitude of the relative critical frequency changes due to lunar drift velocity divergence at 300 km.

also consider the gradient of the eastward component of drift ($\partial v_\varphi/r \sin\theta\partial\varphi$). The new term is equal and opposite to the old (i.e. $\partial v_r/\partial r = -\partial v_\varphi/r \sin\theta\partial\varphi$) so that the total velocity divergence is zero. The derivation given here [(**41**)–(**45**)] shows that this is generally true: the drift divergence due to the electric field gradient is zero because $\operatorname{div}(\mathbf{E}\times\mathbf{H})\equiv 0$. The residual velocity divergence is due to the gradient of the geomagnetic field intensity.

The critical frequency variation given by equation (**45**), for a height of 300 km, τ equal to 4000 sec, and v_r as given in (**23**), is plotted against geomagnetic latitude in Figure 8. It is seen that a critical frequency variation of about $\frac{1}{2}$ per cent. may result near the geomagnetic equator, but that generally the effect is small.

It seems likely, therefore, that electron density variations at moderate latitudes are principally due to the effect of drift in an ionosphere with a height gradient of electron production and recombination.

Let I be the electron production rate,
α be the recombination coefficient,
N_m be the maximum electron density,
h_m be the height at which this occurs,
$N_{m,0}$ be the maximum electron density which would prevail if there were no drift, and
$h_{m,0}$ be the height at which this would occur, and
$N_{e,0}$ be the equilibrium electron density (i.e. $N_{e,0}=\sqrt{(I_0/\alpha)}$) at the height $h_{m,0}$.

Then let

$$n=N_m-N_{e,0},$$
$$h'=h_m-h_{m,0}.$$

h' is the tidal perturbation of h_m, but n, the perturbation of the maximum electron density, will be due partly to the drift and partly to the natural difference between the actual electron density and the equilibrium value at any time, that is,

$$n=N_m-N_{e,0}=(n_m-N_{m,0})+(N_{m,0}-N_{e,0}).$$

Now,

$$\frac{\mathrm{d}N_m}{\mathrm{d}t}=I-\alpha N_m^2, \qquad (46)$$

therefore

$$\frac{\mathrm{d}N_{e,0}}{\mathrm{d}t}+\frac{\mathrm{d}n}{\mathrm{d}t}=I_0+\frac{\partial I}{\partial h}h'-\left(\alpha_0+\frac{\partial \alpha}{\partial h}h'\right)(N_{e,0}+n)^2$$

and

$$\frac{\mathrm{d}n}{\mathrm{d}t}=-\alpha_0(N_m-N_{e,0})n+\frac{\partial}{\partial h}(I-\alpha N_m^2)h'.$$

This relation is precise but to the first order we may write

$$\frac{\mathrm{d}n}{\mathrm{d}t}=-2\alpha Nn+\frac{\partial}{\partial h}(I-\alpha N^2)h',$$

that is,

$$\frac{\mathrm{d}n}{\mathrm{d}t}=-An+Bh'. \qquad (47)$$

If h' varies sinusoidally semi-diurnally, that is,

$$h'=P_2\cos 2\omega t,$$

where ω is the angular velocity of the Earth, (47) has the solution

$$n=n_0\mathrm{e}^{-At}+\frac{BP_2}{\sqrt{(4\omega^2+A^2)}}\cos\left(2\omega t-\arctan\frac{2\omega}{A}\right). \qquad (48)$$

The first term in this expression shows that the electron density recovers exponentially, with a relaxation time $\tau = 1/2\alpha N$, from any initially imposed perturbation n_0.

The second term,

$$n = \frac{BP_2}{\sqrt{(4\omega^2 + 4\alpha^2 N^2)}} \cos\left(2\omega t - \arctan\frac{\omega}{\alpha N}\right). \quad \ldots\ldots \quad (\mathbf{49})$$

gives the tidal perturbation of the electron density.

Now, as in fact equation (**49**) implies, because of the intrinsic stability of an ionospheric layer in which ionization and recombination processes are rapid, we should expect appreciable lunar perturbation of the F_2 only during the late afternoon and evening when the relaxation time is reasonably long, yet the E region is still the most highly conducting region of the ionosphere. This is in fact the case at Canberra, as will be shown in a later paper, and it seems reasonable to assume that at all moderate latitude stations the lunar variation extracted from data averaged over all solar hours is dominated by the behaviour at these times.

Recombination is then the dominant process so that we may write

$$B = -\frac{\partial}{\partial h}(\alpha N^2). \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{50})$$

At the height of the F_2 region at moderate latitudes (250–300 km) the recombination coefficient is proportional to the air density, that is,

$$\alpha = \alpha_0 e^{-h/H},$$

where H is the scale height,

$$B = \frac{\alpha}{H} N^2 \qquad \left(\frac{\partial N}{\partial h} \text{ is zero at the density maximum}\right),$$

and

$$n = \frac{\alpha N^2 P_2}{H\sqrt{(4\omega^2 + 4\alpha^2 N^2)}} \cos\left(2\omega t - \arctan\frac{\omega}{\alpha N}\right). \quad \ldots\ldots \quad (\mathbf{51})$$

Now at night the relaxation time $(1/2\alpha N)$ of the ionosphere can be considered infinite, as its value at any time t after sunset is always greater than t. Hence the variation of critical frequency should lag that of height by 3 hr.

We may estimate the amplitude of the critical frequency (f_0) variation from (**51**)

$$\frac{n}{N} = \frac{\alpha N P_2}{2H\sqrt{(\omega^2 + \alpha^2 N^2)}},$$

therefore

$$\frac{\Delta f_0}{f_0} = \frac{\alpha N P_2}{2H\sqrt{(\omega^2 + \alpha^2 N^2)}},$$

where Δf_0 is the total variation of critical frequency, i.e. twice the amplitude of the variation.

Put $\alpha = 3 \times 10^{-10}$ cm^3/sec (Mitra),
$N = 2 \cdot 5 \times 10^5/\mathrm{cm}^3$,
$P_2 = 1 \cdot 5 \times 10^5$ cm,
$\omega = 2\pi/(24 \times 3600)$ sec^{-1},
$H = 3 \times 10^6$ cm.

Then

$$\Delta f_0/f_0 = 1 \cdot 8 \times 10^{-2}.$$

Martyn found a variation of about 2 per cent. at Canberra, with maximum critical frequency at 09 lunar hours, in good agreement with these results, and other non-equatorial stations show variations of similar amplitude and phase (Table 1).

The arguments above are not applicable to the F_2 region near the magnetic equator, for here, probably as a result of solar tides (Martyn 1955), the day-time F_2 region occurs at the great height of from 350 to 450 km (Maeda 1955). The recombination coefficient at this height is small and thus the electron relaxation time in the F_2 region near the magnetic equator is of the order of hours during the day when the conductivity of the E region is greatest. Vertical diffusion is inhibited by the geomagnetic field. This will lead to large day-time lunar variations of $h^{\max.}F_2$ and f_0F_2, and we should expect these day-time lunar variations to dominate the variations found in data meaned over all 24 solar hours. Figures 1 and 2 show that this is the case.

There is good evidence that at 300 km electron decay is primarily due to attachment and hence proportional to the pressure, but the rate of electron loss cannot fall indefinitely with height as attachment must ultimately cease to be the most important process of electron loss. At 400 km, then, the effective recombination coefficient has probably fallen to a constant low value.

On the other hand 400 km is well above the height of maximum ion production so that the ionization rate will be practically proportional to the pressure, that is,

$$I = I_0 e^{-h/H}. \qquad \ldots\ldots\ldots\ldots \quad (52)$$

For day-time equatorial conditions, therefore, $\partial(I - \alpha N^2)/\partial h$ becomes $-I/H$, $\partial N/\partial h$ being zero at the electron density maximum. Substituting this value in (**49**) we obtain

$$n = -\frac{IP_2}{H\sqrt{(4\omega^2 + 4\alpha^2 N^2)}} \cos\left(2\omega t - \arctan\frac{\omega}{\alpha N}\right),$$

hence

$$\frac{\Delta f_0}{f_0} = \frac{IP_2}{2HN\sqrt{(\omega^2 + \alpha^2 N^2)}}. \qquad \ldots\ldots\ldots\ldots \quad (53)$$

From this we may estimate the expected phase and amplitude of the Huancayo lunar critical frequency variation.

Put $N=10^6$ cm^{-2}, $\alpha=5\times10^{-11}$ cm^3/sec. Then the relaxation time $(1/2\alpha N)$ equals about $2\cdot8$ hr. This is much less than the time for which the sun shines (12 hr) so we should expect the electron density to be approximately equal to the equilibrium value and thus

$$I \simeq \alpha N^2 = 50 \text{ cm}^{-3}\text{sec}^{-1},$$

also, $P_2=5\times10^5$ cm, $H=3\times10^6$ cm, and $\omega=2\pi/(24/3600)$ sec^{-1}.

Substituting these values in (**53**) gives arctan $\omega/\alpha N$ equal to 55°, i.e. $1\cdot9$ hr, and $\Delta f_0/f_0$ equal to $4\cdot7\times10^{-2}$.

Thus, we should expect a lunar critical frequency variation of about 5 per cent. at Huancayo, and maximum critical frequency should occur about 4 hr before the time of maximum height. The phases and amplitudes observed by Martyn (Table 1) are in agreement with these conclusions.

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